Lab 5: Quantum Teleportation

Preliminaries

Refer to the teaching materials in Module 5.

Tasks

Run 'jupyter notebook', and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

We assume the ordering of qubits in Tensor Product follows the Qiskit package.

1. Suppose $q_1 = |1\rangle$ and $q_0 = |0\rangle$. A Bell Circuit is applied to these two qubits with q_0 as the control qubit in CNOT. What's the resulting state of these two qubits after the Bell Circuit? Please use Markdown cells to detail the calculation steps.

2. Suppose $q_1 = \frac{4}{5}|0\rangle - \frac{3}{5}|1\rangle$ and $q_0 = \frac{1}{3}|0\rangle + \frac{2\sqrt{2}}{3}|1\rangle$. A Reverse Bell Circuit is applied to these two qubits with q_0 as the control qubit in CNOT. What's the resulting state of these two qubits after the Reverse Bell Circuit? Please use Markdown cells to detail the calculation steps.

3. In the original Teleportation Protocol, a third party prepares a pair of entangled qubits with the state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$, and then sends Alice one qubit from the pair and sends Bob the other. In this task, you are asked to replace the entangled state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ with $\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$, and make the Teleportation still work.

   To accomplish this task, you should roughly follow the structure in the Jupyter notebook NB05 accompanying our Lecture 5 to conduct the following:

   3.1 Give the protocol and the circuit for it first. You should detail what operations (e.g., gates or measurements) are taken in each step by both Markdown and Code cells. In the Markdown cells, manual calculation should be given to show what will happen in each step by assuming that the qubit to teleport has a general state $a|0\rangle + b|1\rangle$. In the Code cells, the circuit for this Teleportation Protocol should be constructed step by step with barriers introduced between steps.

   Hints: Step 1 needs to be changed to produce $\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$. Steps 2 and 3 don't need changes. Step 4 needs to apply X and/or Z gate differently from the original Teleportation protocol.

   3.2 Then, verify the above circuit by running it under the QasmSimulator, with randomness introduced to the qubit to teleport. The verification technique should be the same as the one used in the notebook NB05.