

데이터 과학 외전

Day 4 – 앙상블 알고리즘

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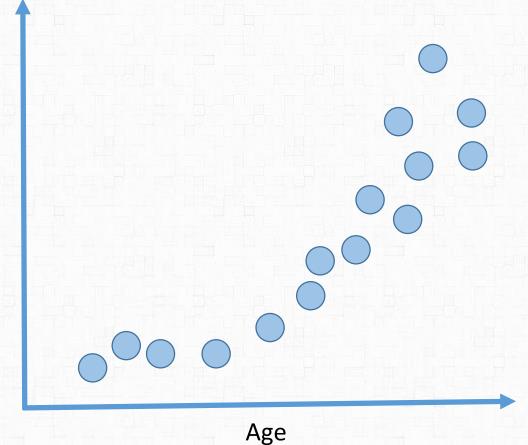


Bias And Variance



사람들의 나이와 연간의료비용을 데이터로 나타내었다

- 나이가 어리거나 젊을 때는 의료비용의 증가가 없거나 거의 미미하다
- 중년 이상이 되면서 의료비용이 서서히 증가하기 시작한다
- 노년 이상이되면서 의료비용의 증가 폭이 더욱 커진다



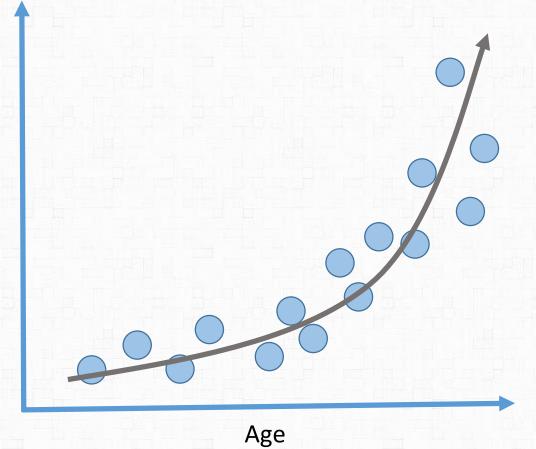


이상적으로는

의료비용과 나이의 관계를 결정하는 법칙 = 수식 = 모델 을 알고자 함

"True" relationship

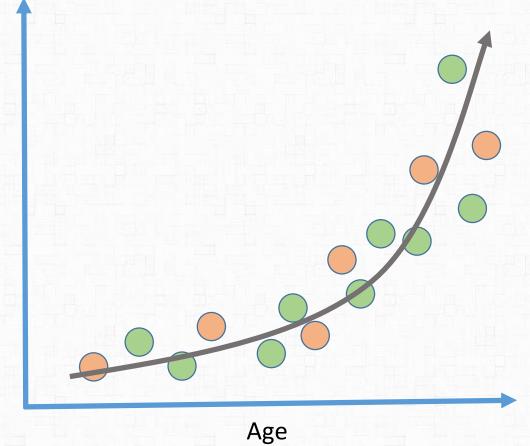
나이와 의료비용의 관계를 sample data로 부터 추정하는 과정 = data modeling





데이터를 모델을 학습(추정)하기 위한 데이터(학습데이터)

모델을 검증하기 위한 데이터(<mark>테스트 데이</mark> 터)로 구분한다

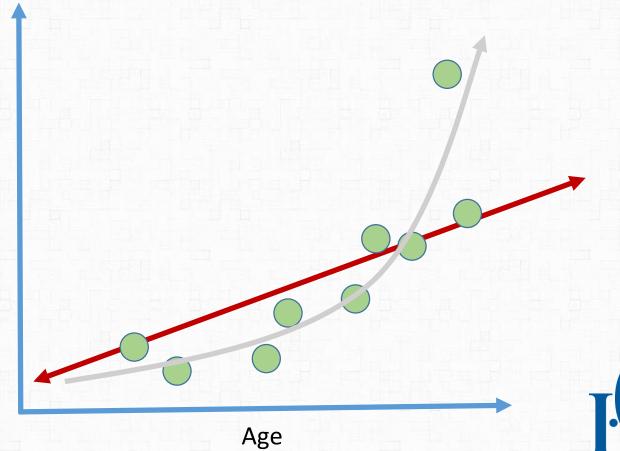




$$y = \alpha_0 + \alpha_1 x$$

y:YME

x : Age

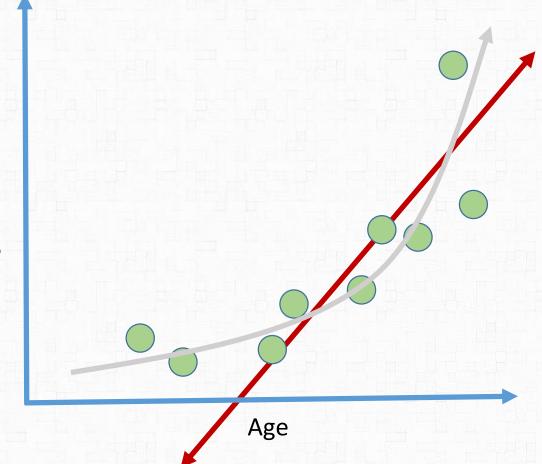




$$y = \alpha_0 + \alpha_1 x$$

y:YME

x : Age

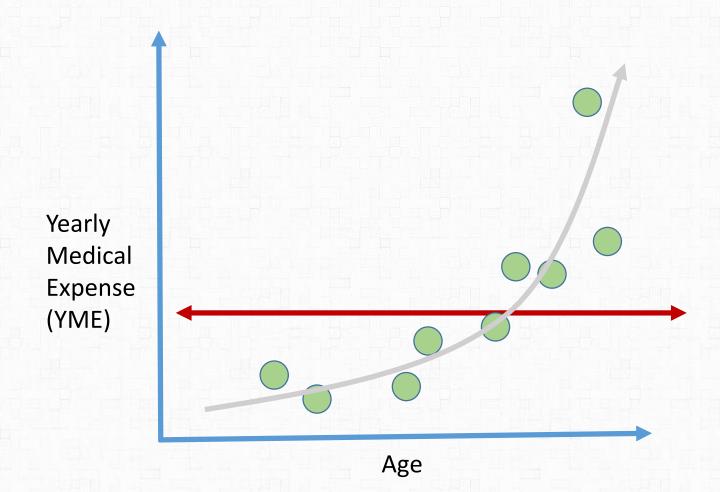




$$y = \alpha_0 + \alpha_1 x$$

y:YME

x : Age





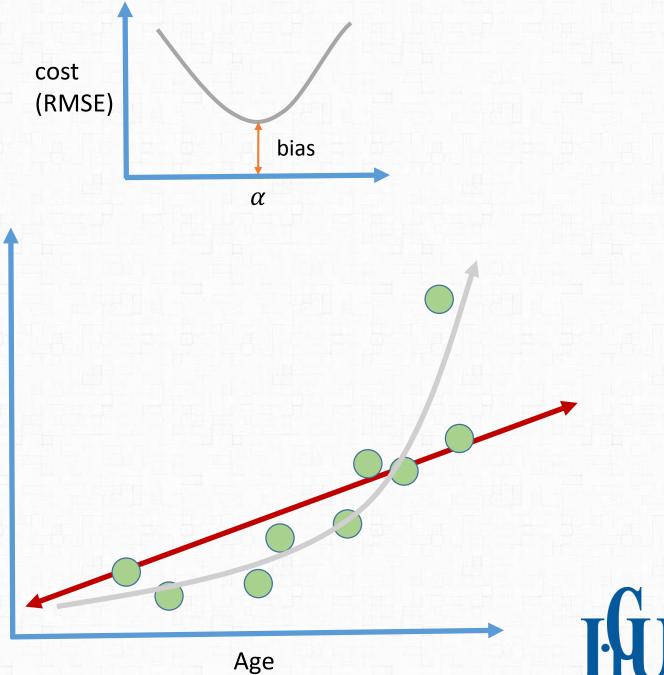
$$y = \alpha_0 + \alpha_1 x$$

y:YME

x : Age

Linear Regression will never capture the "true relationship"

= Bias



Second Try: Polynomial

Regression

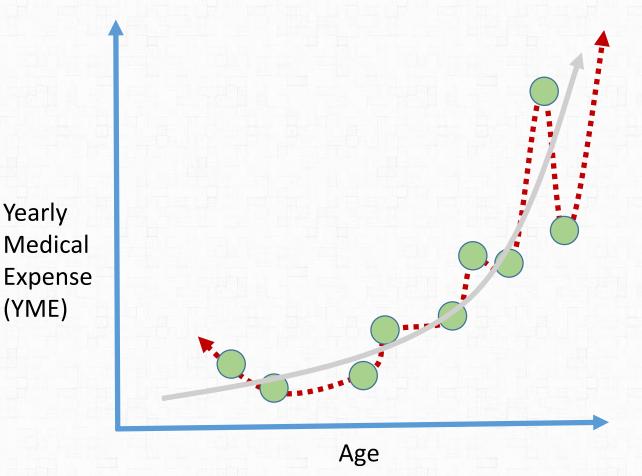
Model is flexible to capture the non-linearity

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4 + \alpha_5 x^5$$

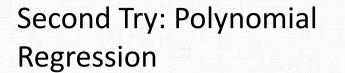
y:YME

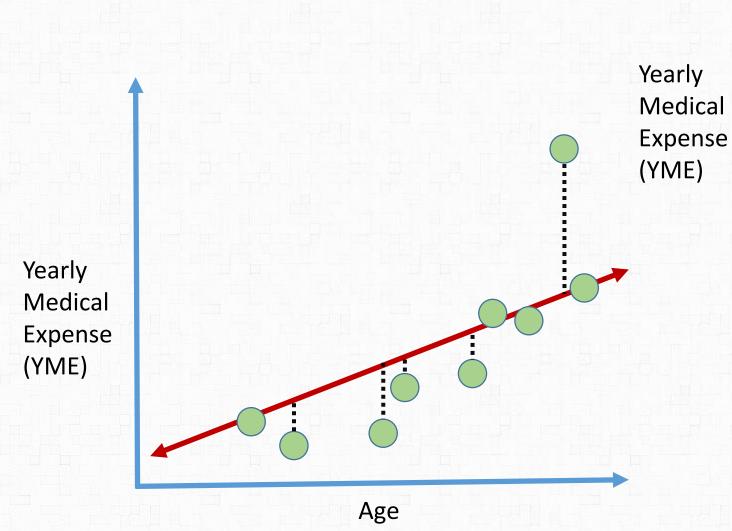
x : Age

(YME)









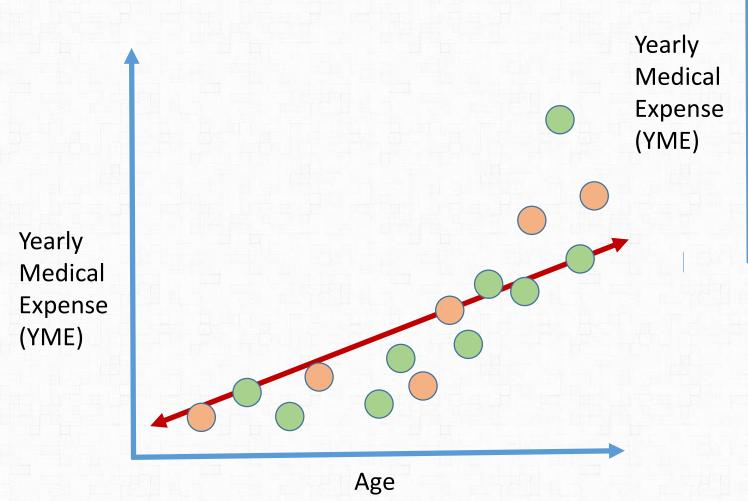
lower bias here!

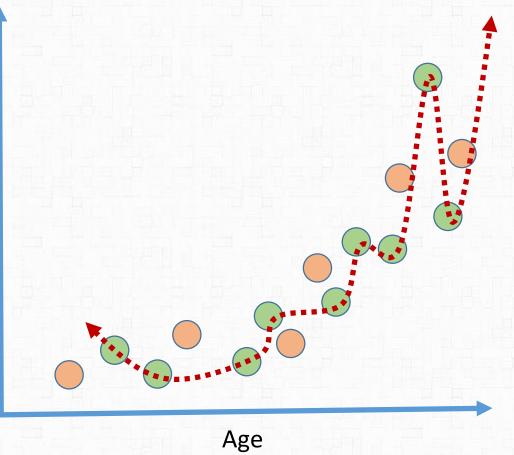
Age



Second Try: Polynomial Regression

what about training set

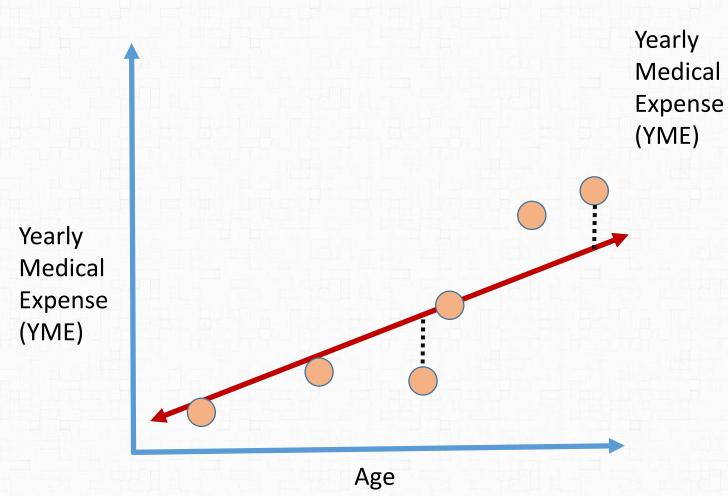


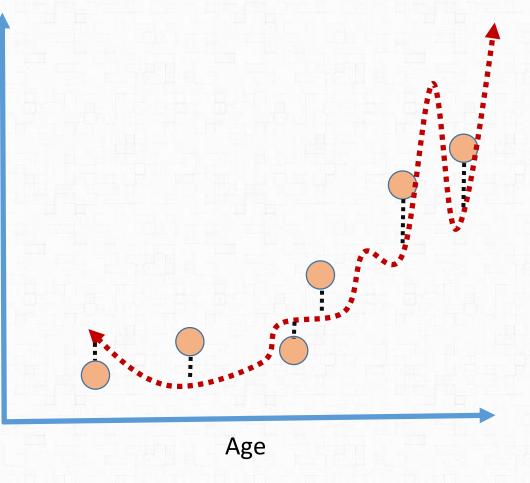




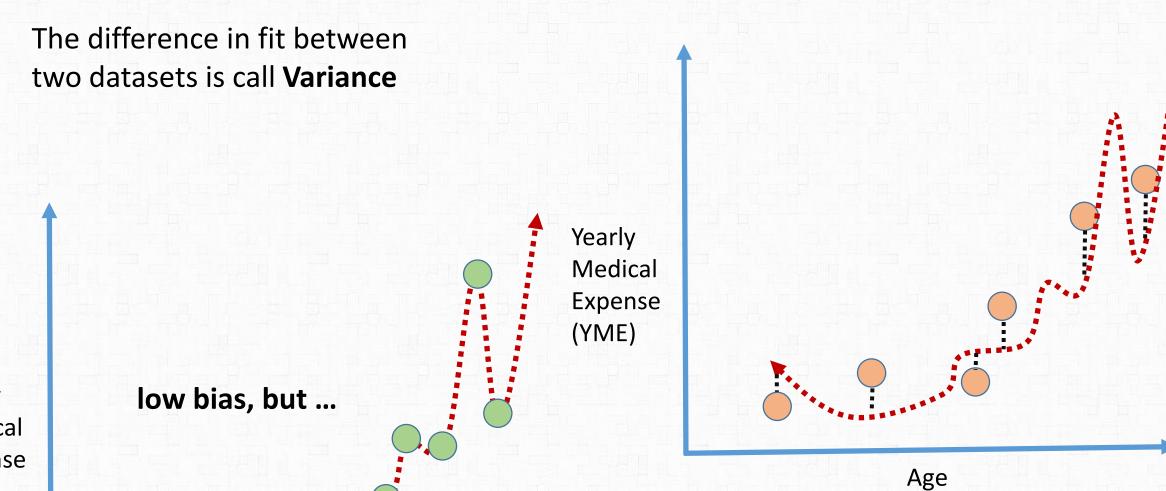
Second Try: Polynomial Regression

what about training set









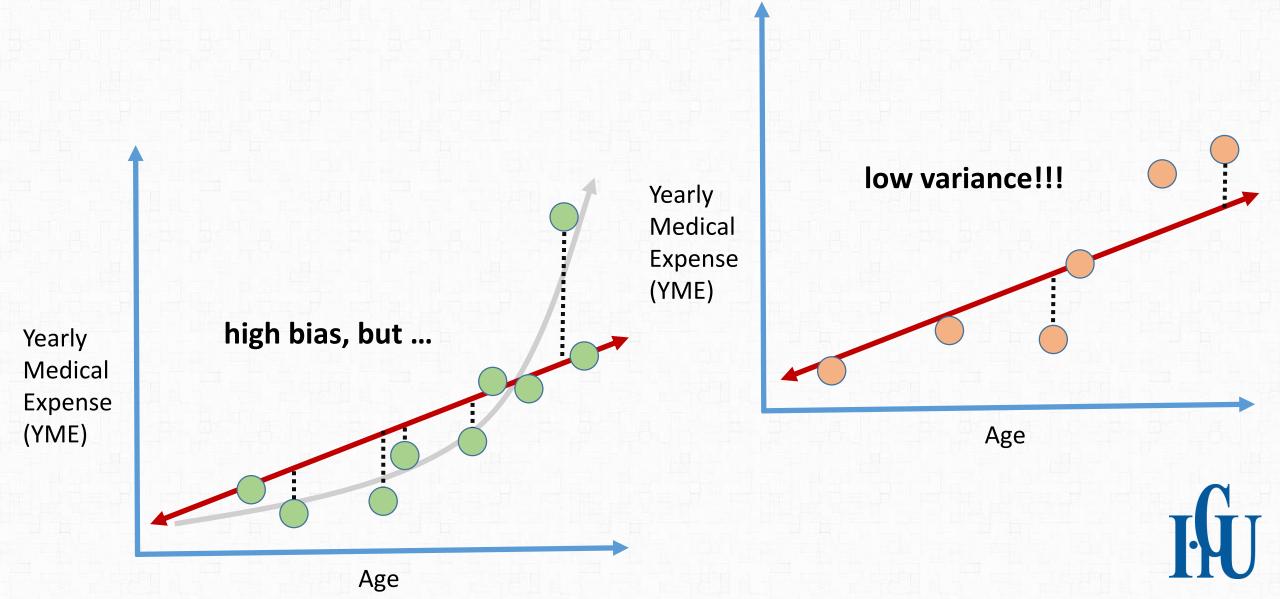
Yearly Medical Expense (YME)

Age

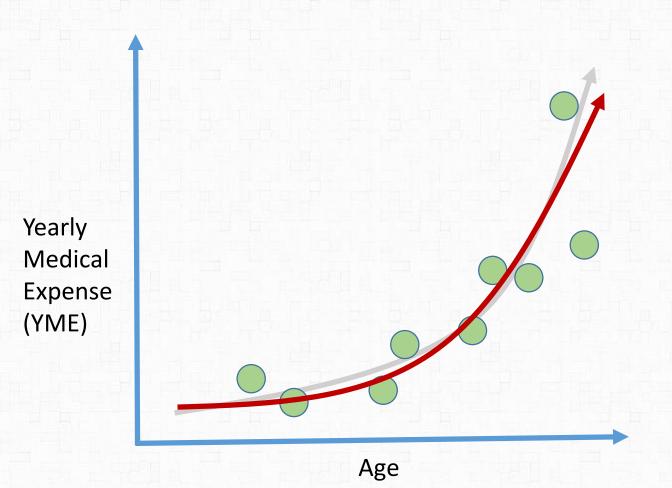
high variance!!



low variance model consistently give a good prediction (even though it is not a great prediction)

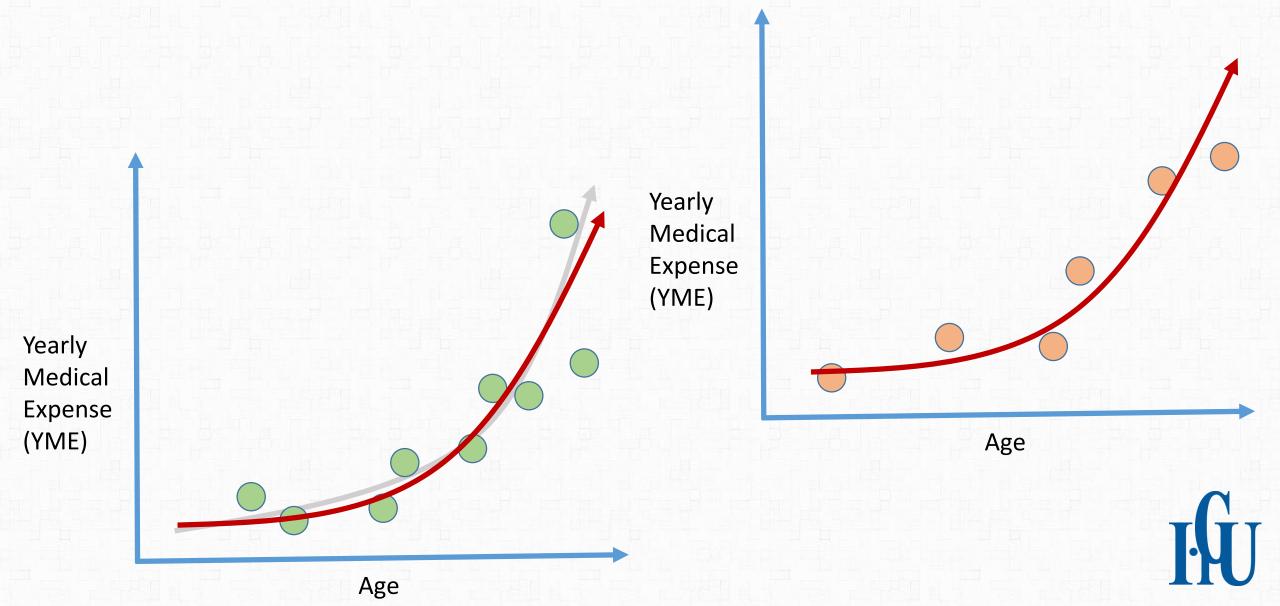


the ideal model has **low bias**, which accurately model the true relationship



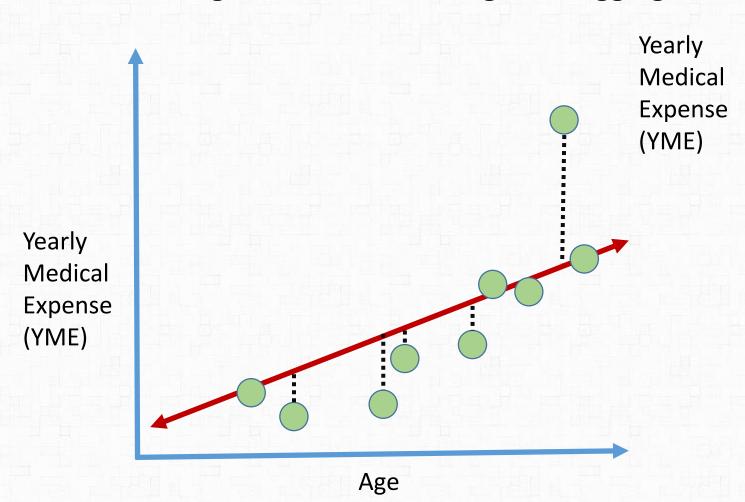


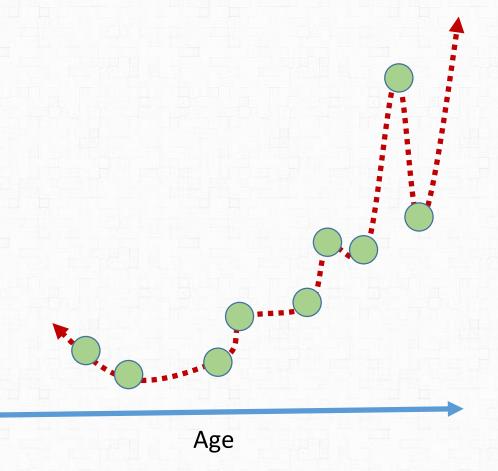
... ant it has **low variance**, by making consistence predictions across different datasets



The key is to find a balance between a simple model and a complex model

The commonly used methods are: regularization, boosting and bagging



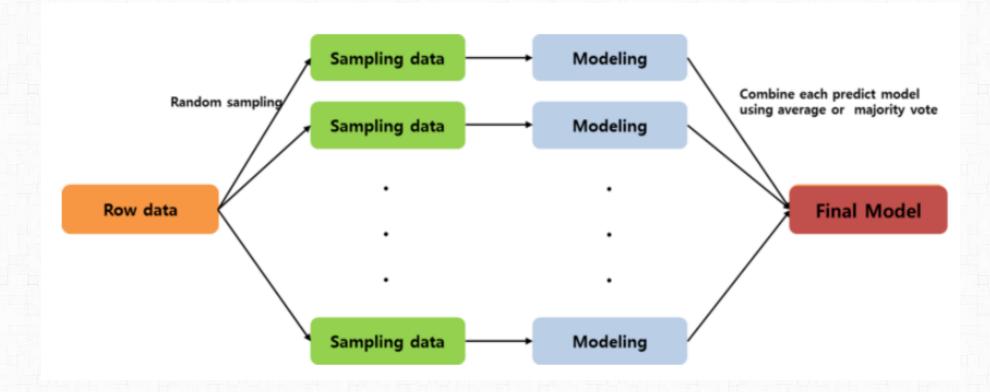




Bagging and Boosting

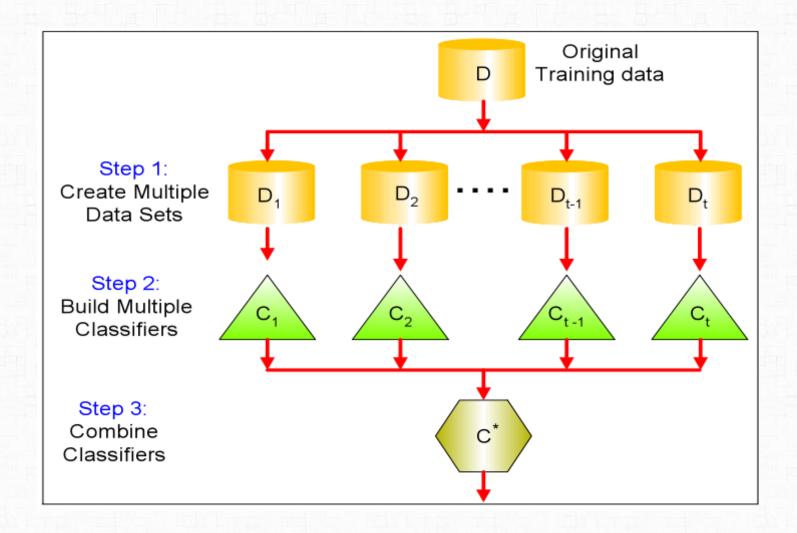


Bagging





Bagging





Example of Bagging

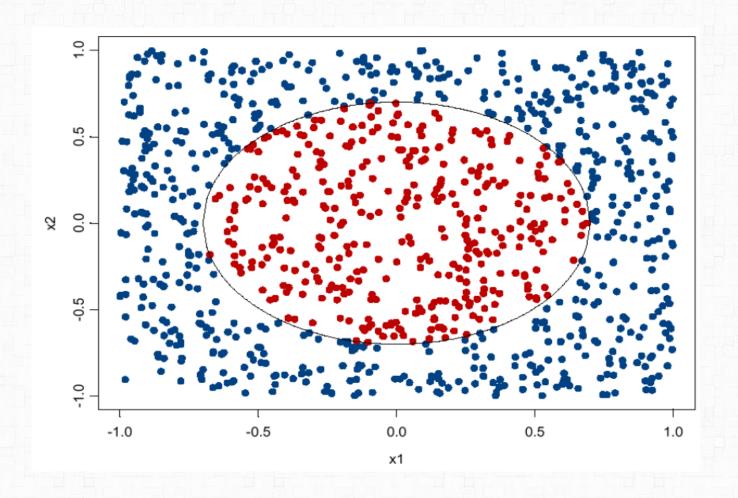
Sampling with replacement

Data ID	Training Data									
Original Data	1	2	3	4	5	6	7	8	9	10
Bagging (Round 1)	7	8	10	8	2	5	10	10	5	9
Bagging (Round 2)	1	4	9	1	2	3	2	7	3	2
Bagging (Round 3)	1	8	5	10	5	5	9	6	3	7

- Build classifier on each bootstrap sample
- Each data point has probability (1 1/n)ⁿ of being selected as test data
- Training data = $1 (1 1/n)^n$ of the original data

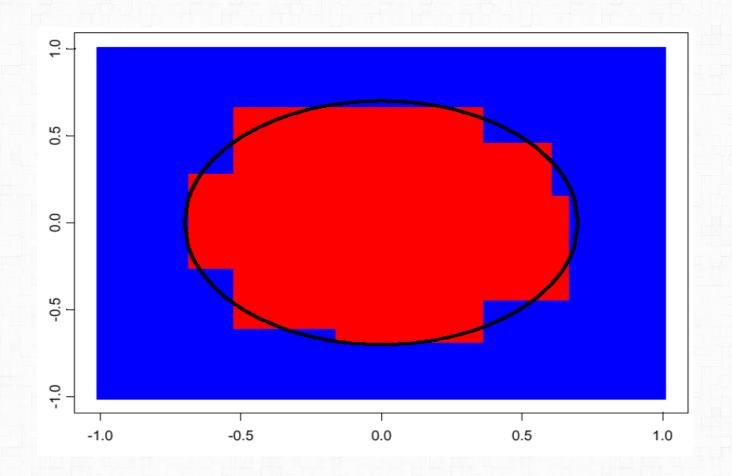


Example of Bagging



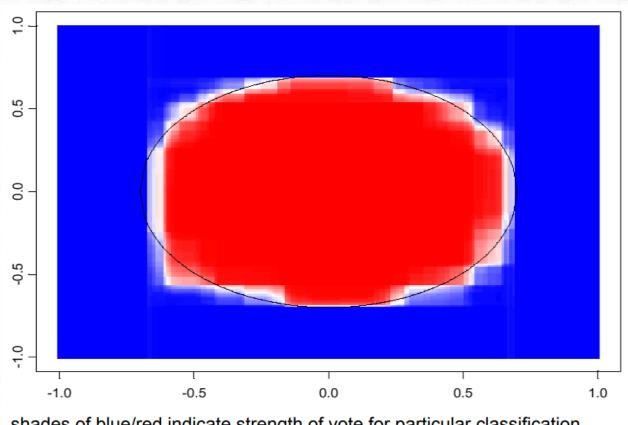


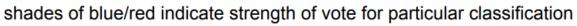
Example of Bagging





100 bagged trees







Bagging / Resampling

- It reduces variance of model complexity
- It builds various models with many different samplings
- It reduces bias b/c it combines all models
- Aggregating all the models, reduce variance



Practice - Bagging

```
spamD <- read.table('spamD.tsv',header=T,sep='\t')</pre>
spamD[1:5, c(1:5,58:59)]
##
     word.freq.make word.freq.address word.freq.all word.freq.3d
## 1
               0.00
                                  0.64
                                                 0.64
               0.21
                                  0.28
                                                 0.50
## 2
                                                 0.71
## 3
               0.06
                                  0.00
## 4
               0.00
                                  0.00
                                                 0.00
               0.00
                                  0.00
## 5
                                                 0.00
##
     word.freq.our spam rgroup
## 1
              0.32 spam
                             52
## 2
              0.14 spam
                             91
              1.23 spam
                             49
## 3
              0.63 spam
                             88
## 4
              0.63 spam
                             73
## 5
                                                # Note 1:
spamTrain <- subset(spamD, spamD$rgroup>=10)
                                                    Load the data and split into training (90% of data)
spamTest <- subset(spamD, spamD$rgroup<10)</pre>
                                                    and test (10% of data) sets.
```

Building Tree model

```
Use all the features and do binary classification,
                                                           where TRUE corresponds to spam documents.
spamVars <- setdiff(colnames(spamD), list('rgroup', 'spam'))</pre>
spamFormula <- as.formula(paste('spam=="spam"',</pre>
                              paste(spamVars,collapse=' + '),sep=' ~ '))
                                                    # Note 4:
                                                       A function to calculate and return various measures
                                                       on the model: prediction accuracy, and f1, which is the
                                                       harmonic mean of precision and recall.
accuracyMeasures <- function(pred, truth, name="model") {
  ctable <- table(truth=truth,
                    pred=(pred>0.5))
                                                        # Note 6:
                                                           Convert the class probability estimator into a
  accuracy <- sum(diag(ctable))/sum(ctable)</pre>
                                                            classifier by labeling documents that score greater
  precision <- ctable[2,2]/sum(ctable[,2])</pre>
                                                        than 0.5 as
  recall <- ctable[2,2]/sum(ctable[2,])
                                                            spam.
  f1 <- 2*precision*recall/(precision+recall)</pre>
  data.frame(model=name, accuracy=accuracy, f1=f1)
```

Note 2:

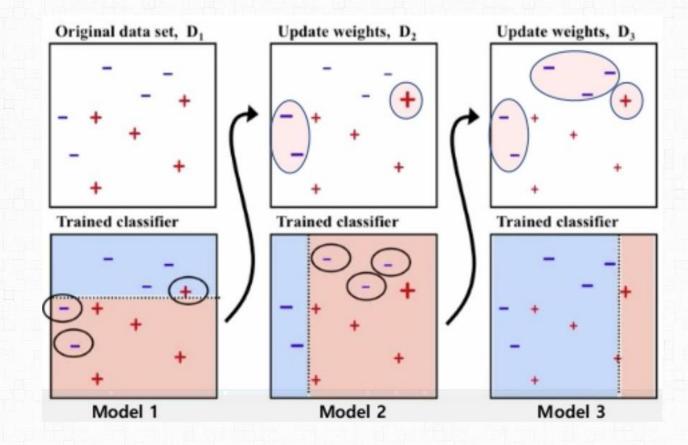
```
library(rpart)
treemodel <- rpart(spamFormula, spamTrain)</pre>
accuracyMeasures(predict(treemodel, newdata=spamTrain),
                 spamTrain$spam=="spam",
                 name="tree, training")
##
              model accuracy
## 1 tree, training 0.9104514 0.88337
accuracyMeasures(predict(treemodel, newdata=spamTest),
                 spamTest$spam=="spam",
                 name="tree, test")
##
          model accuracy
## 1 tree, test 0.8799127 0.8414986
```

```
bagging
                                    # Note 1:
ntrain <- dim(spamTrain)[1]</pre>
                                       Use bootstrap samples the same size as the training
                                        set, with 100 trees.
n <- ntrain
ntree <- 100
                                                   # Note 2:
                                                       Build the bootstrap samples by sampling the row indices
                                                   of spamTrain with replacement. Each
                                                       column of the matrix samples represents the row indices
samples <- sapply(1:ntree,</pre>
                                                   into spamTrain
                                                       that comprise the bootstrap sample.
                     FUN = function(iter)
                        {sample(1:ntrain, size=n, replace=T)})
treelist <-lapply(1:ntree,</pre>
                                                        # Note 3:
                      FUN=function(iter)
                                                           Train the individual decision trees and return them
                                                           in a list. Note: this step can take a few minutes.
                      {samp <- samples[,iter];
                        rpart(spamFormula, spamTrain[samp,])})
```

```
# Note 4:
predict.bag <- function(treelist, newdata) {</pre>
                                                      predict.bag assumes the underlying classifier returns
                                                   decision probabilities, not
  preds <- sapply(1:length(treelist),</pre>
                                                      decisions.
                  FUN=function(iter) {
                    predict(treelist[[iter]], newdata=newdata)})
  predsums <- rowSums(preds)</pre>
  predsums/length(treelist)
accuracyMeasures(predict.bag(treelist, newdata=spamTrain),
                  spamTrain$spam=="spam",
                  name="bagging, training")
##
                  model accuracy
## 1 bagging, training 0.9227613 0.8990536
accuracyMeasures(predict.bag(treelist, newdata=spamTest),
                  spamTest$spam=="spam",
                  name="bagging, test")
##
              model accuracy
  1 bagging, test 0.9126638 0.8809524
```

Boosting (C5.0)







Boosting

Aggregate the result of boosted model

알고리즘	특징	비고
AdaBoost	• 다수결을 통한 정답 분류 및 오답에 가중치 부여	
GBM	• Loss Function의 gradient를 통해 오답에 가중치 부여	gradient_boosting.pdf
Xgboost	 GBM 대비 성능향상 시스템 자원 효율적 활용 (CPU, Mem) Kaggle을 통한 성능 검증 (많은 상위 랭커가 사용) 	2014년 공개 boosting-algorithm-xgboost
Light GBM	 Xgboost 대비 성능향상 및 자원소모 최소화 Xgboost가 처리하지 못하는 대용량 데이터 학습 가능 Approximates the split (근사치의 분할)을 통한 성능 향상 	2016년 공개 light-gbm-vs-xgboost



Bagging v.s. Boosting

비교	Bagging	Boosting			
특징	병렬 앙상블 모델 (각 모델은 서로 독립적)	연속 앙상블 (이전 모델의 오류를 고려)			
목적	Variance 감소	Bias 감소			
적합한 상황	복잡한 모델 (High variance, Low bias)	Low variance, High bias 모 델			
대표 알고리즘	Random Forest	Gradient Boosting, AdaBoost			
Sampling	Random Sampling	Random Sampling with weight on error			

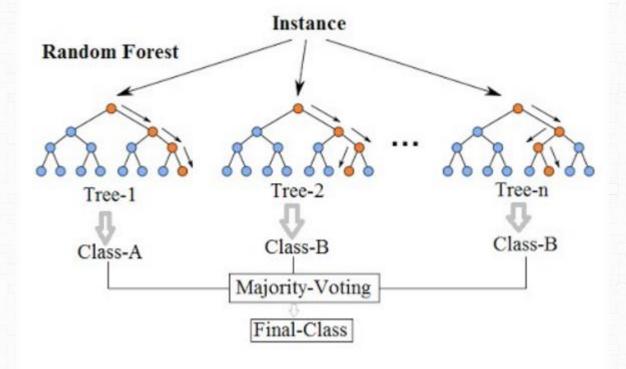


Random Forest



 Tree-based Enssenble model with Bagging

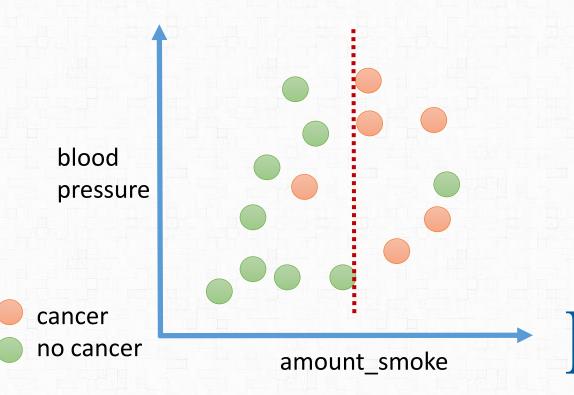
Random Forest Simplified





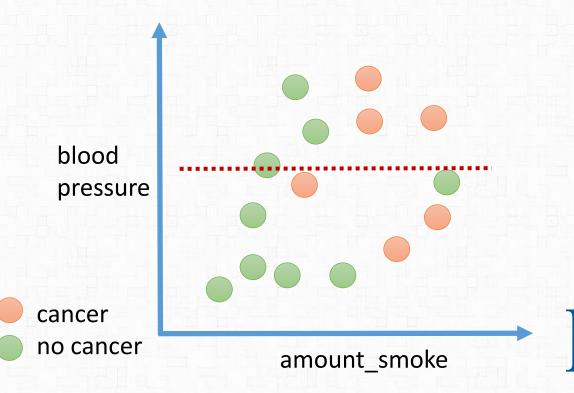
Random Forest - Tree Correlation

- Bagging reduces variance by aggregating many different models
- Bagging samples observations not features
- If certain features have high predictive power then all the tree models look very similar
 - Eg. Smoker variable of insurance dataset
 - High correlation in trees
- Random Forest
 - Select features randomly and build trees



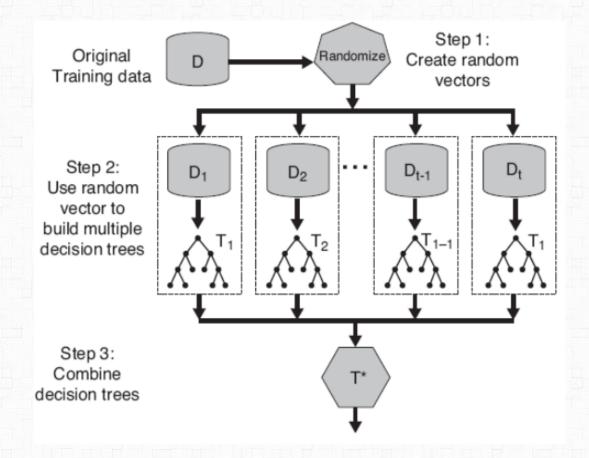
Random Forest - Tree Correlation

- Bagging reduces variance by aggregating many different models
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 - High correlation in trees
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 - Select features randomly and build trees



- Ensemble method specifically designed for decision tree classifiers
- Introduce two sources of randomness:
 - "Bagging" and "Random input vectors"
 - Bagging method: each tree is grown using a bootstrap sample of training data
 - Random vector method: At each node, best split is chosen from a random sample of m attributes instead of all attributes







```
library(randomForest)
set.seed(5123512)
fmodel <- randomForest(x=spamTrain[,spamVars],</pre>
          y=spamTrain$spam,
                         # Note 4:
                           Use 100 trees to be compatible with our bagging
          ntree=100.
                           example. The default is 500 trees.
                           # Note 5:
                              Specify that each node of a tree must have a minimum
          nodesize=7,
                              of 7 elements, to be compatible with the default minimum node size that rpart()
                              uses on this training set.
                              # Note 6:
          importance=T)
                                Tell the algorithm to save information to be used for
                                calculating variable importance (we'll see this later).
```

```
accuracyMeasures(predict(fmodel,
    newdata=spamTrain[,spamVars],type='prob')[,'spam'],
    spamTrain$spam=="spam",name="random forest, train")

## model accuracy f1

## 1 random forest, train 0.9884142 0.9851943

accuracyMeasures(predict(fmodel,
    newdata=spamTest[,spamVars],type='prob')[,'spam'],
    spamTest$spam=="spam",name="random forest, test")

## model accuracy f1

## 1 random forest, test 0.9497817 0.9340974
```



Estimated Error Rate

- 1. At each bootstrap iteration, predict the data not in the bootstrap sample (what Breiman calls "out-of-bag", or OOB, data) using the tree grown with the bootstrap sample.
- 2. Aggregate the OOB predictions. (On the average, each data point would be out-of-bag around 36% of the times, so aggregate these predictions.) Calcuate the error rate, and call it the OOB estimate of error rate.



Variable importance

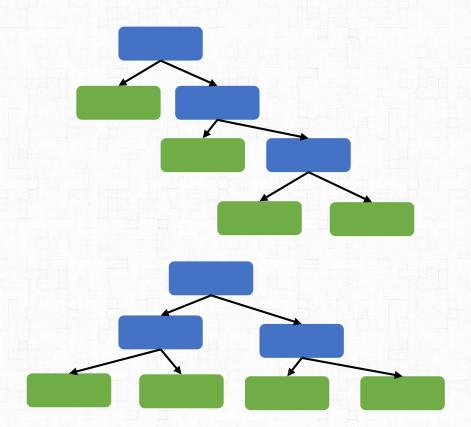
- Make permutation on certain variable p of OOB data while other variables unchanged
- Compare prediction accuracy on permuted data and unchanged data



Adaboost and XGBoost



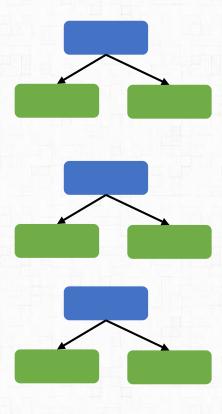
uses a set of fully grown trees



Adaboost

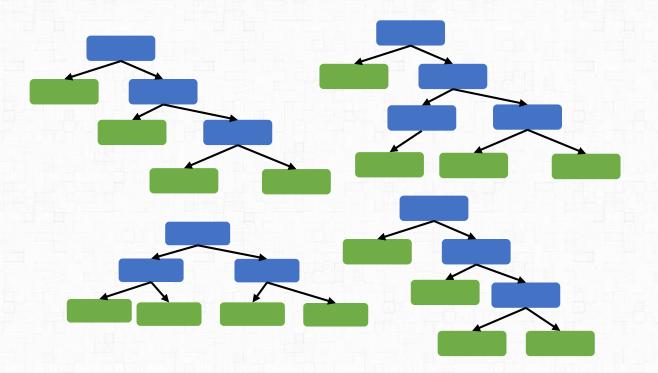
uses a set of stumps

- simplified decision tree (weak learner)



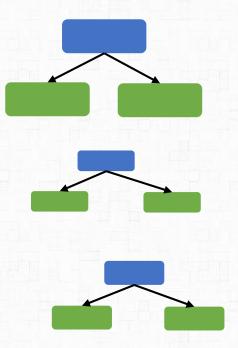


all trees are equally important to make a final decision



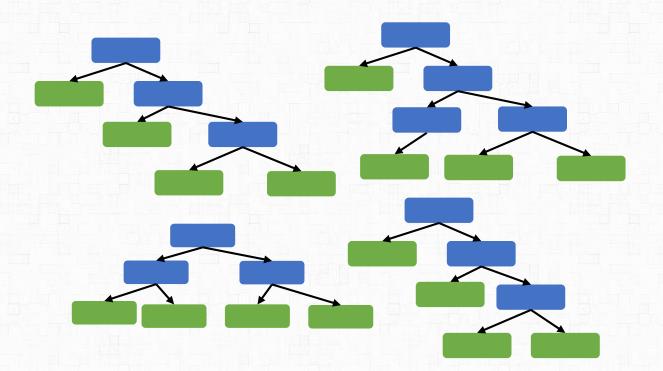
Adaboost

some stumps are more important than others when making a final decision



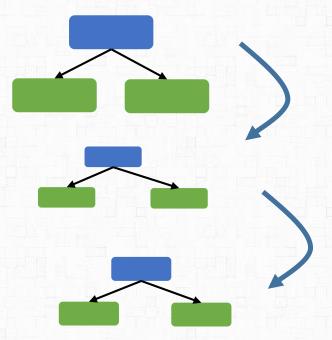


makes all tree independently



Adaboost

makes trees sequentially





Chest Pain	Blocked Arteries	Patient Weight	Heart Disease
Yes	Yes	205	Yes
No	Yes	180	Yes
Yes	No	210	Yes
Yes	Yes	167	Yes
No	Yes	156	No
No	Yes	125	No
Yes	No	168	No
Yes	Yes	172	No

Sample Weight	
1/8	
1/8	
1/8	
1/8	
1/8	
1/8	
1/8	
1/8	



Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
Yes	Yes	205	Yes	 1/8
No	Yes	180	Yes	1/8
Yes	No	210	Yes	 1/8
Yes	Yes	167	Yes	 1/8
No	Yes	156	No	1/8
No	Yes	125	No	1/8
Yes	No	168	No	1/8
Yes	Yes	172	No	1/8

Chest Pain

Yes Heart Disease

correct : 3

incorrect: 2

No Heart Disease

correct : 2

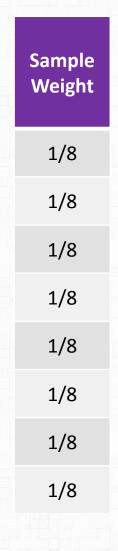
incorrect: 1

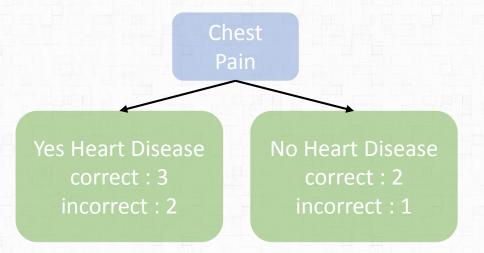
G.I. =
$$\frac{5}{8} \left[1 - \left\{ \left(\frac{3}{5} \right)^2 + \left(\frac{2}{5} \right)^2 \right\} \right] + \frac{3}{8} \left[1 - \left\{ \left(\frac{2}{3} \right)^2 + \left(\frac{1}{3} \right)^2 \right\} \right]$$

= 0.47



Chest Pain	Blocked Arteries	Patient Weight	Heart Disease
Yes	Yes	205	Yes
No	Yes	180	Yes
Yes	No	210	Yes
Yes	Yes	167	Yes
No	Yes	156	No
No	Yes	125	No
Yes	No	168	No
Yes	Yes	172	No





Blocked

Arteries

Yes Heart Disease

correct: 3

incorrect: 3

No Heart Disease

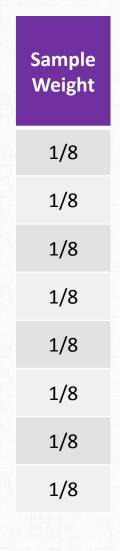
correct: 1

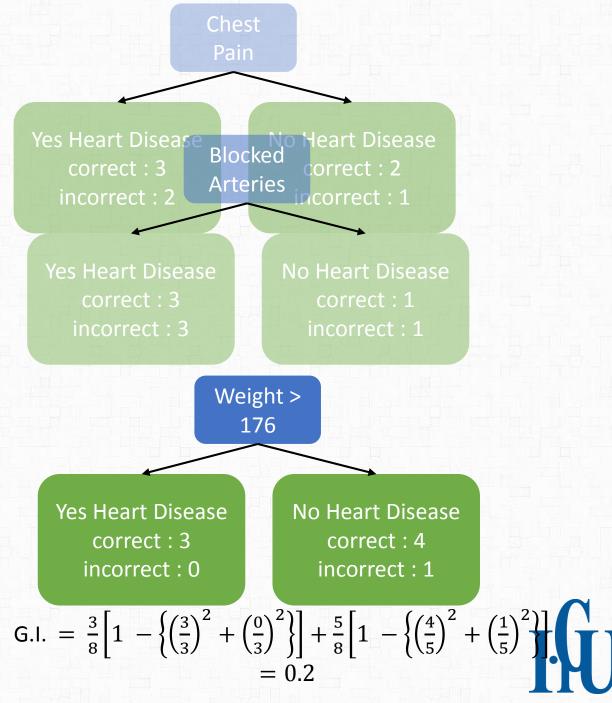
incorrect: 1

G.I. =
$$\frac{6}{8} \left[1 - \left\{ \left(\frac{3}{6} \right)^2 + \left(\frac{3}{6} \right)^2 \right\} \right] + \frac{2}{8} \left[1 - \left\{ \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right)^2 \right\} \right]$$

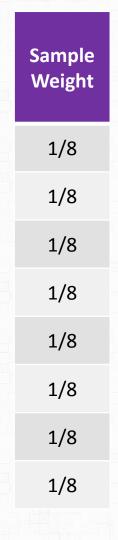
= 0.5

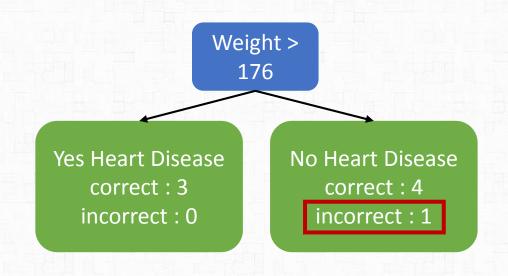
Chest Pain	Blocked Arteries	Patient Weight	Heart Disease
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Chest Pain	Blocked Arteries	Patient Weight	Heart Disease
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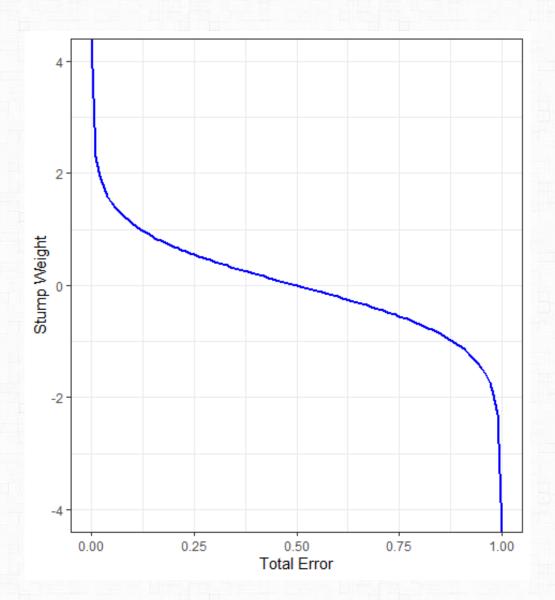
Total Error: the sum of weights for *incorrectly* classified samples In the example, Total Error is 1/8

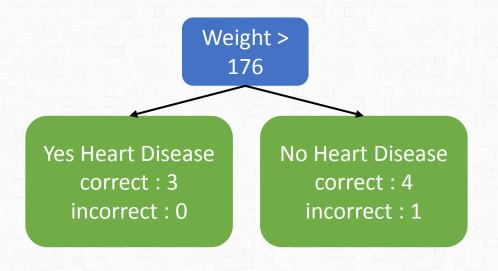
Stumps Weight:

Amount of weight that the stump contributes to the final decision

$$\frac{1}{2}\log(\frac{1-total\ error}{total\ error})$$







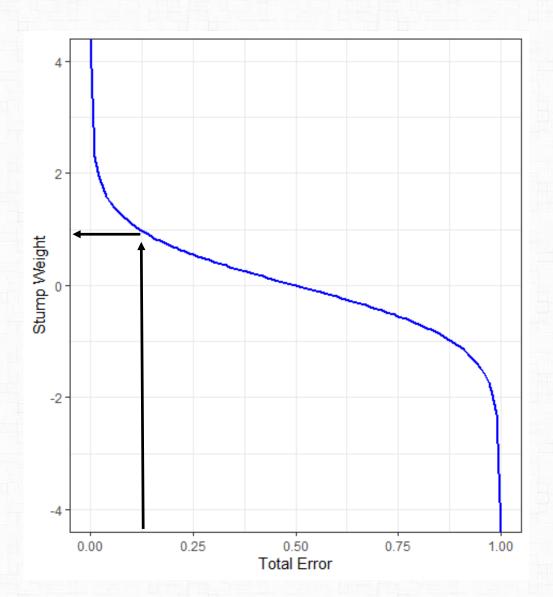
Total Error: the sum of weights for *incorrectly* classified samples In the example, Total Error is 1/8

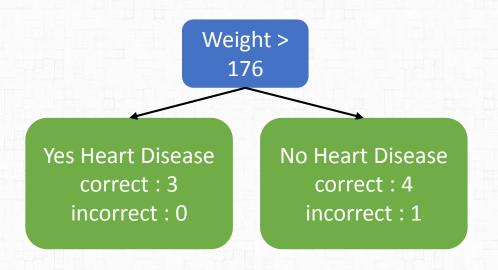
Stumps Weight:

Amount of weight that the stump contributes to the final decision

$$\frac{1}{2}\log(\frac{1-total\ error}{total\ error})$$







Total Error: the sum of weights for *incorrectly* classified samples In the example, Total Error is 1/8

Stumps Weight:

Amount of weight that the stump contributes to the final decision

$$\frac{1}{2}\log\left(\frac{1-\frac{1}{8}}{\frac{1}{8}}\right) = 0.97$$



Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
Yes	Yes	205	Yes	1/8
No	Yes	180	Yes	1/8
Yes	No	210	Yes	1/8
Yes	Yes	167	Yes	1/8
No	Yes	156	No	1/8
No	Yes	125	No	1/8
Yes	No	168	No	1/8
Yes	Yes	172	No	1/8

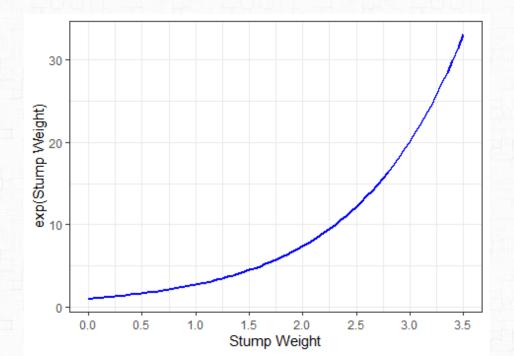
New Sample Weight:

Incorrectly classified samples:

sample weight
$$\times e^{stump \text{ weight}}$$

$$= \frac{1}{8} \times e^{0.97} = \frac{1}{8} \times 2.64 = 0.33$$

Correctly classified samples: sample weight $\times e^{stump \ weight}$





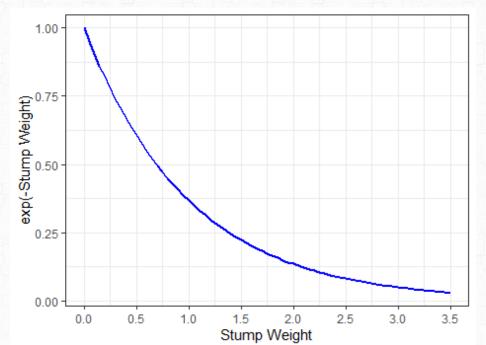
Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
Yes	Yes	205	Yes	1/8
No	Yes	180	Yes	1/8
Yes	No	210	Yes	1/8
Yes	Yes	167	Yes	1/8
No	Yes	156	No	1/8
No	Yes	125	No	1/8
Yes	No	168	No	1/8
Yes	Yes	172	No	1/8

New Sample Weight:

Incorrectly classified samples: $sample \ weight \times e^{stump \ weight}$ $= \frac{1}{8} \times e^{0.97} = \frac{1}{8} \times 2.64 = 0.33$

Correctly classified samples : $sample weight \times e^{stump weight}$

$$= \frac{1}{8} \times e^{-0.97} = \frac{1}{8} \times 0.38 = 0.05$$





Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight	Norm. Sample Weight
Yes	Yes	205	Yes	0.05	0.07
No	Yes	180	Yes	0.05	0.07
Yes	No	210	Yes	0.05	0.07
Yes	Yes	167	Yes	0.33	0.49
No	Yes	156	No	0.05	0.07
No	Yes	125	No	0.05	0.07
Yes	No	168	No	0.05	0.07
Yes	Yes	172	No	0.05	0.07

New Sample Weight:

Incorrectly classified samples: $sample \ weight \times e^{stump \ weight}$ $= \frac{1}{8} \times e^{0.97} = \frac{1}{8} \times 2.64 = 0.33$

Correctly classified samples: $sample weight \times e^{stump weight}$

$$= \frac{1}{8} \times e^{-0.97} = \frac{1}{8} \times 0.38 = 0.05$$

sum of new sample weights = 0.68 normalize the new weight dividing by 0.68



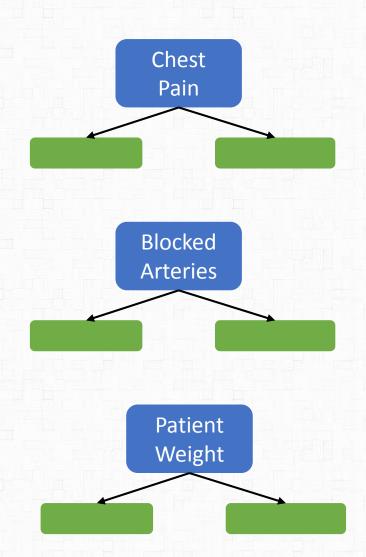
Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
Yes	Yes	205	Yes	0.07
No	Yes	180	Yes	0.07
Yes	No	210	Yes	0.07
Yes	Yes	167	Yes	0.49
No	Yes	156	No	0.07
No	Yes	125	No	0.07
Yes	No	168	No	0.07
Yes	Yes	172	No	0.07

make a new sample with the same sample size with respect to sample weight

Chest Pain	Blocked Arteries	Patient Weight	Heart Disease
No	Yes	156	No
Yes	Yes	167	Yes
No	Yes	125	No
Yes	Yes	167	Yes
Yes	Yes	167	Yes
Yes	Yes	172	No
Yes	Yes	167	Yes
Yes	Yes	167	Yes

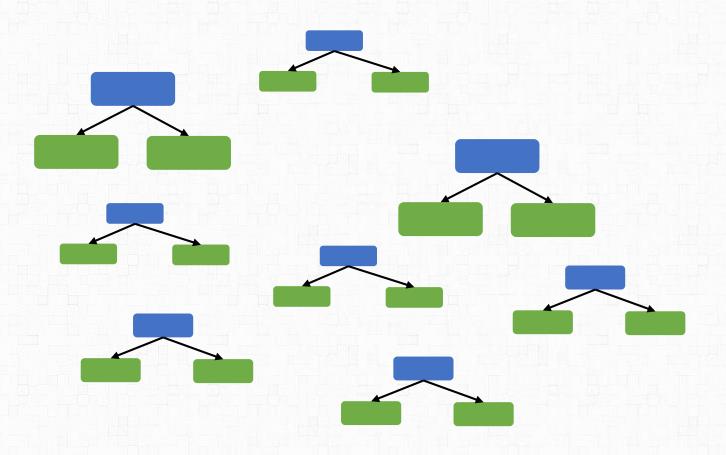


Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
No	Yes	156	No	1/8
Yes	Yes	167	Yes	1/8
No	Yes	125	No	1/8
Yes	Yes	167	Yes	1/8
Yes	Yes	167	Yes	1/8
Yes	Yes	172	No	1/8
Yes	Yes	167	Yes	1/8
Yes	Yes	167	Yes	1/8





Now we have a forest of stumps...

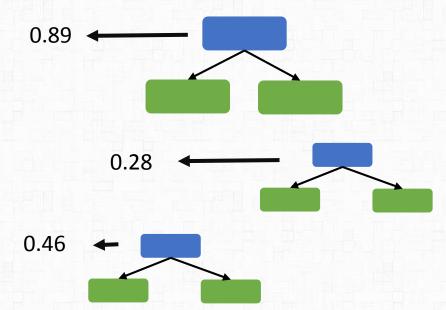




Heart Disease!

0.42 0.93 0.33 0.41 0.39

No Heart Disease!

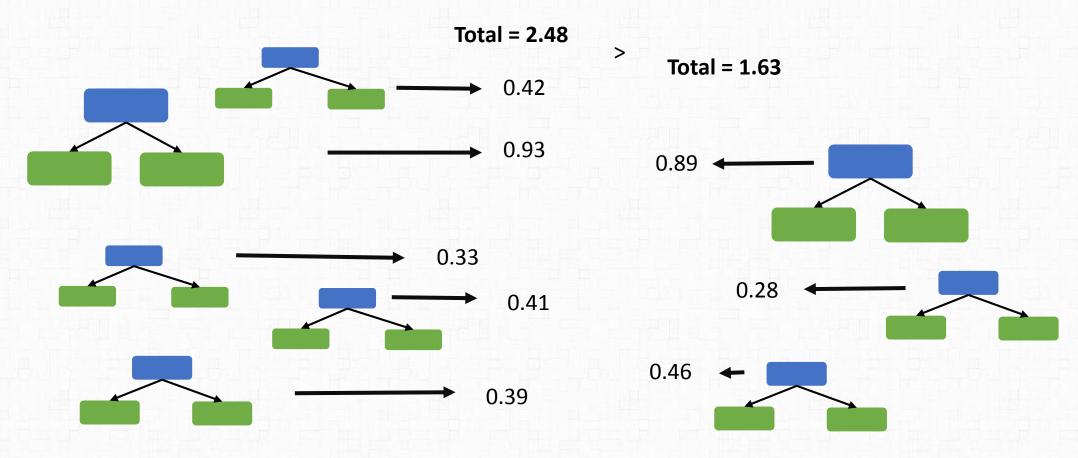




Final Call The patient has Heart Disease!

Heart Disease!

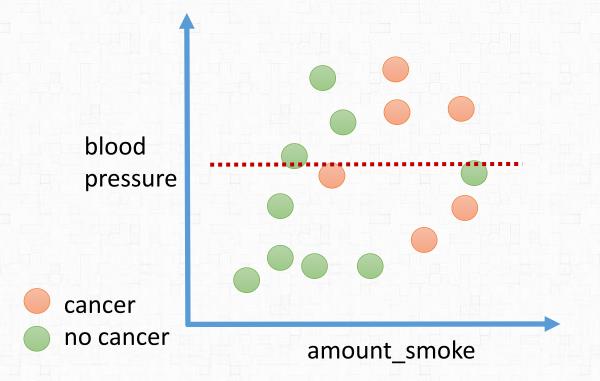
No Heart Disease!

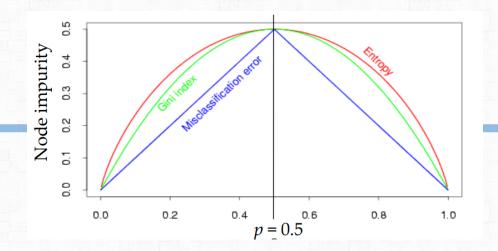


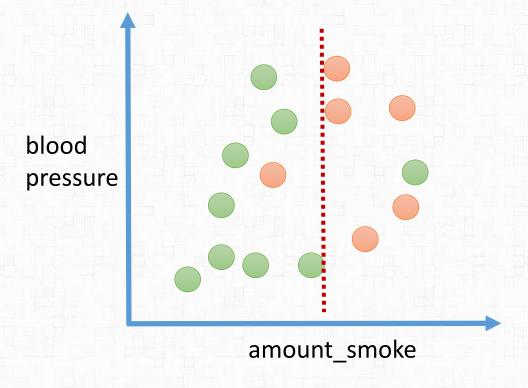


Gini Index (G. I.)

$$G.\,I(A) = \sum_{i=1}^d \left(R_i\left(1 - \sum_{k=1}^m p_{ik}^2
ight)
ight)$$

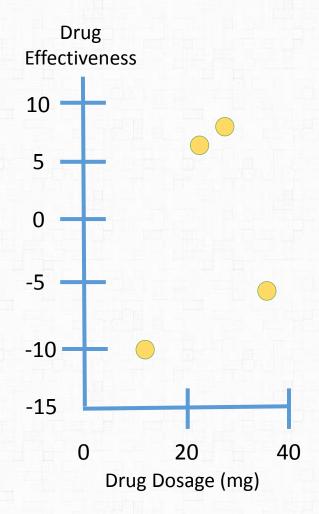




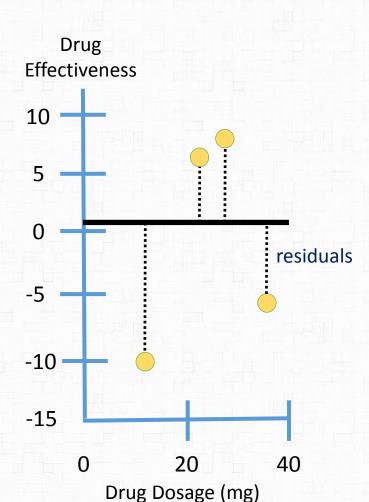




Initial Prediction: 0.5







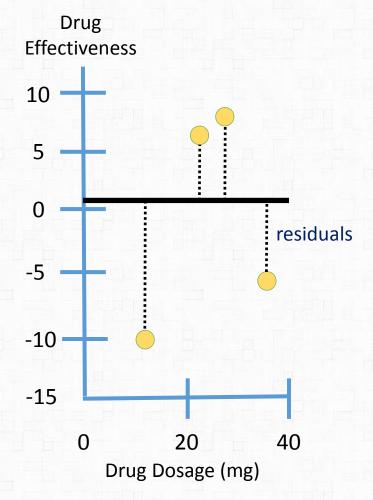
Initial Prediction: 0.5

-10.5, 6.5, 7.5, -7.5

From residuals,
We build the first **XGBoost Tree**







-10.5, 6.5, 7.5, -7.5

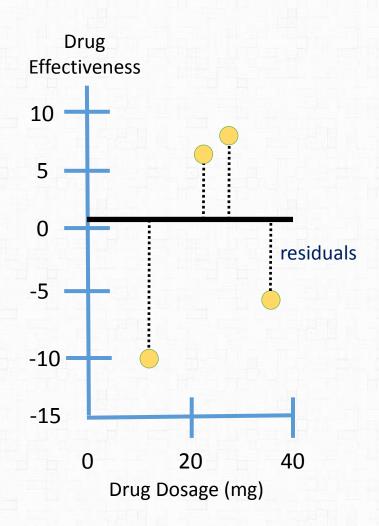
From residuals,
We build the first **XGBoost Tree**

 $Simliarity Score = \frac{Sum of Residual, Squared}{Number of Residuals + \lambda(lambda)}$

 λ is a regularization term (to avoid overfitting) Let's assume $\lambda = 0$



Initial Prediction: 0.5

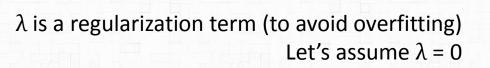


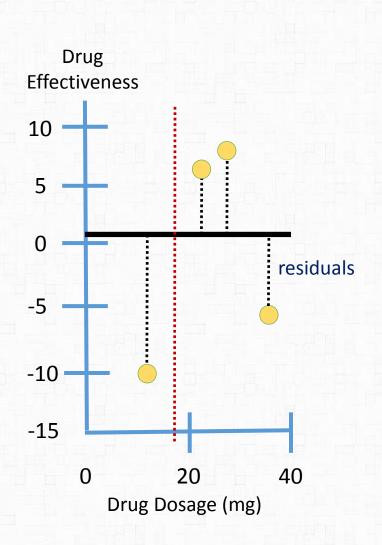
-10.5, 6.5, 7.5, -7.5

Simliarity = 4

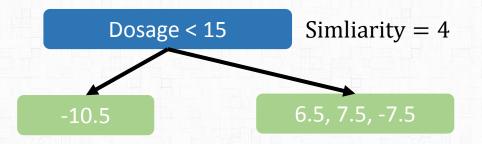
From residuals,
We build the first **XGBoost Tree**

Similarity Score =
$$\frac{(-10.5 + 6.5 + 7.5 + -7.5)^{2}}{4 + 0}$$
$$= \frac{(-4)^{2}}{4 + 0} = 4$$







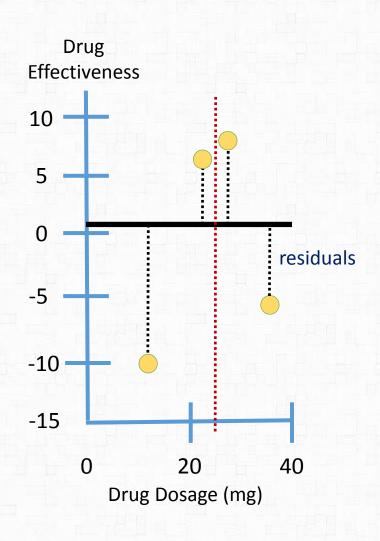


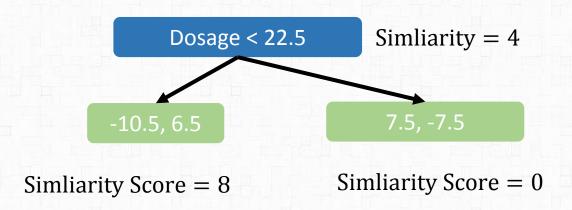
Simliarity Score =
$$\frac{(-10.5)^2}{1+0}$$

= 110.25 Simliarity Score = $\frac{(6.5+7.5+-7.5)^2}{3+0}$
= 14.08

$$Gain = Leftsimiliar_{ity} + Rightsi_{miliarity} - Rootsi_{miliarity} = 120.33$$

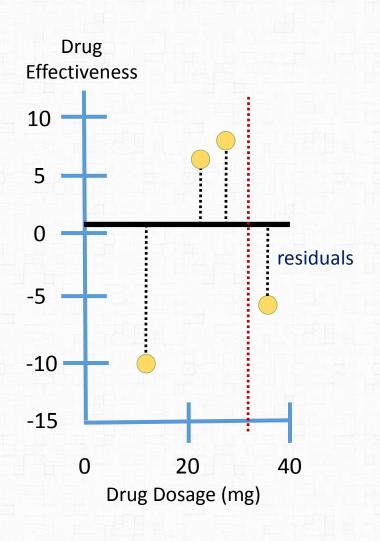


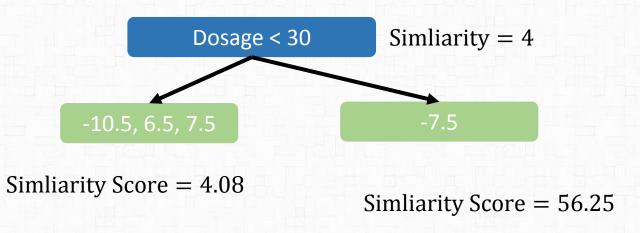




Gain =
$$8 + 0 - 4 = 4$$

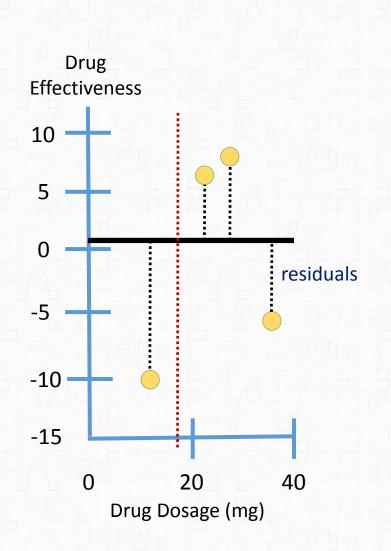


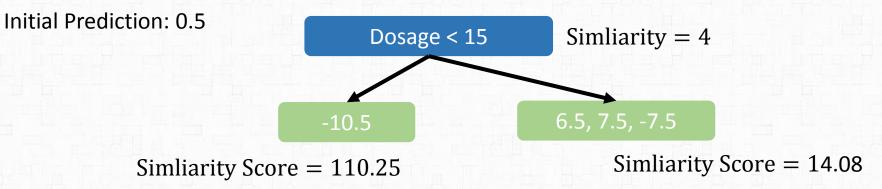




$$Gain = Leftsimiliar_{ity} + Rightsi_{miliarity} - Rootsi_{miliarity} = 56.33$$

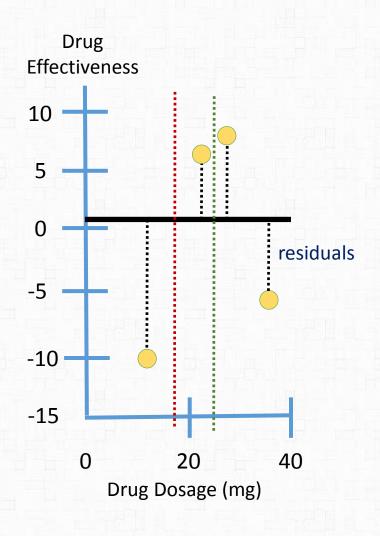


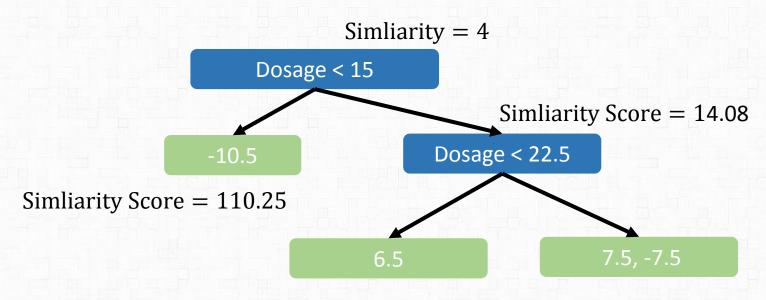




Largest Gain, 120.33



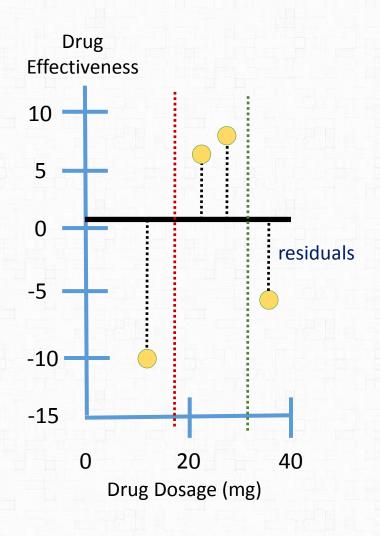


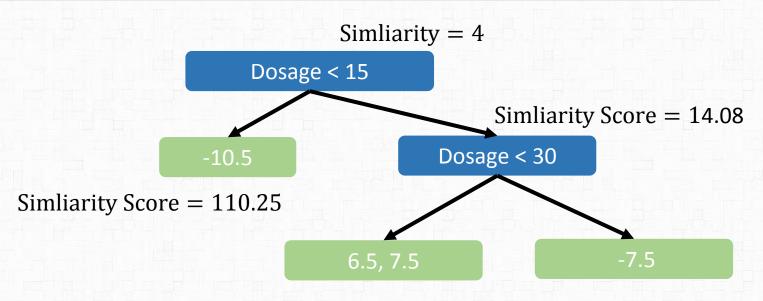


Simliarity Score = 42.25 Simliarity Score = 0

Gain = Leftsimiliar_{ity} + Rightsi_{miliarity} - Rootsi_{miliarity} =
$$42.25 + 0 - 14.08 = 28.17$$



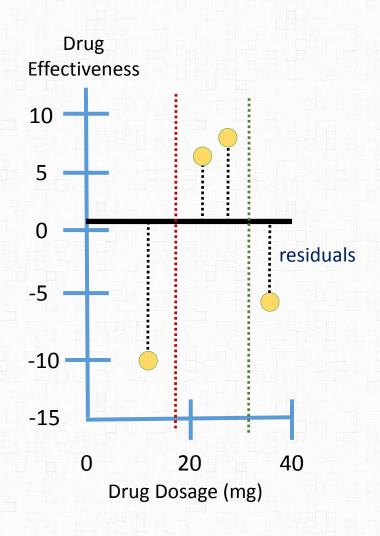


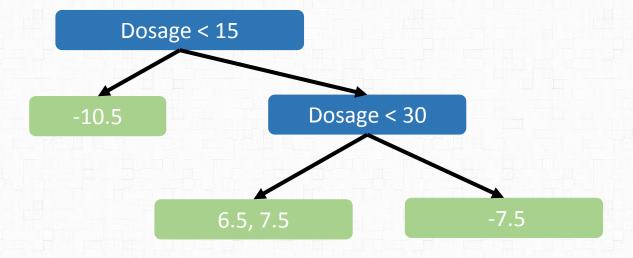


Simliarity Score = 98 Simliarity Score = 56.25

Gain =
$$98 + 56.25 - 14.08 = 140.17$$



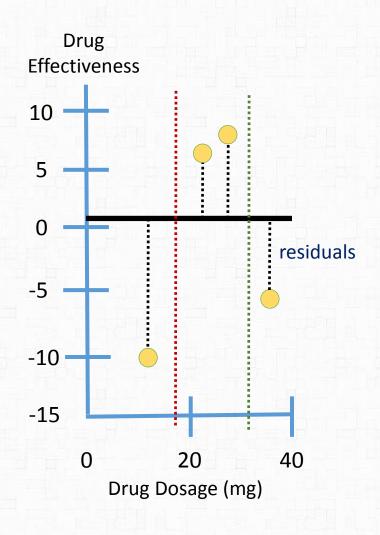


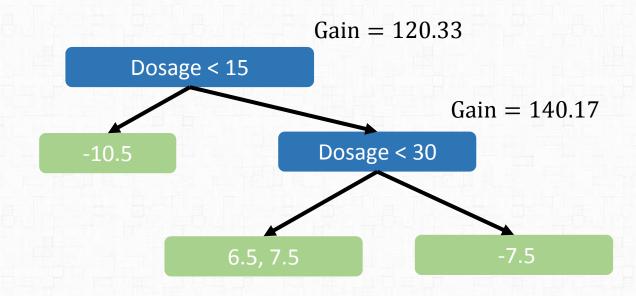


Limiting max depth of XGBoost Tree as 2, We stop branching out the tree here.

By default, max depth is 6 levels



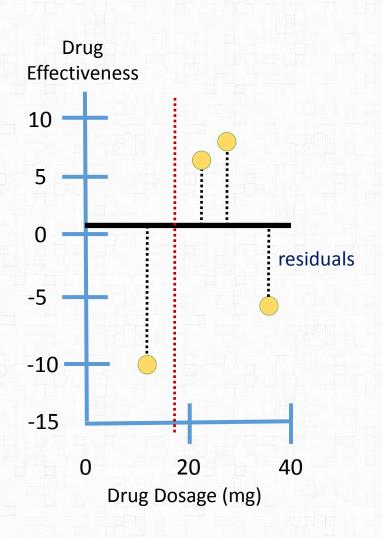


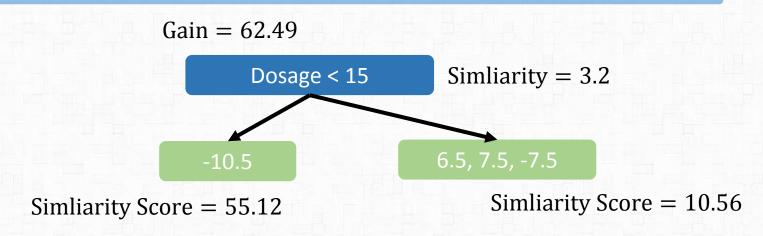


We prune the tree comparing Gain and γ (gamma) (starting from leaf nodes) γ is threshold for Gain when the max gain is smaller than γ , we prune that branch



Regularization term – λ



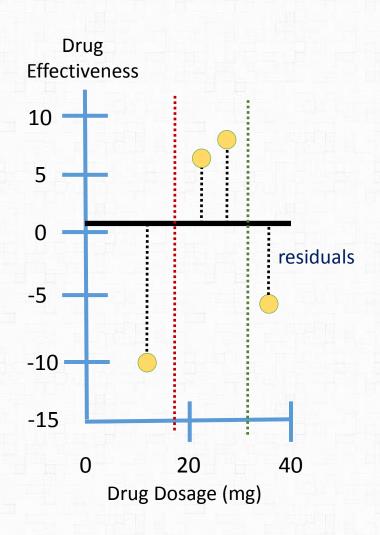


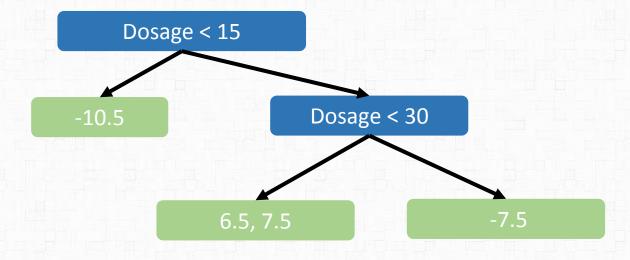
Now let's think about $\lambda > 0$ Let's assume $\lambda = 1$

For the smaller number of residuals, it decrease more of similarity score

Since it gives smaller Gain, Tree gets pruned more easily



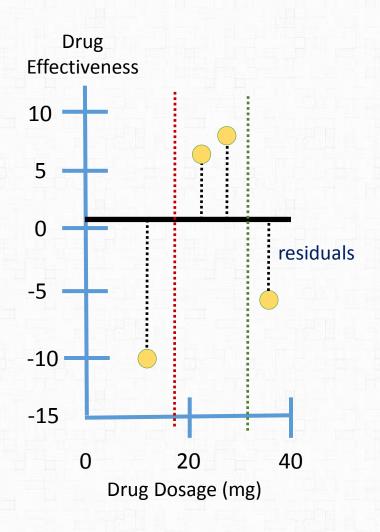


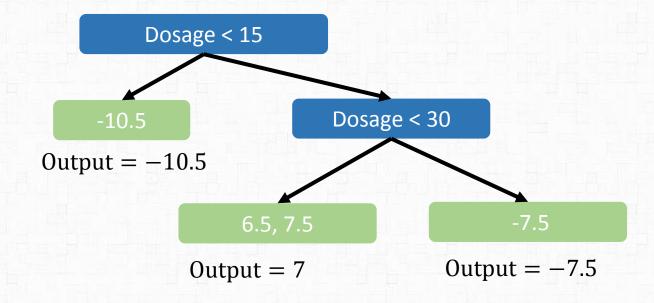


Output Value =
$$\frac{\text{Sum of Residual}}{\text{Number of Residuals} + \lambda}$$

$$Simliarity Score = \frac{Sum of Residual, Squared}{Number of Residuals + \lambda}$$







Output Value =
$$\frac{\text{Sum of Residual}}{\text{Number of Residuals} + \lambda}$$

$$Simliarity Score = \frac{Sum of Residual, Squared}{Number of Residuals + \lambda}$$



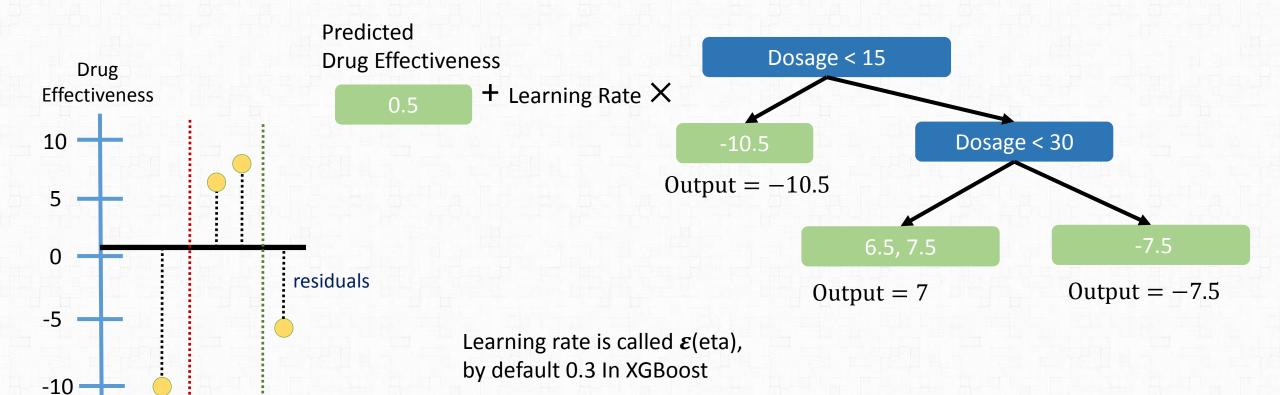
-15

0

20

Drug Dosage (mg)

40





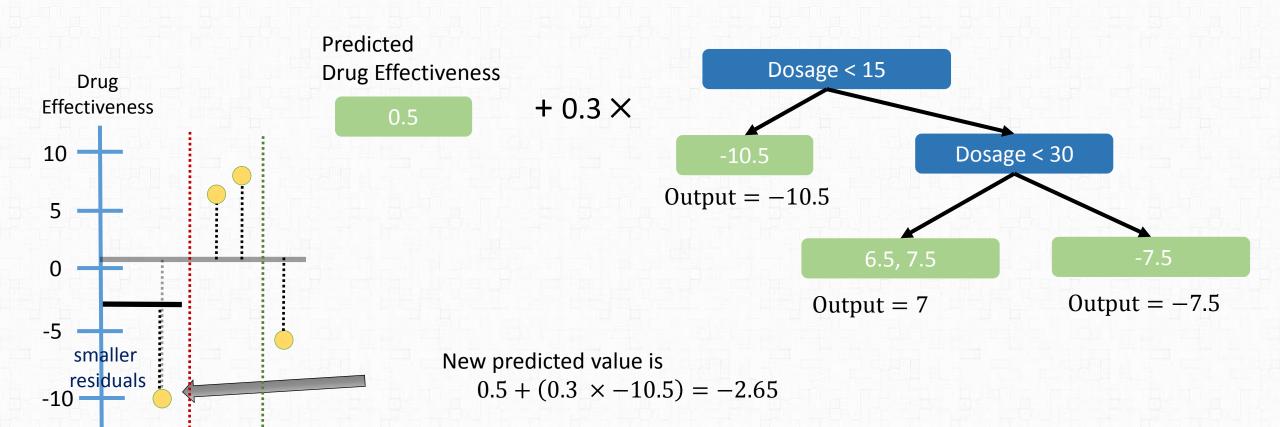
-15

0

20

Drug Dosage (mg)

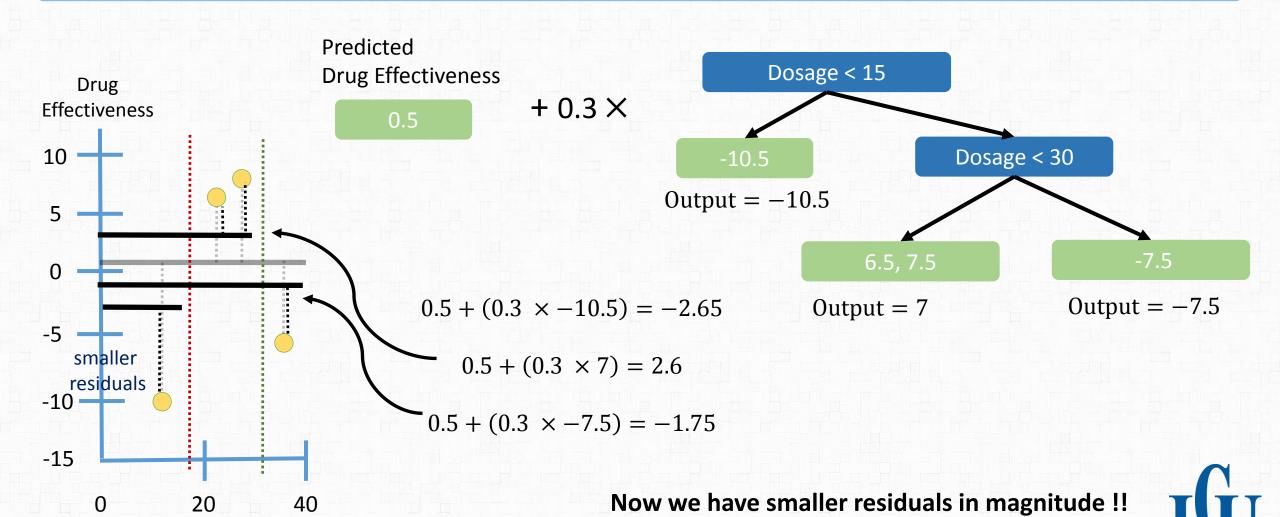
40

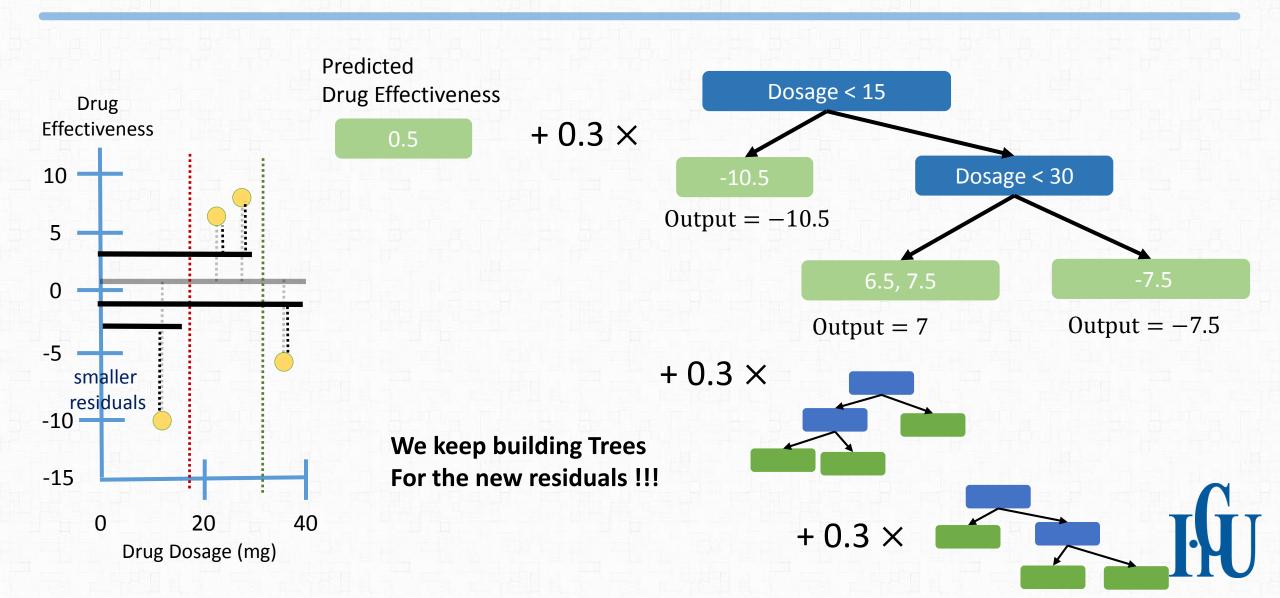


Learning rate is called $oldsymbol{arepsilon}$ (eta), by default 0.3 In XGBoost



Drug Dosage (mg)





What to do next

- XGBoost for Classification, https://youtu.be/8b1JEDvenQU
- Mathematical details, https://youtu.be/ZVFeW798-2I
- Properties, https://youtu.be/oRrKeUCEbq8



References

STATQUEST, https://statquest.org/

