LAB 4

Name: Heja Bibani
Student Number: 16301173

Section 1

Create a quantum circuit with measurement and visualization by following the steps below.

Task 1.1 Import all the neccessary packages

In [1]:

```
import numpy as np
from qiskit import *
from qiskit.providers.aer import QasmSimulator
from qiskit.visualization import plot_histogram
from math import sqrt
%matplotlib inline
```

Task 1.2 Import all the neccessary packages

Create a circuit with two input qubits and 2 output bits. Initialize $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$ and $q_1 = |0\rangle$ by default.

In [16]:

```
# If the QuantumCircuit() function only has one parameter,
# it means the number of input qubits.
qc = QuantumCircuit(2,2)
# Create an initial state of |0>
initial_state = [1/3, -(2*sqrt(2))/3]
qc.initialize(initial_state, 0)
```

Out[16]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x21105fb0240>
```

Task 1.3

Apply h() gate to q_0 , and then apply cx() gate to q_0 and q_1 with q_0 as the control qubit.

In [17]:

```
#Apply h-gate on qubit 0.
qc.h(0)
# Its parameters indicate q0 is the Control Qubit, and q1 is the Target Qubit.
qc.cx(0,1)
```

Out[17]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x21105f98f40>
```

Task 1.4

Measure q_0 and q_1, and store the measurement results into bits 0 and 1 respectively.

In [18]:

```
# Store the measurement results of qubits 0 and 1 into bits 0 and 1 respectively
qc.measure([0,1], [0,1])
```

Out[18]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x21105e1e740>
```

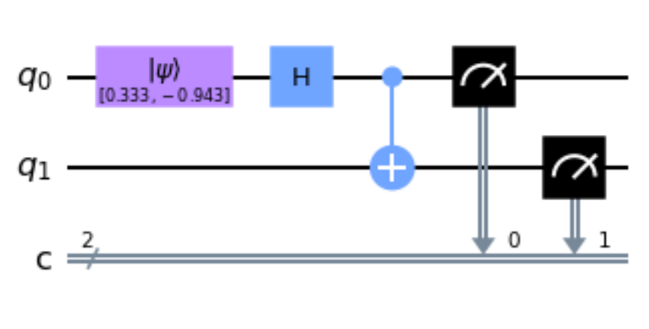
Task 1.5

Visualize this circuit.

In [19]:

```
qc.draw()
```

Out[19]:



Section 2

Run the quantum circuit in Task 1 with a QasmSimulator for 2000 shots and plot the result with a histogram.

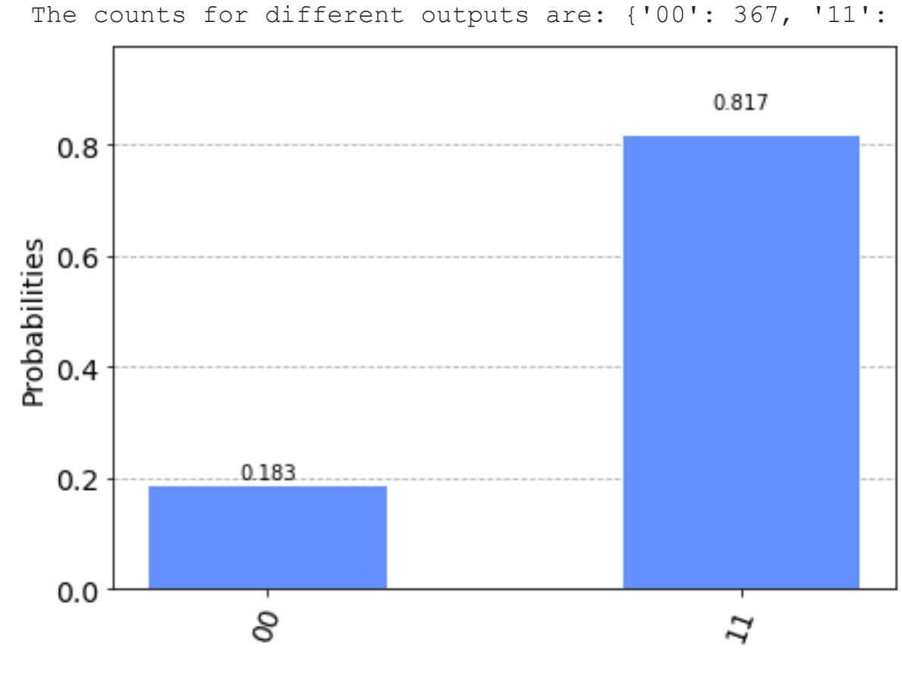
In [23]:

```
simulator = QasmSimulator()
job = execute(qc, backend=simulator, shots=2000)
# Call result() to obtain the result
result = job.result()
# Call get_counts() to obtain the counts of different outputs
counts = result.get_counts(qc)

# Print the counts of different outputs
print("\n The counts for different outputs are:", counts)
# Plot the histogram
plot_histogram(counts)
```

```
 The counts for different outputs are: {'00': 367, '11': 1633}
```

Out[23]:



Section 3

For the quantum circuit in Task 1, manually calculate the probabilities of measuring 00, 01, 10, and 11 respectively in theory.

The values for $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$ and $q_1 = |0\rangle$. Since Qiskit calculates it from q_1 first then we calculate in the same fashion.

First we apply the H-Gate to q_0 :

$$\frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{3} \\ -\frac{2\sqrt{2}}{3} \end{bmatrix} = \begin{bmatrix} \frac{1-2\sqrt{2}}{3\sqrt{2}} \\ \frac{1+2\sqrt{2}}{3\sqrt{2}} \end{bmatrix}$$

We get the tensor product of q_1 and the output of the H-gate:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{1-2\sqrt{2}}{3\sqrt{2}} \\ \frac{1+2\sqrt{2}}{3\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1-2\sqrt{2}}{3\sqrt{2}} \\ \frac{1+2\sqrt{2}}{3\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$$

Applying the C-NOT gate:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} . \times \begin{bmatrix} \frac{1-2\sqrt{2}}{3\sqrt{2}} \\ \frac{1+2\sqrt{2}}{3\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1-2\sqrt{2}}{3\sqrt{2}} \\ 0 \\ 0 \\ \frac{1+2\sqrt{2}}{3\sqrt{2}} \end{bmatrix}$$

This would leave the state in the following:

$$\frac{1-2\sqrt{2}}{3\sqrt{2}}|00\rangle + \frac{1+2\sqrt{2}}{3\sqrt{2}}|11\rangle$$

.

The probability calculations are of the following:

$P(|q_1q_0\rangle \text{ measured as } |00\rangle) = (\frac{1-2\sqrt{2}}{3\sqrt{2}})^2 \approx 0.1857.$

$P(|q_1q_0\rangle \text{ measured as } |11\rangle) = (\frac{1+2\sqrt{2}}{3\sqrt{2}})^2 \approx 0.8143.$

Task 3.1

Is the resulting state entangled? Why?

Yes it is entangled since:

Probability ru = $\frac{1-2\sqrt{2}}{3\sqrt{2}} \times \frac{1+2\sqrt{2}}{3\sqrt{2}} = -\frac{7}{18}.$

Probability st = 0

Therefore, ru $\neq$ st.

Task 3.2

Do your probability results match the ones in the histogram of Task 2?

The manual probability was measured at:

$P(|q_1q_0\rangle \text{ measured as } |00\rangle) = (\frac{1-2\sqrt{2}}{3\sqrt{2}})^2 \approx 0.1857.$

$P(|q_1q_0\rangle \text{ measured as } |11\rangle) = (\frac{1+2\sqrt{2}}{3\sqrt{2}})^2 \approx 0.8143.$

In the histogram the probabilities are almost identical:

The probability for $|00\rangle$ is measured at 0.183 and the probability for $|11\rangle$ is measured at 0.817.

Section 4

Create a second quantum circuit with measurement and visualization by following the steps below.

Task 4.1 Import all necessary packages.

This is done in Section 1.1.

Task 4.2

Create a circuit with 3 input qubits and 3 output bits. Initialize $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle, q_1 = \frac{5}{13}|0\rangle - \frac{12}{13}|1\rangle$ and $q_2 = |0\rangle$ by default.

In [25]:

```
# If the QuantumCircuit() function only has one parameter,
# it means the number of input qubits.
qc1 = QuantumCircuit(3,3)
# Create an initial state of |0>
initial_state = [1/3, -(2*sqrt(2))/3]
qc1.initialize(initial_state, 0)
initial_state = [5/13, -12/13]
qc1.initialize(initial_state, 1)
```

Out[25]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x21108d25100>
```

Task 4.3

Apply Toffoli gate to q_0, q_1 and q_2 with q_0 and q_1 as control qubits.

In [26]:

```
# The ccx() function is used to apply a CCNOT-gate to 3 qubits.
# The first two parameters indicate Control Qubits,
# and the last parameter the Target Qubit.
qc1.ccx(0, 1, 2)
```

Out[26]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x21108e113c0>
```

Task 4.4

Measure q_0, q_1 and q_2, and store the measurement results into bits 0, 1, and 2 respectively.

In [28]:

```
# Store the measurement results of qubits 0 and 1,2 into bits 0, 1 and 2 respectively
qc1.measure([0,1,2], [0,1,2])
```

Out[28]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x2110777b3c0>
```

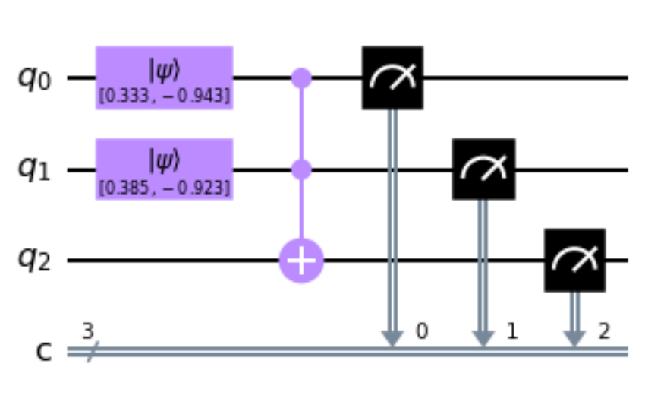
Task 4.5

Visualize this circuit.

In [29]:

```
qc1.draw()
```

Out[29]:



Section 5

Run the quantum circuit in Task 4 with a QasmSimulator for 2000 shots and plot the result with a histogram.

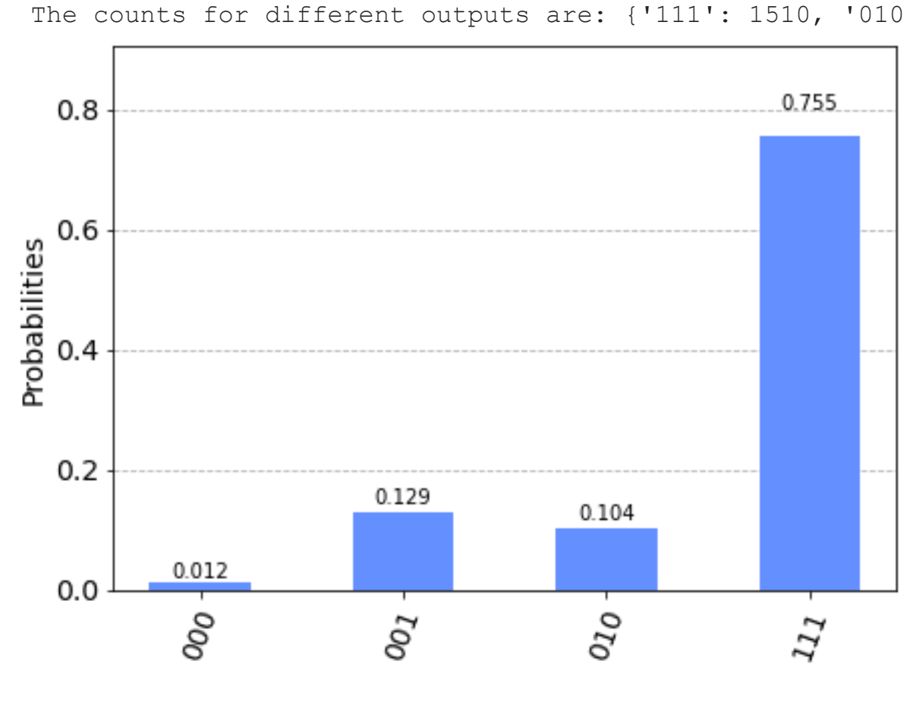
In [30]:

```
simulator = QasmSimulator()
job = execute(qc1, backend=simulator, shots=2000)
# Call result() to obtain the result
result = job.result()
# Call get_counts() to obtain the counts of different outputs
counts = result.get_counts(qc1)

# Print the counts of different outputs
print("\n The counts for different outputs are:", counts)
# Plot the histogram
plot_histogram(counts)
```

```
 The counts for different outputs are: {'111': 1510, '010': 208, '001': 258, '000': 24}
```

Out[30]:



Section 6

For the quantum circuit in Task 4, manually calculate the probabilities of measuring 000, 001, 010, 011, 100, 101, 110, and 111 respectively in theory. Do your results match the ones in the histogram of Task 5?

We know that qiskit applies the products from the top to the bottom. Therefore we calculate the tensor product:

$$q_2 \otimes q_1 \otimes q_0$$

The calculation will be of the following:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{3} \\ -\frac{2\sqrt{2}}{3} \end{bmatrix}$$

We begin with the first tensor product:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13} \end{bmatrix} = \begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13} \\ 0 \\ 0 \end{bmatrix}$$

The second tensor product:

$$\begin{bmatrix} \frac{5}{13} \\ -\frac{12}{13} \\ 0 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{3} \\ -\frac{2\sqrt{2}}{3} \end{bmatrix} = \begin{bmatrix} \frac{5}{39} \\ -\frac{10\sqrt{2}}{39} \\ -\frac{12}{39} \\ \frac{24\sqrt{2}}{39} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Then we apply the matrix multiplication of the Troffoli gate:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \frac{5}{39} \\ -\frac{10\sqrt{2}}{39} \\ -\frac{12}{39} \\ \frac{24\sqrt{2}}{39} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{39} \\ -\frac{10\sqrt{2}}{39} \\ -\frac{12}{39} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{24\sqrt{2}}{39} \end{bmatrix}$$

The manual probability was measured at:

$P(|q_2q_1q_0\rangle \text{ measured as } |000\rangle) = (\frac{5}{39})^2 \approx 0.0164.$

$P(|q_2q_1q_0\rangle \text{ measured as } |001\rangle) = (-\frac{10\sqrt{2}}{39})^2 \approx 0.131.$

$P(|q_2q_1q_0\rangle \text{ measured as } |010\rangle) = (-\frac{12}{39})^2 \approx 0.095.$

$P(|q_2q_1q_0\rangle \text{ measured as } |111\rangle) = (\frac{24\sqrt{2}}{39})^2 \approx 0.757.$

Our results indicate that the values that were calculated manually are relatively the same as the output completed in task 5.

For $|000\rangle : 0.0164 \approx 0.012$

For $|001\rangle : 0.131 \approx 0.129$

For $|010\rangle : 0.095 \approx 0.104$

For $|111\rangle : 0.757 \approx 0.755$

Section 7

Create a third quantum circuit with measurement and visualization by following the steps below.

Task 7.1 Import all necessary packages.

This is done in Section 1.1.

Task 7.2

Create a circuit with 4 input qubits and 4 output bits. Initialize $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle, q_1 = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle, q_2 = |+\rangle$ and $q_3 = |0\rangle$ by default.

In [32]:

```
# If the QuantumCircuit() function only has one parameter,
# it means the number of input qubits.
qc2 = QuantumCircuit(4,4)
# Create an initial state of |0>
initial_state = [1/3, -(2*sqrt(2))/3]
qc2.initialize(initial_state, 0)
initial_state = [3/5, -4/5]
qc2.initialize(initial_state, 1)
initial_state = [1/sqrt(2), 1/sqrt(2)]
qc2.initialize(initial_state, 2)
```

Out[32]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x211091cbf00>
```

Task 7.3

Apply MCT gate to q_0, q_1, q_2, q_3 with q_0 ,q_1 and q_2 as control qubits.

In [33]:

```
qc2.mct([0, 1, 2], 3)
```

Out[33]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x21108db5880>
```

Task 7.4

Measure q_0, q_1, q_2, q_3 a q_0 ,q_1 and store the measurement results into bits 0, 1, 2, and 3 respectively.

In [34]:

```
# Store the measurement results of qubits 0 and 1,2 into bits 0, 1 and 2 respectively
qc2.measure([0,1,2,3], [0,1,2,3])
```

Out[34]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x211091b7b00>
```

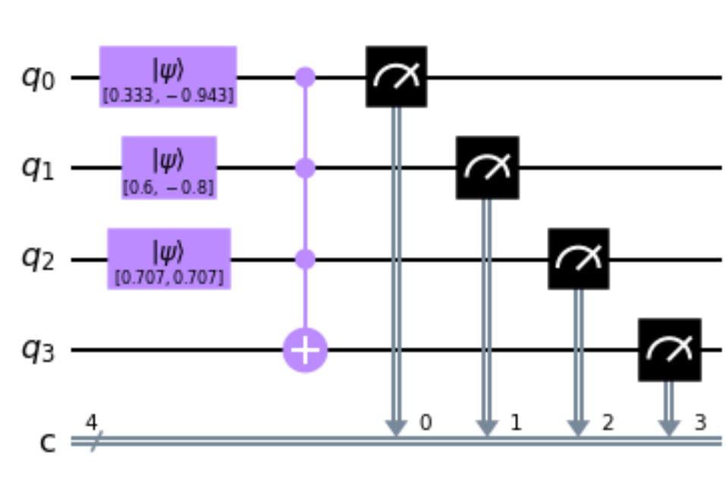
Task 7.5

Visualize this circuit.

In [35]:

```
qc2.draw()
```

Out[35]:



Section 8

Run the quantum circuit in Task 7 with a QasmSimulator for 2000 shots and plot the result with a histogram.

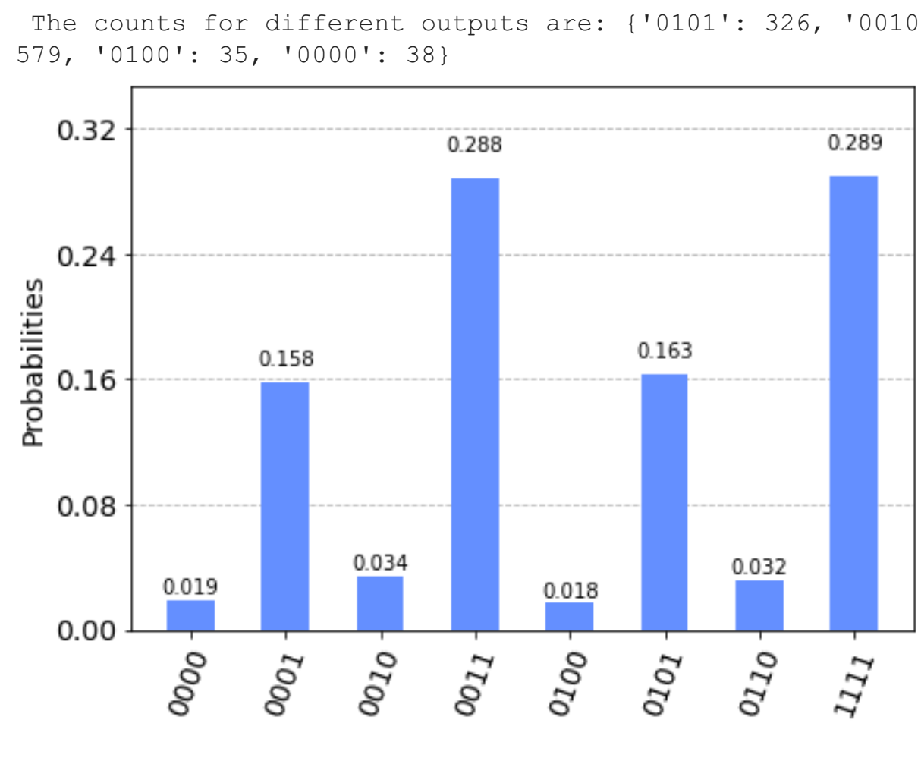
In [37]:

```
simulator = QasmSimulator()
job = execute(qc2, backend=simulator, shots=2000)
# Call result() to obtain the result
result = job.result()
# Call get_counts() to obtain the counts of different outputs
counts = result.get_counts(qc2)

# Print the counts of different outputs
print("\n The counts for different outputs are:", counts)
# Plot the histogram
plot_histogram(counts)
```

```
 The counts for different outputs are: {'0101': 326, '0010': 67, '0001': 316, '0011': 576, '0110': 63, '1111':
579, '0100': 35, '0000': 38}
```

Out[37]:



Section 9

For the quantum circuit in Task 7, manually calculate the probabilities of measuring 0000, 0001, ..., 1110, and 1111 respectively in theory. Do your results match the ones in the histogram of Task 8?

We know that qiskit applies the products from the top to the bottom. Therefore we calculate the tensor product:

$q_3 \otimes q_2 \otimes q_1 \otimes q_0$

The calculation will be of the following:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{3} \\ -\frac{2\sqrt{2}}{3} \end{bmatrix}$$

The first tensor product:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$$

The second tensor product:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix} = \begin{bmatrix} \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} \\ \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The third tensor product:

$$\begin{bmatrix} \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} \\ \frac{3}{5\sqrt{2}} \\ -\frac{4}{5\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{3} \\ -\frac{2\sqrt{2}}{3} \end{bmatrix} = \begin{bmatrix} \frac{3}{15\sqrt{2}} \\ -\frac{6}{15} \\ -\frac{4}{15\sqrt{2}} \\ \frac{8}{15} \\ \frac{3}{15\sqrt{2}} \\ -\frac{6}{15} \\ -\frac{4}{15\sqrt{2}} \\ \frac{8}{15} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

After applying the gate:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \frac{3}{15\sqrt{2}} \\ -\frac{6}{15} \\ -\frac{4}{15\sqrt{2}} \\ \frac{8}{15} \\ \frac{3}{15\sqrt{2}} \\ -\frac{6}{15} \\ -\frac{4}{15\sqrt{2}} \\ \frac{8}{15} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{15\sqrt{2}} \\ -\frac{6}{15} \\ -\frac{4}{15\sqrt{2}} \\ \frac{8}{15} \\ \frac{3}{15\sqrt{2}} \\ -\frac{6}{15} \\ -\frac{4}{15\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{8}{15} \end{bmatrix}$$

The manual probability was measured at:

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0000\rangle) = (\frac{3}{15\sqrt{2}})^2 \approx 0.02.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0001\rangle) = (-\frac{6}{15})^2 \approx 0.16.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0010\rangle) = (-\frac{4}{15\sqrt{2}})^2 \approx 0.037.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0011\rangle) = (\frac{8}{15})^2 \approx 0.284.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0100\rangle) = (\frac{3}{15\sqrt{2}})^2 \approx 0.02.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0101\rangle) = (-\frac{6}{15})^2 \approx 0.16.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |0110\rangle) = (-\frac{4}{15\sqrt{2}})^2 \approx 0.037.$

$P(|q_3q_2q_1q_0\rangle \text{ measured as } |1111\rangle) = (\frac{8}{15})^2 \approx 0.284.$

Our results indicate that the values that were calculated manually are relatively the same as the output completed in task 7.

For $|0000\rangle : 0.019 \approx 0.02$

For $|0001\rangle : 0.158 \approx 0.16$

For $|0010\rangle : 0.034 \approx 0.037$

For $|0011\rangle : 0.288 \approx 0.284$

For $|0100\rangle : 0.018 \approx 0.02$

For $|0101\rangle : 0.163 \approx 0.16$

For $|0110\rangle : 0.032 \approx 0.037$

For $|1111\rangle : 0.289 \approx 0.284$

Section 10

In Lecture 4, we proved the No-Cloning Theorem by using $|0\rangle$ as the ancilla qubit. In this task, you are asked to prove this theorem by using $|1\rangle$ as the ancilla qubit.

There are three equations that we need to consider:

Equation 1:

$G(|1\rangle \otimes |0\rangle) = |0\rangle \otimes |0\rangle = |00\rangle$

Equation 2:

$G(|1\rangle \otimes |1\rangle) = |1\rangle \otimes |1\rangle = |11\rangle$

Equation 3:

$$\begin{aligned} G(|1\rangle \otimes (\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle)) &= (\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \otimes (\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \\ &= \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle \end{aligned}$$

We can also derive equation 3 as follows:

$$G(|1\rangle \otimes (\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle)) = G(\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle)$$

Since matrix multiplication satisfies distributivity:

$$\begin{aligned} G(\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle) &= G(\frac{1}{\sqrt{2}}|10\rangle) + G(\frac{1}{\sqrt{2}}|11\rangle) \\ &= \frac{1}{\sqrt{2}}G(|10\rangle) + \frac{1}{\sqrt{2}}G(|11\rangle) \end{aligned}$$

From equation 1 and 2 we know that:

$$\frac{1}{\sqrt{2}}G(|10\rangle) + \frac{1}{\sqrt{2}}G(|11\rangle) = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

The results form a different equation from equation 3 above. Thus demonstrating the contradiction.