Lab 3: Multi-Qubit States and Entanglement

Preliminaries

Refer to the teaching materials in Module 3.

Tasks

Run 'jupyter notebook', and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

1. Manually calculate the Tensor Product of the following two vectors $\begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$ in two ways.
   1.1 Conduct calculation by their vector forms.
   1.2 Convert these two vectors into their basis forms and then conduct calculation by their basis forms.

2. Factor the joint state $\frac{4}{5\sqrt{2}}|00\rangle - \frac{3}{5\sqrt{2}}|01\rangle - \frac{4}{5\sqrt{2}}|10\rangle + \frac{3}{5\sqrt{2}}|11\rangle$ into two independent qubit states.
   2.1 Verify this joint state is not entangled.
   2.2 Include the detailed steps for the factoring.

3. You are given three vectors: $|x\rangle = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$ and $|y\rangle = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $|z\rangle = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$. Use them to demonstrate the following properties of Tensor Product with Python code:
   3.1 Distributivity: $|x\rangle(|y\rangle + |z\rangle) = |x\rangle|y\rangle + |x\rangle|z\rangle$.
   3.2 Associativity: $(|x\rangle|y\rangle)|z\rangle = |x\rangle(|y\rangle|z\rangle)$.
   3.3 Non-commutativity: $|x\rangle|y\rangle \neq |y\rangle|x\rangle$

4. You are given two independent qubits: $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$ and $q_1 = \frac{5}{13}|0\rangle - \frac{12}{13}|1\rangle$. Use Qiskit package to perform the following.

   4.1 Construct a circuit with 2-qubit input and 2-bit output. Initialize q_0 as $\frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$ and initialize q_1 as $\frac{5}{13}|0\rangle - \frac{12}{13}|1\rangle$. Measure these two qubits. Draw this circuit.
   4.2 Use a QasmSimulator to run the above circuit with 2000 shots, and then plot the measurement results.
   4.3 Manually calculate the probabilities of measuring 00, 01, 10, and 11 in theory.
   4.4 Show by calculation that the probabilities obtained in 4.2 roughly match the probabilities obtained in 4.3.