Aalto University
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Gaussian Process for Indoor Positioning

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Contents

Introduction to Indoor Positioning Systems

Why do we need them?

- The Ubiquitous GPS fails indoors due to signal attenuation.
- This leads to uncertainty spanning several rooms or even floors.
- Used technologies for Assisted GPS, MEMS inertial sensors, LED's, WiFi, Visual light communication, Bluetooth Low Energy (BLE)/ Bluetooth Smart etc.
- Now solutions use **signals of opportunity** like magnetic field, pressure, light, sound intensity, GSM mobile signals, GNSS etc.

Now again, why do we need this?

Introduction to Indoor Positioning Systems

Business Value

- By 2018, this is going to be market worth 4 billion US dollars. [**?**]
- Interesting applications:
  - Asset/Personal Tracking
  - Product flow optimization
  - Product recommendation

Introduction to Indoor Positioning Systems

Signal Types

- The different signal types used in Indoor positioning are:
  - *Radio Frequency Signal*: Signal's ranging in the frequency 2 and 5 GHz. Mainly includes Wireless Fidelity (Wi-Fi).
  - *Light*: Includes visible and infrared.
  - *Sound*: Includes audible and ultrasonic.
  - *Magnetic Field*: Includes both natural and artificially produced magnetic fields.

Introduction to Indoor Positioning Systems

Treatment of Signal

- The different ways how different signal's are received and analyzed:
  - *Active*: Here the device generates signal's.
    For example, Asset Tracking.
  - *Passive*: Here the device receives the signal.

> The signals could also be categorized based on embedded information.

Introduction to Indoor Positioning Systems

Technologies

- The different technologies used in Indoor positioning are:
  - *Optical Technologies*: Infrared technology, Visible Light Communication (VLC)
  - *Sound Based Technology*: Ultrasound, Audible sound.
  - *Radio Frequency Technologies*: Wi-Fi, Bluetooth, ZigBee, RFID, Ultrawideband.
  - *Passive/ without Embedded Information Technologies*: Magnetic field, Inertial technology, passive sound-based technologies, passive visible light, computer vision.

We focus on fingerprinting based localization methods.

(because they work the best!)

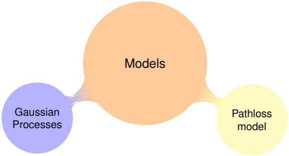
Types of Indoor Positioning systems

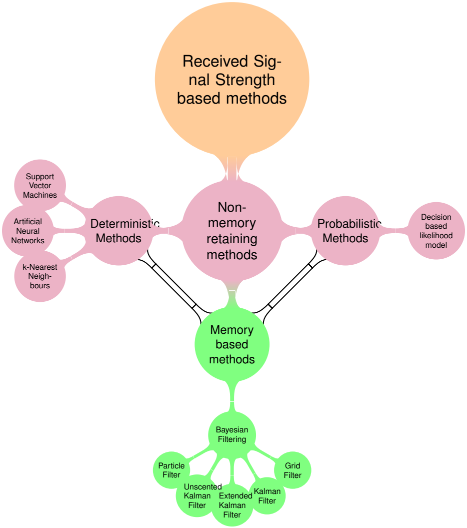
(Thesis work under progress)

- *Non-memory based methods*
  - These are deterministic methods and they try to relate the spatial with the signal space.
  - Methods include Triangulation, Trilateration, k-Nearest Neighbors, ANN's, etc.
  - These methods estimate the location based on Received Signal Strength Indicator (RSSI) values, and have no prior knowledge.
- *Memory based methods*
  - These methods just needs addition of a dynamic model or prior knowledge to the non-memory based methods. Hence, we would arrive at better estimate of the posterior.
  - The non-memory based methods are used as measurement models. We can make use of (Recursive Bayesian) Filtering methods.

Types of Indoor Positioning systems

Received Signal Strength Based methods (Thesis)





Recent advances in Indoor Positioning systems

Comments

- [**?**] was the first to give mathematical formulation and theoretical basis for radio-map.
- Gaussian Process Latent Variable Models are the state-of-the-art in the fingerprinting based methods.
- Pareto Smoothed Importance Sampling Particle Filter can also be used instead of conventional particle filter. (work under progress)

Recent advances in Indoor Positioning systems

Comments on RSSI values

- The distribution of RSSI values is dependent not only on the location **but also on the time.** [?]
- **Gaussian** or **log-normal** used to measure the variation in RSSI measurement.
- Variation in RSSI is due to:

  Some factors are problem specific

  - user's presence
  - smart-phones (quality of Bluetooth chip)
  - orientation of the phone
  - material of the luminaire
  - orientation of Bluetooth module
  - calibration time
  - calibration points
  - malfunctioning BLE chips

Recent advances in Indoor Positioning systems

Comments on RSSI values

Other challenges:

- Multipath and fading.
- User/ Indoor setting movement causes fluctuation called fading. Even in movement of door can add some bias.
- Signal interference 2.4 GHz license-free frequency.
- Signal attenuation also dependent on the number of people indoors.
- The access points would be unheard, the data is sparse.

Why learn variations in RSSI values?

Introduction to Gaussian Processes

Advantages of Gaussian Processes [ferris_06]

1. **Continuous locations**
   - Any arbitrary location would work. Doesn't need cell based or any pre-specified points.
   - Fingerprinting radiomap, takes some effort so we need to engineer the selection of points.
2. **Arbitrary likelihood models**
   - Given the non-parametric nature, the GP's can approximate wide range of non-linear signal propagation models.
3. **Correct uncertainty handling**
   - Uncertainty estimates available at every location. How do we use it?
4. **Consistent parameter estimation**
   - Hyperparameter estimation. They include the spatial correlation between measurements and the measurement noise.

How's GPs been used?

1. **Radiomaps**
   - To interpolate to other known locations in the space. Conventional GP regression is used. [**?**]
2. **Meaurement model likelihood**
   - GPs provide the likelihood of the measured signal strength values to the radio map learnt above. (Simpler, parametric models like Path loss could also be used.)
   - It's just Gaussian of the measurement with GP predicted mean and variance from the regression.
3. **GP Latent Variable Model**
   - Maps the high dimensional signal strength values (depends on the no. of access points) to the low dimensional space (here, 2D coordinates of the person).

Design Matrix

Reference Table

$D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ where the x's are the locations and y's are the measured received signal strengths indicator values (RSSI). The RSSI values usually range from -50 dBm to -100 dBm

Variables - reference_map

reference_map

20x42 double

	1	2	3	4	5	6	7	8	9	10	1
1	1.5000	0	-72.3333	-80	-90.6667	-92.8571	-94.5000	-91.5000	-90.8333	-88.1250	-86
2	3	2.3500	-93	-93	-93	-93	-93	-94.2500	-93	-93.2000	
3	-3	1.1700	-93	-93	-93	-93	-93	-92	-93	-98	
4	6	2.1500	-60.2857	-88.9000	-90.8333	-96	-93.6250	-89.0714	-94.1250	-87.4545	-89
5	8	0	-67.4000	-80.8462	-90.0769	-85.0833	-91	-92.3333	-88.6667	-94	
6	10.5000	2.3500	-76.2222	-68.0667	-74.8000	-79	-84	-75.0909	-84.3333	-79.5833	-63
7	13	0.3000	-69	-65.5000	-76.8750	-85.4286	-86	-80.2308	-78.5385	-72.8667	-71
8	15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Figure: The first two columns denote x and rest of the columns are y

Design Matrix
Reference Table

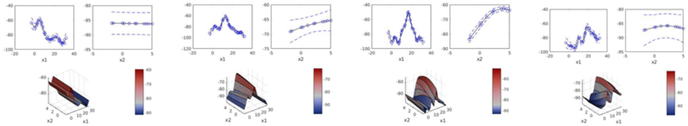
- [**?**] refers to reference table as **radiomap**. But, we give an other definition.
- If the floor space is divided in to cells, then given the signal strength values from each of the heard access points, the *i*'th row of the reference table is given by,

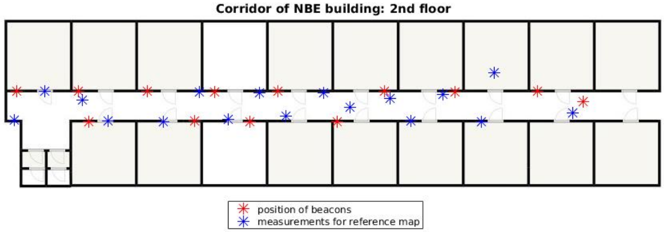
$$M_i = \Big(B_i, \{a_{ij} | j \in N_i, \theta_i\}\Big), \qquad i = 1, \ldots, m,$$

where B_i is the ith cell,
a_{ij} contains the RSSI value from jth AP,
θ_i can contain any other information.

Radiomaps

- Radiomaps are the interpolation of the points in reference table of the other parts of the space.





Radiomaps

Mathematics behind it

The noisy process is:

$$y_i = f(x_i) + \epsilon \tag{1}$$

where x_i is the location, R^2.

y_i is the target RSSI value or observation for an AP.

ϵ is the zero mean, additive Gaussian noise.

Radiomaps

Mathematics behind it

- We model the GP over the function which relates location to noiseless signal strength values $f(x_i)$.
- The conventional GP prediction for a location x_*

$$p(f(x_*)|x_*, \mathbf{X}, \mathbf{y}) = N(f(x_*)|\mu_{x_*}, \sigma^2_{x_*}) \tag{2}$$

- Here, $\mu_{x_*} = {k_*}^T(K + \sigma_n^2 I)^{-1} y$ and
  $\sigma^2_{x_*} = k(x_*, x_*) - {k_*}^T(K + \sigma_n^2 I)^{-1} k_*$

We’ve a problem here!

Radiomaps

Problem with radiomap

- GP modeling is generally zero mean, so as the we go away from the region of data.
- We need to either use a constant covariance function to get the offset or use a mean function.
- Logical step would be to use the deterministic signal propagation model [**?**] or use some linear model [**?**].

Gaussian Process Latent Variable Model (GP-LVM)

Introduction

- As already told, this method maps the high dimensional data space to low dimension latent space.
- The constraints used to solve this problem are:
  1. **Location to signal strength**: Similar locations would have similar signal strength values.
  2. **Signal strength to signal strength**: Similar signal strengths observed were from similar locations.
  3. **Locations to locations**: Locations that follow in the data stream (data log file) are near each other.

Gaussian Process Latent Variable Model (GP-LVM)

Mathematics

- GP-LVM's tries to solve the SLAM problem and the main task here is to get rid of dependency on the locations in the design matrix.
- We would treat them as latent variables. Instead of the considering the measurements from 1-D space, here we club to the dimension of the number of the AP's.
- Let $X \in R^{n \times q}$ and $Y \in R^{n \times d}$. Our process model looks like,

$$y_{ij} = f(x_i; w_j) + \epsilon \tag{3}$$

- where y_{ij} are the elements of $\mathbf{Y}$, x_i is the i-th of $\mathbf{X}$ matrix, w_j are the parameters of the non-linear function and ϵ is zero mean noise term.

Gaussian Process Latent Variable Model (GP-LVM)

Mathematics

- Now, the probabilistic relation implies between the data and the latent variables is $p(y_{ij}|x_i, w_j)$.

$$p(\mathbf{Y}|\mathbf{X}, \mathbf{W}) = \prod_{ij} p(y_{ij}|x_i, w_j) \tag{4}$$

- Assuming the functional relationship between the data and the latent variables, it can recovered through optimization of the marginal likelihood.

$$f(x_i; w_j) = w_j^T x_i; \tag{5}$$

Gaussian Process Latent Variable Model (GP-LVM)

Mathematics

- The marginal likelihood takes the form:

$$p(\mathbf{Y}|\mathbf{W}) = N(\mathbf{Y}; \mathbf{0}, \mathbf{W}\mathbf{W}^T + \sigma_n^2 I) \tag{6}$$

- Maximizing the above likelihood, results in the **W** which spans through the principle sub-space of the data. Clearly our function f is non-linear and marginalization to get 6 can't be done without reverting to approximations.

$$f(x_i; w_j) = w_j^T x_i; \tag{7}$$

- Alternatively, we can marginalize the parameters and reach, assuming standard Normal prior for the parameters.

$$p(\mathbf{Y}|\mathbf{W}) = \prod_j N(\mathbf{Y}; \mathbf{0}, \mathbf{X}\mathbf{X}^T + \sigma_n^2 I) \tag{8}$$

Gaussian Process Latent Variable Model (GP-LVM)

GP-LVM for WiFi-SLAM

- The probabilistic model of measurements and locations factorize as:

$$p(\mathbf{X}, \mathbf{Y}) = p(\mathbf{Y}|\mathbf{X})p(\mathbf{X}) \tag{9}$$

- The likelihood is modeled as GP and $p(X)$ denotes the latent space dynamic model

$$f(x_i; w_j) = w_j^T x_i; \tag{10}$$

Gaussian Process Latent Variable Model (GP-LVM)

GP-LVM for WiFi-SLAM

- The dynamic model is constrained in terms of
  - Distance between successive points.
  - Change in orientation between successful positions.
  - Alignment of the parallel line segments.
- Extra information is fed as we are not giving the location data.

Gaussian Process Latent Variable Model (GP-LVM)

GP-LVM for WiFi-SLAM

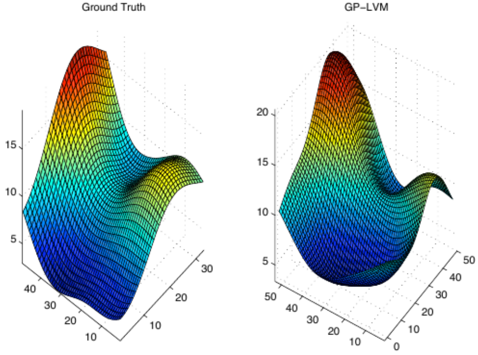


Figure 2: Comparison of ground truth Gaussian process versus that constructed by GP-LVM.