EPFL

Fin404 Derivatives

Master in Financial Engineering

Spring 2022

Project 1: Convertible Bonds

The goal of this homework is to study convertible bonds (CB) and their variants which constitute a flexible source of financing for corporations. Throughout the project you are asked to produce various deliverables that are as documents for potential clients of the investment bank where you work.

Part 1. Documentation

Provide a basic text presentation (maximum 5 pages) of convertible bonds. This part of your report is meant as an introductory briefing and thus should not include equations or technical arguments, but it can include numbers and graphics if necessary.

You are free to structure and organize your presentation as you see fit. However, your report should at the minimum address the following points:

1. Definition and structure of the product(s): convertible bonds, callable convertible bonds, mandatory convertibles, contingent convertibles, exercize priorities, etc...
2. Why do such products exist? What do they allow investors to achieve? What do they allow issuing compagnies to achieve?
3. Capitalization, number of issuances/year, location, past performance, etc...
4. From the point of view of an investor: What are the benefits and drawbacks of the callable convertible bond relative to the plain vanilla convertible and to the mandatory convertible bond?

5. From the point of view of a company seeking financing: What are the benefits of callable convertible bonds relative to i) standard bonds, ii) standard convertibles and iii) mandatory convertible bonds.

Part 2. Preliminary analysis

Assume that time is indexed by $t \in \{0, 1, ..., T\}$ where t represents the number of elapsed periods of length Δ since the issuance of bond and T represents the total number of periods between the issuance and the maturity of the bond.

Denote by S_t the *ex-dividend* price of one share of stock of the issuer and consider first a *mandatory* convertible bond (mCB) with the following characteristics:

- Face value F
- Coupons $c \times F$ at dates $\mathscr{C} = (T_j)_{j=1}^N$
- Mandatory conversion to M_T shares of stock at $T_N = T$ where

$$M_T = \begin{cases} \alpha & \text{if } \alpha S_T < F \\ F/S_T & \text{if } F/\alpha \leq S_T \leq F/\beta \\ \beta & \text{if } \beta S_T > F \end{cases}$$

  for some constants $\alpha \geq \beta$.

1. Describe the position of the holder of the mCB as a portfolio that includes a coupon bond and a combination of European stock options.

Now consider a plain vanilla (i.e. non callable) convertible bond (CB) with the following characteristics:

- Face value F
- Coupons $c \times F$ at dates $\mathscr{C} = (T_j)_{j=1}^N$ until conversion
- Principal repayment at date $T_N = T$ unless the bond is converted
- Conversion to stock possible at every coupon date T_j *after* the payment of the coupon but *before* the principal repayment if $T_j = T$.
- Conversion factor $\gamma \geq 0$ so that the bond becomes γ shares of company stock upon exercize of the option to convert.

2. Describe the (pre-conversion) position of the holder of the CB as a portfolio that includes a standard bond and an American derivative to be specified.

3. Assume that markets are arbitrage-free and complete. Provide a formula for the initial prices of the *m*CB and the CB in terms of expectations under the EMM and the value function of an optimal stopping problem to be specified.

Finally, consider a callable convertible bond (*c*CB) that has the same characteristics as the plain vanilla CB but in which the issuer has the option to buy back the bond at a fixed price F_c at each coupon date $T_j \in \mathscr{C}$ after the coupon payment but before the principal repayment if $T_j = T$. Assume that when the issuer calls the bond, the holder retains the option to immediately convert and thus decides whether to receive the call price or shares of stock.

4. Assume that markets are arbitrage-free and complete. Denote by B_t the value of the *c*CB at date t after the payment of the coupon (if any) but before any call or conversion decision, and by

$$C_t = e^{-r_{t+1}\Delta} E_t^{\mathbb{Q}} \left[B_{t+1} + \mathbf{1}_{\{t+1 \in \mathscr{C}\}} cF \right]$$

the *continuation* value of the *c*CB at date $t \in \{0, 1, \dots, T\}$, that is the value of the bond assuming that neither conversion nor call happens at or before date t. Show that the issuer calls the bond at date t if and only if

$$C_t \geq \max\{\psi S_t, F_c\}$$

for some constant $\psi \geq 0$ to be determined.

5. Show that if $t \in \mathscr{C}$ then

$$B_t = \begin{cases} \max\{\psi S_t, F_c\}, & \text{if } C_t \geq \max\{\psi S_t, F_c\} \\ \max\{\psi S_t, C_t\}, & \text{otherwise.} \end{cases}$$

where the constant $\psi \geq 0$ is the same as in the previous question. What is the corresponding formula when $t \notin \mathscr{C}$ is not a coupon date?

Part 3. Valuation code

Use your preferred programming language to write a code that *takes as input* the parameters of a binomial model:

- The number of periods T
- The length Δ of the period
- The annualized interest rate r
- The initial price of the stock S_0
- The set $\mathscr{D} \subseteq \{1, \dots, T\}$ of dividend dates
- The dividend yield δ on dividend dates
- The up and down factors U and D

and the characteristics of the convertible bond:

- The bond type: CB, *m*Cb, or *c*CB
- The face value F
- The coupon dates $\mathscr{C} = (T_i)_{i=1}^N$ expressed in number of periods
- The coupon cF to be paid at each coupon date
- The conversion thresholds $\alpha \geq \beta$ for a *m*CB
- The conversion factor γ for a CB or *c*CB
- The call price F_c for a *c*CB

and *which outputs* the arbitrage-free price of the bond at the initial date of the tree as well as a description of the optimal call and conversion strategies.

Remark 1. Do not forget to properly adapt the model for the fact that the stock of the issuing company pays dividends. If dividends are present, assume that they are paid as a fraction of the cum-dividend price as in the first model of Lecture 6 and that shares received upon conversion are ex–dividend.

Remark 2. We are well aware of the availability of many codes that can be used to do precisely what is asked above. *Please do not copy these codes!* Make sure to

Maturity: 1 Year		
Strike	Call	Put
100	74.7058	0.000819015
110	65.2053	0.0125373
120	55.7561	0.0756443
130	46.4273	0.259109
140	37.5962	0.940308
150	29.4715	2.32795
160	21.9643	4.33301
170	16.1352	8.01623
180	11.1915	12.5849
190	7.5083	18.4139
200	4.85748	25.2754
210	3.01355	32.9438
220	1.76178	41.2043
230	1.0674	50.0222
240	0.502573	58.9697
250	0.319108	68.2985

Table 1: Johnson & Johnson *European* option prices

tell us which language you are using and try to think about Oliver–friendliness when preparing your code. A computer science concept that you may find useful to efficiently compute prices in trees is memoization (no typo).

Part 4. Model calibration

Assume that today is January 1 and that the stock underlying the convertible bond is Johnson & Johnson (JNJ) which is currently worth $S_0 = 175$ ex-dividend. Assume that the annualized interest rate r will remain constant over the life of the bond and that Johnson & Johnson pays dividends on January 1 of each year as a fraction δ of its cum-dividend price.

1. Show that the present value of the dividends paid by the stock over the next $n \in \mathbb{N}$ years is $S_0(1 - \eta^n)$ for some constant η to be determined.

   Hint. Use an argument similar to the one we saw in Lecture 1 but adapted to discrete rather than continuous-time.

2. Use the data in Table 1 (also available in the file `Data-Project1-Fin404.dat`) and the previous result to estimate the values of r and δ.

3. Use the data in Table 1 and the previous results to calibrate a Binomial model with $5 \times 12 = 60$ periods of length one month under the constraint that over one period a price increase is as likely as a price decrease under the EMM.

Part 5. Analysis

Consider the following base case parameters for 5Y different types of convertible bonds issued by JNJ:

- Face value $F = 1000$
- Coupons every 6 months
- Coupon: $cF = 0.02F = 20$
- Conversion thresholds for the *m*CB: $\alpha = 1000$ and $\beta = 4$
- Conversion factor for the CB and *c*CB: $\gamma = 4$
- Call price for the *c*CB: $F_c = 1.05F = 1050$

1. Use your code and the calibrated model to compute the initial price of the CB, the *m*CB, and the *c*CB

2. Describe the optimal exercize policy of the CB holder at the first, second, and third coupon dates

3. Describe the optimal exercize policy of the issuer and the *c*CB holder at the first, second, and third coupon dates. Compare with the results of the previous question and interpet the difference(s).

4. Illustrate graphically how the prices of the three bonds depend on the coupon c, the conversion factor $\beta = \gamma$, and the call price F_c.

5. Use your code to produce a function that computes the par coupon of the bonds taking the other parameters as given, i.e. solve for the values of c so that each bond is initially worth F. Use a bar chart to graphically illustrate how the par coupon rates of the three bonds depend on the conversion factor $\beta = \gamma \in \{1, 2, 3, 4, 5, 6\}$

and how they differ from the par coupon of a standard coupon bond with five year maturity and semiannual coupons. How do you interpret the results?

6. Use your code to produce a function that computes the par conversion factor of the CB and *c*CB taking the other parameters as given, i.e. solve for the values of γ so that each bond is initially worth F. Illustrate graphically how the par conversion factors depend on the coupon c and the call price in the case of the *c*CB. How do you interpret the results?

7. Produce a one page term-sheet that summarizes the conditions your desk can offer JNJ for different types of par-valued 5Y convertible bonds. Briefly compare the different possibilities with a focus on the induced risks for investors. This term-sheet is meant to be sent to clients as a complement to the presentation you created in Part 1 and as such should be as non technical as possible.

Guidelines

1. The project is to be done in groups of **at least 2 and no more than 4 students** without communications between the groups.
2. The name and sciper number of each member of the group should appear clearly on the final report.
3. The final report should be typeset on no more than 15 A4 pages in 12pt font with 1.5cm margins and standard linespacing.
4. The code should be ready to run in one or more standalone files. If there are multiple files please also provide a readme with clear instructions.
5. The code should be commented with clear explanations for each line/block. The comments can be either in the code itself (preferred option) or written separately.
6. For the coding parts your are not allowed to use any external libraries or built-in functions for financial engineering tasks. Please develop your own.
7. Your code and the final report should be uploaded on the moodle.
8. Only one final report and one code archive per group should be uploaded. No need to also send your work by email.

9. Make sure to cite your sources and to not simply copy existing material. Your work will automatically be submitted to an antiplagiarism software.