CSE 3318 Notes 1: Algorithmic Concepts

(Last updated 8/1/22 2:58 PM)

CLRS, Chapters 1 & 2

Pseudocode Conventions (p. 21-24)

Array Subscripts:

Book: 1 . . . n
Notes/C/Java Code: 0 . . . n – 1

1.A. QUADRATIC TIME SORTS:

Selection Sort (CLRS exercise 2.2-2)

```
void selection(Item a[], int ell, int r)
{
  int i, j;
  for (i = ell; i < r; i++)
  {
    int min = i;
    for (j = i+1; j <= r; j++)
      if (less(a[j], a[min]))
        min = j;
    exch(a[i], a[min]);
  }
}
```

Always uses $\sum_{i=2}^{n}(i-1)=\sum_{i=1}^{n-1}i=\frac{n(n-1)}{2}\approx\frac{n^2}{2}$ comparisons and is **not** *stable* (CLRS, p. 210).

(Aside: `https://www.americanscientist.org/article/gausss-day-of-reckoning`)

Insertion Sort (CLRS p.19, `https://ranger.uta.edu/~weems/NOTES3318/insertionSort.c`)

```
void insertionSort(Item *a,int N) // Guaranteed stable
{
int i,j;
Item v;

for (i=1; i<N; i++)
{
  v=a[i];
  j=i;
  while (j>=1 && less(v,a[j-1]))
  {
    a[j]=a[j-1];
    j--;
  }
  a[j]=v;
}
}
```

Maximum ("worst case") number of times that body of `j`-loop executes
for a particular value of `i`?

Maximum number of times that body of `j`-loop executes over entire sort?

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} = ?$$

Expected ("average") number of times that body of `j`-loop executes for a particular value of `i`?

Expected number of times that body of `j`-loop executes over entire sort?

1.B. DIVIDE AND CONQUER (Decomposition)

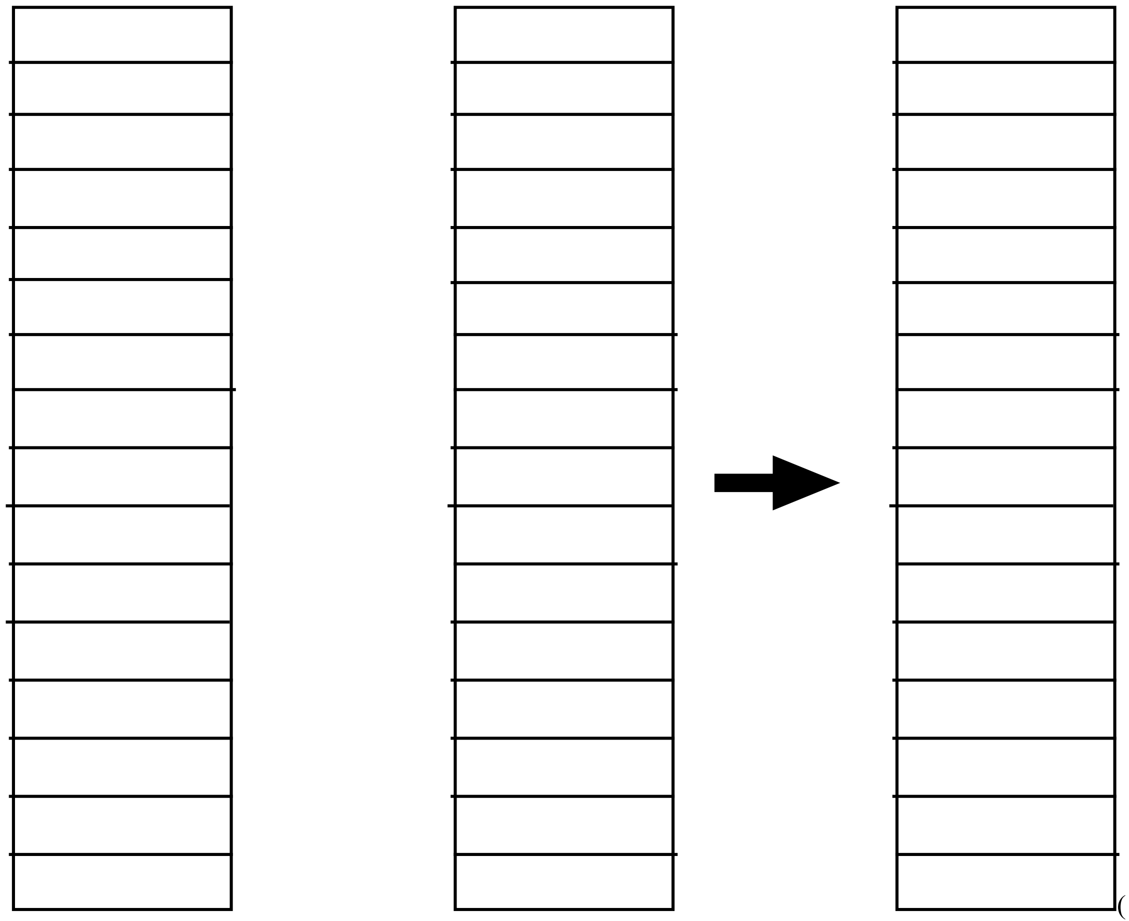
1. Divide into subproblems (unless size allows a trivial solution).
2. Conquer the subproblems.
3. Combine solutions to subproblems.

(Binary) Mergesort – An "Optimal" Key-Comparison Sort (`https://ranger.uta.edu/~weems/NOTES3318/mergesort.new.c`)

1. Split (copy) array into two sub-arrays (unless n<2).
2. Call Mergesort recursively for each sub-array.
3. Merge together the two ordered sub-arrays.

```
void mymergesort(int *arr,int *work,int n)
{
int nleft,nright,i;

if (n<2)
  return;
nleft=n/2;
nright=n-nleft;
for (i=0;i<n;i++)
  work[i]=arr[i];
mymergesort(work,arr,nleft);
mymergesort(work+nleft,arr+nleft,nright); // pointer arithmetic
merge(work,work+nleft,arr,nleft,nright); // pointer arithmetic
}
```



```
int merge(int *in1,int *in2,int *out1,int in1Size,int in2Size)
// Merge (union with duplicates) for two ordered tables in1 and in2
// to give out1.
{
int i,j,k;

i=j=k=0;
while (i<in1Size && j<in2Size)
  if (in1[i]<in2[j])
    out1[k++]=in1[i++];
  else
    out1[k++]=in2[j++];
if (i<in1Size)
  for ( ;i<in1Size;i++)
    out1[k++]=in1[i];
else
  for ( ;j<in2Size;j++)
    out1[k++]=in2[j];
return k;
}
```

How are items with identical keys (“duplicates”) handled?

[Write body of while-loop with ? : expression. Code for linked lists, files, streams, etc.]

Fall 2009 Test Problem Applying Merge Concept

Two `int` arrays, `A` and `B`, contain `m` and `n` `int`s each, respectively. The elements within each of these arrays appear in ascending order without duplication (i.e. each table represents a set). Give Java code for a $\Theta(m+n)$ algorithm to find the **symmetric difference** by producing a third array `C` (in ascending order) with the values that appear in **exactly** one of `A` and `B` **and** sets the variable `p` to the final number of elements copied to `C`. (Details of input/output, allocation, declarations, error checking, comments and style **are unnecessary**.)

```
i=j=p=0;

while (i<m && j<n)
  if (A[i]<B[j])
    C[p++]=A[i++];
  else if (A[i]>B[j])
    C[p++]=B[j++];
  else
  {
    i++;
    j++;
  }

for ( ; i<m; i++)
  C[p++]=A[i];
for ( ; j<n; j++)
  C[p++]=B[j];
```

How much work (time) in worse case? ($T(n)$ – a *recurrence*)

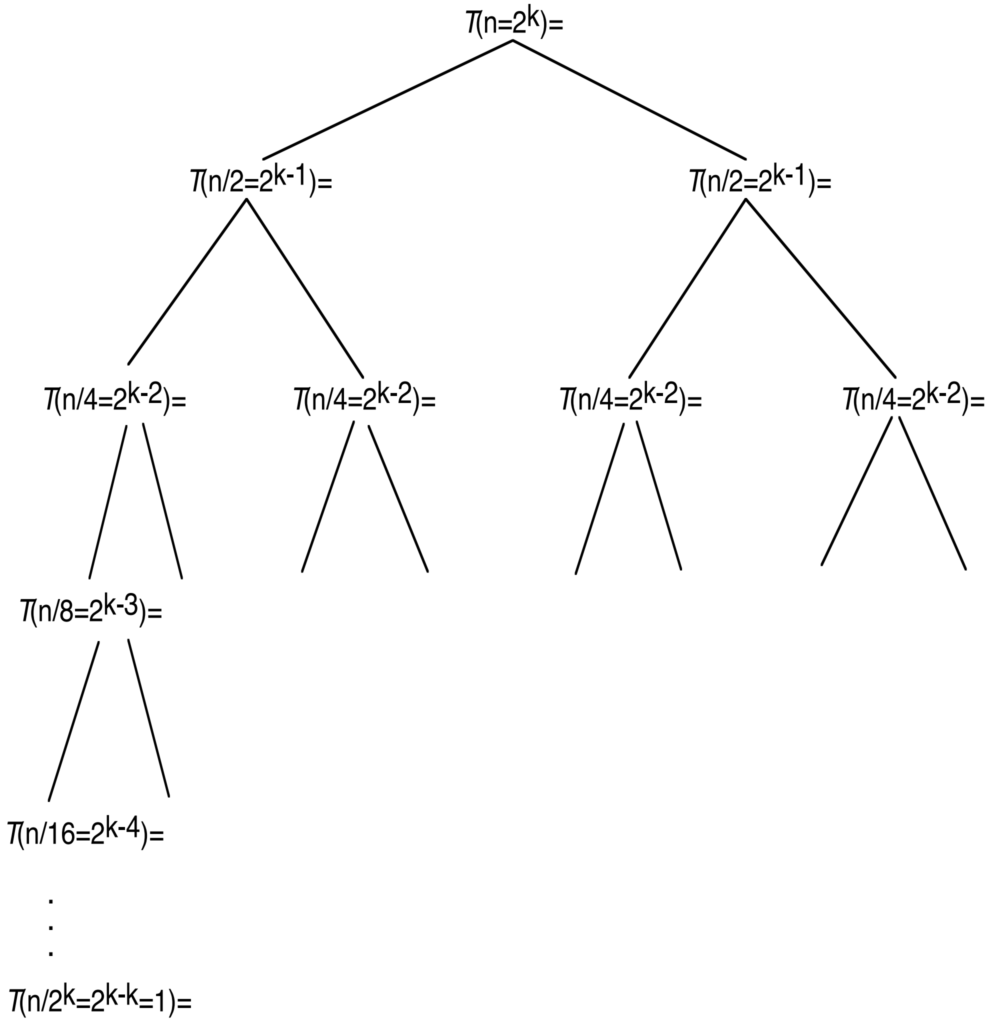
1. Split: n steps. [Can reduce to constant time by pointer arithmetic.]
2. Call recursively:

$$T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right)$$

3. Merge together (n steps)

$$T(n) = c_1 n + T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right) + c_2 n = cn \log_? n$$

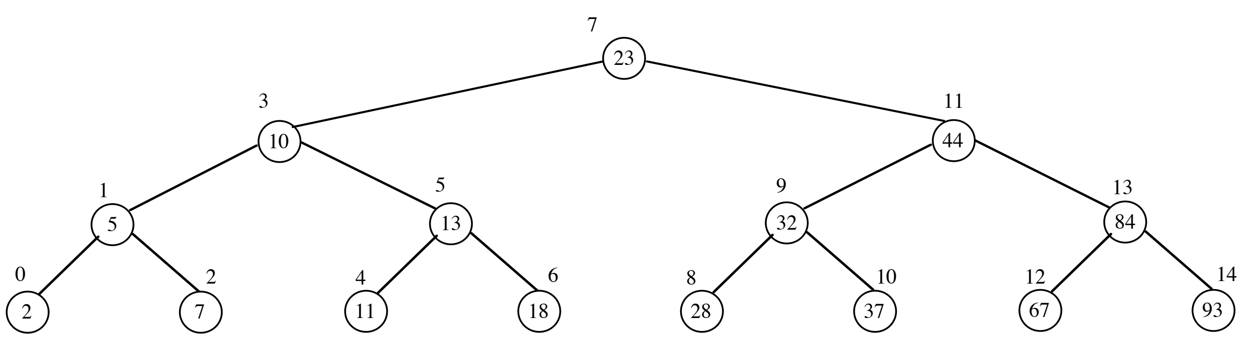
Recursion Tree



[Don't generalize from this example of a recursion tree. More of these in Notes 04.]

1.C. BINARY SEARCH - "Optimal" Search of an Ordered Table (or "Space")

Concept – search *ordered* table in logarithmic time. Consider table with $2^k - 1$ slots.



(`https://ranger.uta.edu/~weems/NOTES3318/binarySearch.c`)

```
int binSearch(int *a,int n,int key)
// Input: int array a[] with n elements in ascending order.
//        int key to find.
// Output: Returns some subscript of a where key is found.
//         Returns -1 if not found.
// Processing: Binary search.
{
  int low,high,mid;
  low=0;
  high=n-1;
// subscripts between low and high are in search range.
// size of range halves in each iteration.
  while (low<=high)
  {
    mid=(low+high)/2;
    if (a[mid]==key)
      return mid; // key found
    if (a[mid]<key)
      low=mid+1;
    else
      high=mid-1;
  }

  return (-1); // key does not appear
}
```

Recursive binary search?

Multiple occurences of keys (`https://ranger.uta.edu/~weems/NOTES3318/binarySearchRange.c`)

Find `i` such that `a[i-1] < key <= a[i]`

```
int binSearchFirst(int *a,int n,int key)
// Input: int array a[] with n elements in ascending order.
//        int key to find.
// Output: Returns subscript of the first a element >= key.
//         Returns n if key>a[n-1].
// Processing: Binary search.
{
  int low,high,mid;
  low=0;
  high=n-1;
// Subscripts between low and high are in search range.
// Size of range halves in each iteration.
// When low>high, low==high+1 and a[high]<key and a[low]>=key.
  while (low<=high)
  {
    mid=(low+high)/2;
    if (a[mid]<key)
      low=mid+1;
    else
      high=mid-1;
  }
  return low;
}
```

Relationship of `low` and `high` on `return`?

Find `i` such that `a[i] <= key < a[i+1]`

```
int binSearchLast(int *a,int n,int key)
{
// Input: int array a[] with n elements in ascending order.
//        int key to find.
// Output: Returns subscript of the last a element <= key.
//         Returns -1 if key<a[0].
// Processing: Binary search.
  int low,high,mid;
  low=0;
  high=n-1;
// subscripts between low and high are in search range.
// size of range halves in each iteration.
// When low>high, low==high+1 and a[high]<=key and a[low]>key.
  while (low<=high)
  {
    mid=(low+high)/2;
    if (a[mid]<=key)
      low=mid+1;
    else
      high=mid-1;
  }
  return high;
}
```

Relationship of `low` and `high` on `return`?

Partial output from `binarySearchRange.c` (`count` is `last-first+1`)

```
-- table --        key  first  last  count
  0   0             -1      0    -1      0
  1   1              0      0     0      1
  2   1              1      1     3      3
  3   1              2      4     4      1
  4   2              3      5     4      0
  5   4              4      5     6      2
  6   4              5      7     6      0
  7   6              6      7     9      3
  8   6              7     10     9      0
  9   6              8     10     9      0
 10  10              9     10     9      0
 11  12             10     10    10      1
 12  12             11     11    10      0
 13  12             12     11    14      4
 14  12             13     15    14      0
 15  15             14     15    14      0
 16  15             15     15    16      2
 17  17             16     17    16      0
 18  17             17     17    18      2
 19  18             18     19    19      1
                    19     20    19      0
                    20     20    19      0
```