Lab 9: The Deutsch-Jozsa Algorithm

Preliminaries

Refer to the teaching materials in Module 9.

Tasks

Run 'jupyter notebook', and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

1. Let matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ and matrix $B = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$. Use these two matrices to show that the Kronecker Product of matrices may not satisfy the commutativity: $A \otimes B = B \otimes A$.

2. Let matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$, matrix $B = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ and matrix $C = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$. Use these three matrices to show that the Kronecker Product of matrices may not satisfies the Associativity: $(A \otimes B) \otimes C = A \otimes (B \otimes C)$.

3. Consider the 2-Boolean-variable function f_0 below, which is a constant function.

f_0	**Input:**	00	01	10	11
	Output:	0	0	0	0

   3.1 Derive the input/output table for its oracle F_0. (Search the Internet to learn how to do a table with Markdown)
   3.2 Derive the gate matrix for F_0.
   3.3 Use a Quantum Circuit to implement the oracle F_0, and draw this circuit. (Note: no need to measure or run this circuit.)

4. Consider the 2-Boolean-variable function f_4 below, which is a neither constant nor balanced function.

f_4	**Input:**	00	01	10	11
	Output:	0	1	0	0

   4.1 Derive the input/output table for its oracle F_4.
   4.2 Derive the gate matrix for F_4.
   4.3 Use a Quantum Circuit to implement the oracle F_4, and draw this circuit. (Note: no need to measure or run this circuit.)

5. Consider the 2-Boolean-variable function f_5 below, which is a balanced function.

f_5	**Input:**	00	01	10	11
	Output:	0	1	0	1

5.1 Derive the input/output table for its oracle F_5.
5.2 Derive the gate matrix for F_5.
5.3 Use a Quantum Circuit to implement the oracle F_5, and draw this circuit. (Note: no need to measure or run this circuit.)

6. Use the QasmSimulator to implement the Deutsch-Jozsa Algorithm with the oracles F_0, F_4, and F_5 above respectively. For each oracle, the implementation should roughly follow the structure of the notebook accompanying our lecture, consisting of measuring the outcomes with 1000 shots and plotting the results with histograms.