CHAPTER 5

- Example: 3 patients given vaccine.
  Probability of getting disease = 0.03

① $P(NDN) = P(DNN) = P(NND)$
$= (0.03)(0.97)(0.97)$
$= \underbrace{0.028227}_{(0.03)^1(1-0.03)^2}$

$(0.03)^1(1-0.03)^2 \longrightarrow \binom{3}{1}(0.028227)$
$= 0.084681$

② $P(NDD) = P(DDN) = P(DND)$
$= (0.03)(0.03)(0.97)$
$= \underbrace{0.000873}_{(0.03)^2(1-0.03)^1}$

$(0.03)^2(1-0.03)^1 \longrightarrow \binom{3}{2}(0.000873)$
$= 0.002619$

③ $P(NNN)$
$= (0.97)(0.97)(0.97)$
$= \underbrace{0.912673}_{(0.03)^0(1-0.03)^3}$

$(0.03)^0(1-0.03)^3 \longrightarrow \binom{3}{0}(0.912673)$
$= 0.912673$

④ $P(DDD)$
$= (0.03)(0.03)(0.03)$
$= \underbrace{0.000027}_{(0.03)^3(1-0.03)^0}$

$(0.03)^3(1-0.03)^0 \longrightarrow \binom{3}{3}(0.000027) = 0.000027$

- Example:

$Y \sim Binomial(n=10, p=0.372)$

$$P(Y=2) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \binom{10}{2} (0.372)^2 (1-0.372)^{10-2}$$

$$= 0.1507$$

- Exenise:

Probability a patient recovers from infection is 0.4. For group of 15 patients → use table A1 or manually find probability

(1) at least 10 recover

$X \sim Binomial(n=15, p=0.4)$

$$P(X \geq 10) = P(X < 9)$$

$$= 1 - P(X \leq 9)$$

$$= 1 - 0.9662 = 0.0338$$

By hand: $\sum_{x=10}^{15} \binom{15}{x} (0.4)^x (1-0.4)^{15-x}$

$$= 0.02449 + 0.00742 + 0.00165 + 0.00025 + 0.000024 + 0.00000107\,4$$

$$= 0.03384$$

(2) Between 3 and 8 Recover

$$P(3 \leq x \leq 8) = P(X \leq 8) - P(X \leq 3)$$

$$= 0.9050 - \underbrace{0.0271}_{0.0905}$$

$$= 0.8779$$

cont →

③ Exactly 5 Recover

$P(X=5) = P(X \leq 5) - P(X \leq 4)$
$= 0.4032 - 0.2173$
$= 0.1859$

By hand: $\binom{15}{5}(0.4)^5(1-0.4)^{15-5} = 0.1859$

- Example: Candy comes in Red, Green, Blue, Orange, & Yellow.

Colors occur w/ frequency of:
0.4, 0.2, 0.2, 0.1, 0.1

Sample 20 → what is prob you have 4 of each color?

$$\frac{20!}{4!\,4!\,4!\,4!\,4!}(0.4)^4(0.2)^4(0.2)^4(0.1)^4(0.1)^4$$

$= 0.0002$

- Example: # of radioactive particles passing thru a counter in 1 milisecond is 4. What is prob that 6 particles enter counter in a given milisecond?

$\lambda = 4 \,;\, t = 1 \rightarrow P(X=6) = \dfrac{e^{-4} \cdot 4^6}{6!} = 0.1042$

→ 6 particles in 2 seconds?

$\lambda = 4 \,;\, t = 2 \rightarrow P(X=6) = \dfrac{e^{-(4)(2)}(4 \cdot 2)^6}{6!} = 0.1221$

- Exenise: # of cases of a certain disease reported in a year follows a Poisson with mean 2.5

Find probability that:

1. No cases are reported in a given year.
   Table A.2 $\rightarrow r = 0, \mu = 2.5 \rightarrow 0.08208$

2. Exaclety 3 cases reported in 1 year
   Table A.2 $\rightarrow P(r = 3; \mu = 2.5) - P(r = 2; \mu = 2.5)$
   $= 0.7576 - 0.5438$
   $= 0.2138$

3. Atleast 3 in 1 year
   $P(X \geq 3) = 1 - P(X \leq 2)$
   $= 1 - \underbrace{0.5438}_{\text{Table A.2}}$
   $= 0.4562$

4. atmost 3 in 1 year
   $P(X \leq 3) = 0.7576$

5. Between 2 & 5 in 6 months
   $\lambda = \mu = 2.5$ & $t = \frac{1}{2}$ year

   $\lambda t = 2.5(0.5) = 1.25$

   $X = 2: \dfrac{e^{-1.25}(1.25)^2}{2!} = 0.22383$

   $X = 3: \dfrac{e^{-1.25}(1.25)^3}{3!} = 0.09326$

cont $\rightarrow$

$X=4: \dfrac{e^{-1.25}(1.25)^4}{4!} = 0.02914$

$X=5: \dfrac{e^{-1.25}(1.25)^5}{5!} = 0.00729$

$\Sigma = 0.3535$

OR $P(X \leq 5) - P(X \leq 1)$ (Table)

$= 0.99816 - 0.64464$

$= 0.3535$