This collection of exam questions and solutions is intended to help you study for this semester's Math 234 courses.

Chapter 1

2025 Spring Exams

1.1 Math234Sp25-Midterm1

1. Let $\vec{a} = \langle 2, 1, -3\rangle$ and $\vec{b} = \langle 4, -1, -2\rangle$.

(a) Find a vector of length 2 that points in the opposite direction of $\vec{a}$.

(b) Find the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.

2. Find the equation of a line that is perpendicular to the plane $2x - 3y + 7z = 12$ and that contains the point $(8, 9, -2)$.

3. Find an equation for the plane that contains the points $A = (2, -1, 3)$, $B = (1, 2, -1)$ and $C = (0, 4, 3)$.

4. Consider the figure shown. In this picture, $P = (1, 0, 0)$, $Q = (1, 6, 0)$, and $R = (2, 4, 3)$.

(a) Compute the vector $\overrightarrow{PR}$. Write your answer in the provided answer box.

(b) Compute the vector $\overrightarrow{PQ}$. Write your answer in the provided answer box.

(c) Compute the vector $\overrightarrow{PM}$ (that is, compute the projection of the vector $\overrightarrow{PR}$ onto the vector $\overrightarrow{PQ}$). Write your answer in the provided answer box. You WILL be graded for your work, so make sure to justify your answer.

(d) Compute the vector $\overrightarrow{MR}$. Write your answer in the provided answer box. You WILL be graded for your work, so make sure to justify your answer.

5. Consider the surface $y = x^2 + 9z^2$.

(a) Sketch the traces in the vertical planes $x = k$, with $k = 0, 1$. Label your traces.

(b) Sketch the traces in the vertical planes $y = k$, with $k = 0, 1$. Label your traces.

6. Consider the curve $\vec{r}(t) = \langle 3t - 2, \cos(\pi t), t^2 + 1\rangle$.

(a) Write down an integral that represents the length of the curve from the point $(4, 1, 5)$ to the point $(7, -1, 10)$. You do NOT need to compute the integral.

(b) Find an equation for the tangent line to the curve at the point $(4, 1, 5)$.

7. Consider the surfaces $z = 2x + 2$ and $z = x^2 + y^2 - 1$. Find a vector function that represents the curve of intersection of the given surfaces.

8. Consider the curve given by $\vec{r}(t) = \langle 3\cos(t), 4t, 5 - 3\sin(t)\rangle$

(a) Find the unit tangent vector, $\vec{T}(t)$.

(b) Find the unit normal vector, $\vec{N}(t)$.

(c) Find the curvature, $\kappa(t)$.

9. An object's velocity is given by

$$\vec{v}(t) = \langle 6t^2 + 5, 2 + 3\sqrt{t}, 4e^{2t}\rangle.$$

Its initial position is $\vec{r}(0) = \langle 2, \pi, 0\rangle$. Find a vector equation $\vec{r}(t)$ for the position of the object at time t.

10. Consider the function $f(x, y) = \dfrac{\sqrt{x - y}}{\ln(2 + x^2 + y^2)}$.

(a) Find the domain of the function.

(b) Sketch the domain of the function. Shade the region(s) that are part of the domain. Use dotted lines when sketching a curve that is NOT in the domain, and solid lines when sketching curves that are.

11. If the limit exists, compute it (making sure to justify your answer). If the limit does not exist, justify why it doesn't exist. If you use a theorem, state clearly which theorem you are using. Make sure to use correct limit notation.

$$\lim_{(x,y)\to(0,0)} \frac{4xy^3}{2x^4+3y^4}$$

1.2 Math234Sp25-Midterm2

1. Let
$$f(x,y) = e^{2x+3y} + \frac{x^2y^3}{12} + 6.$$

(a) Find the tangent plane to f at $(3, -2)$.

(b) Use the tangent plane to f at $(3, -2)$ to approximate the value of $f(2.9, -2.1)$.

2. Let $z(x, y)$ be defined implicitly by
$$x^2z^2 + 2yz^3 = x^2e^y + 2z$$
Compute $\frac{\partial z}{\partial x}$.

3.

(a) Suppose $z = f(s, t)$ where s and t are functions of x, y and u. Write an expression for $\frac{\partial z}{\partial u}$ using the Chain Rule.

(b) Let $z = f(s, t)$, where $s = x + 2y + 3u$, $t = 4xy^2u^3$. Suppose $\frac{\partial z}{\partial s} = 3s^2 + 2t$, and $\frac{\partial z}{\partial t} = 2t + 2s$. Compute $\frac{\partial z}{\partial u}$ when $x = 1$, $y = -1$ and $u = 2$.

4. Let $f(x, y) = x^2y + \cos(\pi xy)$. Find the directional derivative of the function $f(x, y)$ at the point $(1, 2)$ in the direction of the point $(4, -3)$. That is, find the directional derivative at $(1, 2)$ as a particle is moving from $(1, 2)$ to $(4, -3)$.

5. Consider the function
$$f(x, y) = 2x^2 - 8xy + y^4 - 4y^3.$$

(a) Find all critical point(s) of $f(x, y)$.

(b) Classify each critical point you found in (a) as a local maximum, local minimum, or saddle point.

6. Find the absolute maximum and minimum values of the function $f(x, y) = y^2 - 4x^2$ subject to the constraint $x^2 + 2y^2 = 4$.

7. Compute the following integrals.

(a) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dx\, dy$

(b) Compute $\displaystyle\int_{-4}^0 \int_{-\sqrt{16-x^2}}^0 \cos(2x^2 + 2y^2)\, dy\, dx$

8. Write down triple integrals that represent the volume of the following solids. You do **NOT** need to evaluate the integrals.

(a) The solid bounded above by the cone $z = \sqrt{x^2 + y^2}$, below by the cone $z = -\sqrt{x^2 + y^2}$, and enclosed between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 7$.

(b) The solid in the first octant that lies within the cylinder $x^2 + y^2 = 4$, above the paraboloid $z = x^2 + y^2 + 1$, and below the plane $z = 9$.

9. Consider the solid in the first octant that is bounded by the coordinate planes and the plane $4x + 2y + 6z = 12$.

(a) Write down an integral that represents the volume of the solid.

(b) Compute the volume of the solid by computing the triple integral found in part (a).

1.3 Math234Sp25-FinalExam

1. Compute the following integrals. If you use a theorem, clearly state which theorem you are using.

(a) $\int_{\mathcal{C}} xy\,dy$, where $\mathcal{C}$ is the portion of the parabola $y = x^2 + 1$ with $0 \leq x \leq 1$, traversed starting at $(0,1)$ and ending at $(1,2)$.

(b) $\int_{\mathcal{C}} (x^3 + 3y)\,dx + (6x - \sin(y^2))\,dy$ where C is the rectangle with corners at $(0,0)$, $(2,0)$, $(2,1)$ and $(0,1)$ traversed counterclockwise.

(c) $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where $\vec{F}$ is the conservative vector field given by $\vec{F}(x) = \langle 2xy, x^2 \rangle$, and $\mathcal{C}$ is the circle of radius 3 centered at the point $(-1,0)$, traversed counterclockwise.

(d) $\int_0^8 \int_{y^{1/3}}^2 \sin(x^4)\,dx\,dy$

2. An object's acceleration is given by

$$\vec{a}(t) = \langle e^{3t} - 2, t + \sin(t), t^2 \rangle.$$

Its initial velocity is $\vec{v}(0) = \langle 3, 4, 2 \rangle$. Find a vector equation $\vec{v}(t)$ for the velocity of the object at time t.

3. Let $z(x,y)$ be defined implicitly by

$$4x^3y^2 + 3z^2x = \cos(z^2) + 4yz^3$$

Compute $\frac{\partial z}{\partial y}$.

4. Use Stoke's Theorem to compute $\int_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F}(x,y,z) = \langle e^{x^8} + 4y, \sqrt{y^4 + 1} + 2x, 2y + \sin(z^4) \rangle,$$

and C is the boundary of the plane $4x + 3y + z = 12$ in the first octant, oriented counterclockwise when viewed from above.

5. Consider the vector field $\vec{F}(x,y) = \langle 2xe^{x^2-y} + 2y + 4, -e^{x^2-y} + 2x - 3 \rangle$.

(a) Show that $\vec{F}$ is conservative.

(b) Find a potential function for $\vec{F}$.

6. Compute $\iint_S \vec{F} \cdot d\vec{S}$, for the vector field

$$\vec{F}(x,y,z) = \langle 2x + 3y + 4\cos(z), -x + y + e^{xz}, \sin(x^2) + e^{3y} - 2z \rangle,$$

outward through the surface S of the box

$$E = \{(x,y,z) | -3 \leq x \leq 0, -2 \leq y \leq 1, 1 \leq z \leq 6\}.$$

If you use a theorem, clearly state which theorem you are using.

7. Let $\vec{r}(t) = \langle 5\cos(t), 2 + 3\sin(t), 4\sin(t) + 1 \rangle$.

(a) Compute the unit tangent vector at $t = \pi$, that is, $\vec{T}(\pi)$.

(b) Compute the unit normal vector at $t = \pi$, that is, $\vec{N}(\pi)$.

(c) Compute the curvature at $t = 1$, that is, $\kappa(\pi)$.

8. Consider the function

$$f(x,y) = x^3 - 12xy + 8y^3.$$

(a) Find all critical point(s) of $f(x,y)$.

(b) Compute the second derivatives of the function and use them to write down a formula for $D(x, y)$. You do NOT need to evaluate D at the points found on part (a) for this part.

(c) Classify each critical point you found in (a) as a local maximum, local minimum, or saddle point.

9. Compute

$$\iint_R (x-y)^2 \, dA,$$

where R is the region in the xy-plane bounded by the lines

$$x - y = 0, \quad x - y = 2, \quad x + y = 2, \quad x + y = 4,$$

by applying the transformation $u = x - y$ and $v = x + y$.

10.

(a) Find an equation for the cone $z = \sqrt{\frac{x^2}{3} + \frac{y^2}{3}}$ in spherical coordinates.

(b) Write

$$\iiint_E (2y + 3z) dV$$

as an integral in spherical coordinates. Here, E represents the solid that lies below the cone $z = \sqrt{x^2 + y^2}$, above the cone $z = \sqrt{\frac{x^2}{3} + \frac{y^2}{3}}$, and within the sphere $x^2 + y^2 + z^2 = 4$. You do **NOT** need to evaluate the integrals.

11. Evaluate

$$\iint_S (x^2 + y^2) \, dS,$$

where S is the portion of the cylinder $x^2 + y^2 = 4$ bounded between the planes $z = 0$ and $z = 5$.

Chapter 2

2024 Fall Exams

2.1 Math234Fa24-Midterm1

1. Let $\vec{a} = \langle 2, -3, 1\rangle$ and $\vec{b} = \langle 1, 4, -2\rangle$.

(a) Find the unit vector in the direction of $\vec{a}$.

(b) Find the vector projection of $\vec{b}$ onto $\vec{a}$.

2. Find an equation for the plane that contains the points $A = (1, 0, 1)$, $B = (3, 2, 0)$ and $C = (4, 1, 2)$.

3. Find an equation for the **line segment** connecting the points $A = (1, 4, 2)$ and $B = (5, 3, 8)$.

4. Consider the surface $x^2 = 4y^2 + z^2$.

(a) Sketch the traces of the form $x = k$, with $k = 0, 1$ in the yz-plane where the y axis is horizontal. Label your traces.

(b) Sketch the traces of the form $y = k$, with $k = 0, 1$ in the xz-plane where the x axis is horizontal. Label your traces.

5. If the limit exists, compute it (making sure to justify your answer). If the limit does not exist, justify why it doesn't exist. If you use a theorem, state clearly which theorem you are using. Make sure to use correct limit notation.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{2x^2 + y^4}$$

6. At what point(s) does the curve $\vec{r}(t) = \langle 4t^2 + 6t, 0, t\rangle$ intersect the paraboloid $x = y^2 + z^2$? Make sure to give the coordinate(s) of the point(s) in your final answer.

7. Consider the curve given by $\vec{r}(t) = \langle t, t^2, 2t + t^3\rangle$.

(a) Find a tangent vector to the curve at the point $(2, 4, 12)$.

(b) Write down an integral that represents the length of the curve, with $1 \le t \le 2$. You do NOT need to evaluate the integral.

(c) Find an equation for the tangent line to the curve at the point $(2, 4, 12)$.

(d) Find an equation for the normal plane to the curve at the point $(2, 4, 12)$. That is, find an equation for the plane that is perpendicular to the tangent line to the curve at the given point.

8. Consider the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 2+x$. Find a vector function that represents the curve of intersection of the given surfaces.

9. Consider the curve given by $\vec{r}(t) = \langle 3 + 2\sin(t), 2t, 2\cos(t)\rangle$

(a) Find the unit tangent vector, $\vec{T}(t)$.

(b) Find the unit normal vector, $\vec{N}(t)$.

(c) Find the curvature, $\kappa(t)$.

10. Consider the function $f(x, y) = \dfrac{\cos(y + x)}{e^{x^2-y}\sqrt{y^3 - x}}$.

(a) Find the domain of the function.

(b) Sketch the domain of the function. Shade the region(s) that are part of the domain. Use dotted lines when sketching a curve that is NOT in the domain, and solid lines when sketching curves that are.

11. An object's acceleration is given by

$$\vec{a}(t) = \langle \sin(\pi t), e^{-t}, 2t + 3\rangle.$$

Its initial velocity is $\vec{v}(0) = \langle 1, 0, 2\rangle$. Find a vector equation $\vec{v}(t)$ for the velocity of the object at time t.

2.2 Math234Fa24-Midterm2

1. Let $f(x, y) = 4xy + \cos(\pi x) + y^3$.

(a) Find the tangent plane to f at $(2, -1)$.

(b) Use the tangent plane to f at $(2, -1)$ to approximate the value of $f(2.1, -0.9)$.

2. Suppose $f(u, v)$ is differentiable and $u = x - y$ and $v = y - x$. Find

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}.$$

$w = 1$, $x = 2$ and $y = -1$.

3. Let $f(x, y) = e^{x^2 - y} + 2\sqrt{y}$.

(a) Find the directional derivative of the function $f(x, y)$ at the point $(2, 4)$ in the direction of the point $(3, -1)$. That is, find the directional derivative at $(2, 4)$ as a particle is moving from $(2, 4)$ to $(3, -1)$.

(b) Based on your answer from (a), is the function $f(x, y)$ increasing (that is, is the value of the function getting larger), or decreasing (that is, is the value of the function getting smaller) in that direction at the point $(2, 4)$?

A. Increasing

B. Decreasing

C. Staying constant

D. Cannot be determined with the information provided

(c) In what direction does f have the maximum rate of change at the point $(2, 4)$? Write your answer as a **unit** vector. Make sure to write your final answer in the provided answer box. Answer:

(d) What is the maximum rate of change of f at $(2, 4)$? Make sure to write your final answer in the provided answer box. Answer:

4. Consider the function

$$f(x, y) = 3x^2 y + y^3 - 3y.$$

(a) Find all critical point(s) of $f(x, y)$.

(b) Classify each critical point you found in (a) as a local maximum, local minimum, or saddle point.

5. Find the absolute maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $x^2 + 4y^2 = 2$.

6. Compute the following integrals.

(a) $\displaystyle\int_{-2}^{1} \int_{0}^{y} x e^{y^3} \, dx \, dy$

(b) $\displaystyle\int_{0}^{1} \int_{y}^{\sqrt{2-y^2}} \sqrt{x^2 + y^2} \, dx \, dy$

(c) $\displaystyle\int_{0}^{\pi/2} \int_{x}^{\pi/2} \frac{\sin(y)}{y} \, dy \, dx$

7. Write down triple integrals that represent the volume of the following solids. You do **NOT** need to evaluate the integrals. Please note that this question has three parts, (a) and (b) on this page, and (c) on the next page.

(a) The solid in the first octant that is bounded by the coordinate planes and the plane $x + y + z = 3$.

(b) The solid that lies in the first octant, above the paraboloid $z = x^2 + y^2$, and below the paraboloid $z = 8 - x^2 - y^2$.

(c) The solid that lies below the cone $z = -\sqrt{x^2 + y^2}$, and between the spheres $x^2 + y^2 + z^2 = 9$ and

$x^2 + y^2 + z^2 = 16.$

2.3 Math234Fa24-FinalExam

1. Consider the curve $\vec{r}(t) = \langle 2t+1, \sin(\pi t), t^2-2\rangle$.

(a) Write down an integral that represents the length of the curve from the point $(1,0,-2)$ to the point $(5,0,2)$. You do NOT need to compute the integral.

(b) Find an equation for the tangent line to the curve at the point $(5,0,2)$.

2. Write

$$\iiint_E (x+2z)dV$$

as an integral in the specified coordinate system. Here, E represents the solid that lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=9$, with $x>0$. You do **NOT** need to evaluate the integrals.

(a) Cylindrical Coordinates

(b) Spherical Coordinates

3. Compute $\displaystyle\iint_S \operatorname{curl}(\vec{F})\cdot d\vec{S}$, where

$$\vec{F}(x,y,z) = \langle -y+2z-4, xe^{z-2}, \ln(z^2+4)\rangle$$

and S is the portion of the sphere $x^2+y^2+(z-2)^2=4$ that lies below the plane $z=2$, oriented inward (that is, the normal vector at the origin points in the direction of the positive z-axis). If you use a theorem, clearly state which theorem you are using.

4. Rewrite the double integral

$$\int_0^2\int_0^{2-x} \sqrt{y-3x}(x+y)^8\,dydx$$

as a double integral in u and v over an appropriate region by applying the transformation $u=x+y$ and $v=y-3x$. You do NOT need to evaluate the integral.

5. Consider the surface that is the part of the paraboloid $y=x^2+z^2$ that lies within the cylinder $x^2+z^2=4$.

(a) Write down a parametrization for the surface. Make sure to include bounds for your parameters.

(b) Find the surface area of the surface.

6. Compute the following integrals.

(a) $\displaystyle\iint_D e^{x^2+y^2}\,dA$ where the region D is between the curves $x^2+y^2=4$ and $x^2+y^2=16$, below the line $y=x$, and below the line $y=0$. where the region R is the region enclosed by the curves $x^2+y^2=4$, $x^2+y^2=16$, the line $y=x$ and the line $y=-x$, with $y<0$.

(b) $\displaystyle\int_0^1\int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dxdy$

(c) $\displaystyle\int_{\mathcal{C}} xy^2 ds$, where the curve $\mathcal{C}$ is the portion of the circle of radius 3 centered at the origin in the first quadrant traversed counterclockwise.

7. Consider the vector field $\vec{F}(x,y) = \langle 2xy+ye^{xy}, x^2+xe^{xy}+2y\rangle$.

(a) Show that $\vec{F}$ is conservative.

(b) Find a potential function for $\vec{F}$.

(c) Evaluate the integral $\displaystyle\int_{\mathcal{C}} \vec{F}\cdot d\vec{r}$, where $\mathcal{C}$ is parametrized by

$$\vec{r}(t) = \langle t\ln(5-t), t^2-3t\rangle,$$

with $0\le t\le 4$.

8. Compute $\iint_S \vec{F} \cdot d\vec{S}$, for the vector field

$$\vec{F}(x, y, z) = \langle \ln(y^2 + z^2 + 1) + 2x, y - e^{z^2 x}, \sin(xy) + x^2 y^3 \rangle,$$

outward through the surface S of the box

$$E = \{(x, y, z) | -2 \leq x \leq 1, -1 \leq y \leq 0, 0 \leq z \leq 4\}.$$

If you use a theorem, clearly state which theorem you are using.

9. Compute

$$\int_{\mathcal{C}} \left(e^{x^3}\sqrt{x} + 2y\right) dx + \left(-3x + \sin(y^2)\right) dy$$

where $\mathcal{C}$ is given by $\vec{r}(t) = \langle 2\cos(t), 2\sin(t) \rangle$ with $0 \leq t \leq 2\pi$. If you use a theorem, clearly state which theorem you are using.