Lab 8: The Deutsch Algorithm

Preliminaries

Refer to the teaching materials in Module 8.

Tasks

Run ‘jupyter notebook’, and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

1. Prove that the XOR operation satisfies Associativity: $(A \oplus B) \oplus C = A \oplus (B \oplus C)$.
   Hint: Enumerate all the eight ways of assigning Boolean values to A, B, and C. Show that, in each way, the left-hand side equals the right-hand side.

2. In our lecture, we didn’t derive the gate matrix for the oracle F_3. In this task, you are asked to:
   2.1 Derive its gate matrix.
   2.2 Show this gate matrix is its own inverse.
   2.3 Use a Quantum Circuit to implement the oracle F_3, and draw this circuit. (Note: no need to measure or run this circuit.)

3. Use the QasmSimulator to implement the following variant of the Deutsch Algorithm: The circuit should be the same as described in our lecture, but both of the input qubits take the state of $|1\rangle$.
   3.1 Show by calculation that this variant can also tell if an oracle is constant or balanced by one run of the circuit. Especially, please state what measurement outcome of the top qubit indicates a constant oracle and what measurement outcome indicates a balanced oracle.
   3.2 Implement this variant with F_2 as the oracle.
   3.3 Implement this variant with F_3 as the oracle.

   Hint for 3.1: Follow the same steps in the lecture to conduct the calculation.

   Note for 3.2 and 3.2: The implementations should roughly follow the structure of the notebook accompanying our lecture, consisting of measuring the outcomes with 1000 shots and plotting the results with histograms.