Lab 10: The Grover's Algorithm

Preliminaries

Refer to the teaching materials in Module 10.

Tasks

Run 'jupyter notebook', and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

1. Suppose $V = [1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8]^T$ is a column vector (NB: 'T' means transpose here). You should use Python code to complete the following:
   1.1 Construct the matrix $2A - I$ as defined in the lecture. It should be of the shape 8×8.
   1.2 Calculate $V_1 = (2A - I) \times V$.
   1.3 Apply the formula $2a - v$ defined in the lecture to each entry in V to obtain a new vector V_2.
   1.4 Compare V_1 and V_2. Do they contain the same entries?

   Note: As discussed in the notebook NB02a before, the Numpy in Python doesn't distinguish between row vectors and column vectors, but it can figure out which form of vector to use automatically.

2. Suppose in a Grover's circuit, the probability for the measuring outcome to be '101' is 0.945, and four independent measurements are conducted on this circuit. Then, what is the probability for obtaining '101' at least twice among these four measurements? (NB: This task should be done by Python code.)

   Hint: Review the probability calculations involved in the Binomial Distribution. An exemplar good resource is: https://www.mathsisfun.com/data/binomial-distribution.html .

3. Suppose $f(x_1, x_0)$ is a 2-Boolean-variable function. It outputs 1 only when the input is $x_1 = 0$ and $x_0 = 0$, otherwise it outputs 0. In this task, you are asked to implement a Grover's circuit that reveals which input makes $f(x_1, x_0)$ output 1.
   3.1 Roughly follow the notebook accompanying Lecture 10 to implement this circuit and conduct measurement. Run this circuit for at least 1000 shots and plot the result.
   3.2 Manually calculate the probability for the measurement outcome being '00' given the above circuit. The probability obtained should roughly match the result from 3.1.

4. Suppose $f(x_2, x_1, x_0)$ is a 3-Boolean-variable function. It outputs 1 only when the input is $x_2 = 0$, $x_1 = 0$ and $x_0 = 0$, otherwise it outputs 0. In this task, you are asked to implement a Grover's circuit that reveals which input makes $f(x_2, x_1, x_0)$ output 1.
   4.1 Roughly follow the notebook accompanying Lecture 10 to implement this circuit and conduct measurement. Run this circuit for at least 1000 shots and plot the result.
   4.2 Manually calculate the probability for the measurement outcome being '000' given the above circuit. The probability obtained should roughly match the result from 4.1.