Comp3033 Quantum Computing and Communication

Semester Project

Due: 9pm, Friday, 10 June 2022

This project is of individual work. By working on this project, you promise that you will never ask or offer inappropriate help from/to others.

In this project, you will work on tasks that are more challenging than what you have seen in our lectures and labs. However, the basic skills involved are similar.

Each task has hints provided. You should try to solve the tasks based on the hints by yourself. Those hints are very helpful. If you work hard enough, the tasks are actually not difficult. You should NOT consult the lecturer unless your questions are for clarifications.

You are suggested to read the entire specification first, and then start with the tasks that are already covered by our lectures.

1 A General Proof to the Non-Cloning Theorem [4 marks]

In Lecture 4, we proved the No-Cloning Theorem by using $|0\rangle$ as the ancilla qubit. In this task, you are asked to prove this theorem by using a general qubit $a|0\rangle + b|1\rangle$ as the ancilla qubit. That is, no matter which state the ancilla qubit assumes, cloning is impossible.

> **Hint:** The structure of the proof will be the same as the one in the lecture. You can still pick the following three states for $|x\rangle$: $|0\rangle$, $|1\rangle$, and $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ to construct a contradiction.

2 An alternative Teleportation Protocol [4 marks]

In the original Teleportation Protocol, a third party prepares a pair of entangled qubits with the state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$, and then sends Alice one qubit from the pair and sends Bob the other. In this task, you are asked to replace the entangled state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ with $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$, and make the Teleportation still work.

To accomplish this task, you should roughly follow the structure in the Jupyter notebook NB05 accompanying our Lecture 5 to conduct the following:

2.1 Give the protocol and the circuit for it first. You should detail what operations (e.g., gates or measurements) are taken in each step by both Markdown and Code cells. In the Markdown cells, manual calculation should be given to show what will happen in each step by assuming that the qubit to teleport has a general state $a|0\rangle + b|1\rangle$. In the Code cells, the circuit for this Teleportation Protocol should be constructed step by step with barriers introduced between steps.

2.2 Then, verify the above circuit by running it under the QasmSimulator, with randomness introduced to the qubit to teleport. The verification technique should be the same as the one used in the notebook NB05.

3 Proof of Probability in Ekert Protocol [6 marks]

In the variant of the Ekert Protocol taught in our Lecture 7, if Eve eavesdrops every pair of entangled qubits, then for those entangled pairs where Alice and Bob pick different bases, the probability for them to see the same measurement outcome is $\frac{3}{8}$. In this task, you are asked to prove this fact.

Hints: Since there are three bases $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ for Alice and Bob to pick, there can be six cases where Alice and Bob pick different bases (with Alice's basis first and Bob's second): $(\theta = 0,\ \theta = \frac{2\pi}{3})$, $(\theta = 0,\ \theta = \frac{4\pi}{3})$, $(\theta = \frac{2\pi}{3},\ \theta = 0)$, $(\theta = \frac{2\pi}{3},\ \theta = \frac{4\pi}{3})$, $(\theta = \frac{4\pi}{3},\ \theta = 0)$, $(\theta = \frac{4\pi}{3},\ \theta = \frac{2\pi}{3})$. Since these three bases are equally spaced in the 2D circle, each of these six cases will see the same probability result if Eve eavesdrops. Therefore, you only need to pick one case from the above, and show that, under this case, the probability for Alice and Bob to see the same measurement outcome is $\frac{3}{8}$ if Eve overhears the channel.

Here we suggest you to pick the case $(\theta = \frac{2\pi}{3},\ \theta = \frac{4\pi}{3})$. Meanwhile, Eve will pick any of the three bases $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ to eavesdrop with probability $\frac{1}{3}$. You should conduct the calculation for each basis Eve may choose, and then combine the result to obtain $\frac{3}{8}$.

You should review the multiplication rule for probability calculation if you forget about it. An exemplar good resource for it is the one from Khan Academy: https://www.khanacademy.org/math/ap-statistics/probability-ap/probability-multiplication-rule/a/general-multiplication-rule .

4 A Circuit for Grover's Algorithm [6 marks]

Suppose $f(x_3, x_2, x_1, x_0)$ is 4-Boolean-variable function. It outputs 1 only when the input is $x_3 = 1$, $x_2 = 1$, $x_1 = 0$, and $x_0 = 0$, otherwise it outputs 0. In this task, you should implement a Grover's circuit that reveals which input will cause $f(x_3, x_2, x_1, x_0)$ to output 1.

4.1 Roughly follow the notebook accompanying Lecture 10 to implement this circuit and conduct measurement. Run this circuit for at least 1000 shots and plot the result.

4.2 Manually calculate the probability for the measurement outcome being '1100' given the above circuit. The probability obtained should roughly match the result from 4.1.

Report submission

The project report should be submitted as a pdf, but be initially written as a Jupyter Notebook, which should have the following structure:

- The project title, your student ID, and your full name: included in a Markdown cell.
- Labelled answers to project tasks: Your labels should match the labels in the project tasks. Each label should be introduced by a Markdown cell, which should also contain a brief description of the task. After this Markdown cell with label, you can use as many Markdown or Code cells as needed to tackle the task. If a task contains hierarchical labels, such labels should be introduced in your Notebook as well.

You should submit your report to turnitin via the Project Submission link on vUWS. Turnitin will calculate the similarity percentage of your report to other submissions. (NB: To prevent those tricks of bypassing plagiarism detection, we will not show the similarity percentage to you after your submission.)

If you are detected with plagiarism by turnitin, you will be punished seriously according to university policy.

Marking Criteria

The mark allocation for each task above is indicated beside it. We will conduct marking with a rubric reflecting this mark allocation. This rubric can be viewed by clicking the report submission link on vUWS, and then clicking the rubric icon before the 'Submit' button.

In the rubric, there are four categories of marks for each task: Excellent, Adequate, Incomplete, and Poor. Note that the project report has a higher requirement on writing than the lab reports. Basically, the project report should be written in a tutorial style such that your peers can easily understand it. This higher requirement on writing is reflected in the four categories of marks in the rubric:

- **Excellent (full mark):** (1) The task is answered correctly. (2) All critical steps are included. (3) Sufficient texts are added to explain your steps. The amount of texts should be similar to the lecture slides or the accompanying Jupyter notebooks when they present a proof or a circuit. The math preliminaries given in the lectures can be used directly without derivations. (4) Explanatory comments are included in the Python code. The amount of comments should be similar to the lecturing Jupyter notebooks. All in all, we believe this kind of training on writing is very important for you.
- **Adequate (around 80% of the full mark):** The criterion (1) in ‘Excellent’ is met, but some of the criteria (2)-(4) in ‘Excellent’ are not met.
- **Incomplete (around 40% of the full mark):** (1) The solution is partially correct. (2) More than 40% of the critical steps make sense.
- **Poor (0 mark):** (1) The solution is mostly incorrect. (2) Less than 40% of the critical steps make sense.