3. z-transforms

(a) Delayed Impulse: $h[n] = \delta[n-N]$,

the z-transform is

$$H(z) = z^{-N}$$

(b) Box: $h_{box}[n] = N_{box}^{-1}(u[n] - \delta[n-N_{box}] * u[n])$

according to the given formula

$$g \equiv H(z=+1) \quad \text{and} \quad M_1 = \frac{d}{dz^{-1}} H(z) \Big|_{z=+1}$$

we have

$$H(z) = N_{box}^{-1} \cdot \frac{1 - z^{-N_{box}}}{1 - z^{-1}}$$

Let $z=+1$ and using L'Hopital's rule, we got

$$M_1 = (N_{box} - 1)/2, \quad g = 1$$

(c) Unit Step: $h[n] = u[n]$

$$H(z) = \frac{1}{1-z^{-1}}, \quad g = M_1 = \infty \text{ doesn't exist.}$$

(d) Ema: $h_{ema}[n] = (1-p)p^n u[n]$

$$H(z) = \frac{1-p}{1-pz^{-1}}, \quad g = 1, \quad M_1 = \frac{p}{1-p}$$

Decay rate in terms of M_1:

$$p = \frac{M_1}{M_1 + 1}$$

(e) Ema-poly: $h_{poly}[n] = (1-p)^2(n+1)p^n u[n]$

$$H(z) = \frac{(1-p)^2}{(1-pz^{-1})^2}, \quad g = 1, \quad M_1 = \frac{2p}{1-p}$$

p in terms of M_1:

$$p = \frac{M_1}{M_1 + 2}$$

(f) Lifted Macd-poly:

$$h_{lift}[n] = (2N_{eff}/3)^{-1} u[n] * (h_{ema}(N_{eff}/3)[n] - h_{poly}(N_{eff})[n])$$

Since we can compute $H(z)$ as of

$$H(z) = \frac{g^{-1}}{1-z^{-1}} \left[\frac{1-p_+}{1-p_+ z^{-1}} - \frac{(1-p_-)^2}{(1-p_- z^{-1})^2} \right]$$

therefore the gain is

$$g = \frac{2p_-}{1-p_-} - \frac{p_+}{1-p_+} = \frac{2N_{eff}}{3}$$

(g) Macd: $h_{macd}[n] = h_{ema}(N_{eff+})[n] - h_{ema}(N_{eff-})[n]$

$$H(z) = \frac{(1-p_-)}{1-p_- z^{-1}} - \frac{(1-p_+)}{1-p_+ z^{-1}}$$

then the gain is $g = 0$.

(h) Macd-M1: $h_{macd}^{(M_1)}[n] = \delta[n-N_{eff+}] - \delta[n-N_{eff-}]$.

$$H(z) = z^{-N_{eff+}} - z^{-N_{eff-}}$$

the gain is the same, $g = 0$.