Lab 12: Quantum Error Correction

Preliminaries

Refer to the teaching materials in Module 12.

Tasks

Run 'jupyter notebook', and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

1. Alice needs to send a qubit to Bob. Suppose the quantum channel between Alice and Bob only suffers from the quantum bit-flip error (i.e., $a|0\rangle + b|1\rangle$ becoming $a|1\rangle + b|0\rangle$), and the probability for this error is 0.1 for any qubit. Alice and Bob use the Quantum Bit-Flip Code as the quantum error correction technique. Then, what is the probability that Bob will obtain the qubit from Alice correctly?

   Hint: this probability is the addition of the probabilities of the following two cases.
   (1) In the Quantum Bit-Flip Code, all three qubits are transmitted without error. In this case, there is no need to correct the error.
   (2) In the Quantum Bit-Flip Code, only one qubit is transmitted with error. In this case, the error can be corrected.

2. Implement a variant of the Quantum Bit-Flip Code. This variant differs from the one presented in the lecture only in the following way: checking $q_1 = q_0$ and $q_1 = q_2$ instead of $q_0 = q_1$ and $q_0 = q_2$.
   2.1 Suppose the qubit to send is $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$. Give the circuit implementations for this variant at the both the Alice and Bob sides. The circuits should roughly follow the structures given in the notebook accompanying Lecture 12.
   2.2 Given the above circuit implementations, describe how Bob should recover the error based on the measurement results of q_4 and q_3.
   2.3 Suppose a bit-flip error happens to q_0 during the transmission, and Bob successfully recovers this error using the variant implemented above. Construct a circuit to simulate this situation.