Lab 4: Multi-Qubit Gates

Preliminaries

Refer to the teaching materials in Module 4.

Tasks

Run 'jupyter notebook', and create a notebook for this lab. Write your answers for all tasks into this notebook, and then convert it to pdf for submission to vUWS.

1. Create a quantum circuit with measurement and visualization by following the steps below.
   1.1 Import all necessary packages.
   1.2 Create a circuit with 2 input qubits and 2 output bits. Initialize $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$, and leave q_1 as $|0\rangle$ by default.
   1.3 Apply h() gate to q_0, and then apply cx() gate to q_0 and q_1 with q_0 as the control qubit.
   1.4 Measure q_0 and q_1, and store the measurement results into bits 0 and 1 respectively.
   1.5 Visualize this circuit.

2. Run the quantum circuit in Task 1 with a QasmSimulator for 2000 shots and plot the result with a histogram.

3. For the quantum circuit in Task 1, manually calculate the probabilities of measuring 00, 01, 10, and 11 respectively in theory.
   3.1 Is the resulting state entangled? Why?
   3.2 Do your probability results match the ones in the histogram of Task 2?

4. Create a second quantum circuit with measurement and visualization by following the steps below.
   4.1 Import all necessary packages.
   4.2 Create a circuit with 3 input qubits and 3 output bits. Initialize $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$ and $q_1 = \frac{5}{13}|0\rangle - \frac{12}{13}|1\rangle$. Leave q_2 as $|0\rangle$ by default.
   4.3 Apply Toffoli gate to q_0, q_1 and q_2 with q_0 and q_1 as control qubits.
   4.4 Measure q_0, q_1 and q_2, and store the measurement results into bits 0, 1, and 2 respectively.
   4.5 Visualize this circuit.

5. Run the quantum circuit in Task 4 with a QasmSimulator for 2000 shots and plot the result with a histogram.

6. For the quantum circuit in Task 4, manually calculate the probabilities of measuring 000, 001, 010, 011, 100, 101, 110, and 111 respectively in theory. Do your results match the ones in the histogram of Task 5?

7. Create a third quantum circuit with measurement and visualization by following the steps below.
   7.1 Import all necessary packages.
   7.2 Create a circuit with 4 input qubits and 4 output bits. Initialize $q_0 = \frac{1}{3}|0\rangle - \frac{2\sqrt{2}}{3}|1\rangle$, $q_1 = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$, and $q_2 = |+\rangle$. Leave q_3 as $|0\rangle$ by default.
   7.3 Apply MCT gate to q_0, q_1, q_2, and q_3 with q_0 and q_1 and q_2 as control qubits.
   7.4 Measure q_0, q_1, q_2, and q_3, and store the measurement results into bits 0, 1, 2, and 3 respectively.
   7.5 Visualize this circuit.

8. Run the quantum circuit in Task 7 with a QasmSimulator for 2000 shots and plot the result with a histogram.

9. For the quantum circuit in Task 7, manually calculate the probabilities of measuring 0000, 0001, …, 1110, and 1111 respectively in theory. Do your results match the ones in the histogram of Task 8?

10. In Lecture 4, we proved the No-Cloning Theorem by using $|0\rangle$ as the ancilla qubit. In this task, you are asked to prove this theorem by using $|1\rangle$ as the ancilla qubit.
    Hint: The structure of the proof will remain the same as in the lecture. You can still pick the following three states for $|x\rangle$: $|0\rangle$, $|1\rangle$, and $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ to generate a contradiction.