Deaths in the US Example

$n = 100$

$\bar{x} = 71.8$

$\sigma = 8.9$

$\alpha = 0.05$

Method 1: p-value

$H_0: \mu = 70$

$H_a: \mu > 70$

$\alpha = 0.05$

p-value: $P(\bar{X} \geq 71.8 \mid \mu = 70)$

$= P\left(Z \geq \frac{71.8 - 70}{8.9/\sqrt{100}}\right)$

$= P(Z \geq 2.0225)$

$= 0.0216 < 0.05 \rightarrow$ reject H_0

Method 2: critical value

right-tailed test: $z_\alpha = z_{0.05} = 1.65$

test statistic $= 2.0225 >$ critical value $= 1.65$

$\Rightarrow$ reject H_0

Vacuum Cleaner

$\mu = 46$
$n = 12$
$\bar{x} = 42$
$s = 11.9$
$\alpha = 0.05$

Method 1: p-value

$H_0: \mu = 46$
$H_a: \mu < 46$
$\alpha = 0.05$

$$\begin{aligned}
p\text{-value} &= P(\bar{X} \leq 42 \mid \mu = 46) \\
&= P\left(T \leq \frac{42-46}{11.9/\sqrt{12}}\right) \\
&= P(T \leq -1.1644) \\
&= 0.1344 > 0.05 \rightarrow \text{fail to reject } H_0
\end{aligned}$$

Method 2: Critical value

$t_{1-\alpha} = t_{0.95} = -1.7956$

test statistic $= -1.1644$

since $-1.1644 > -1.7956 \rightarrow$ fail to reject H_0

Case 1: population variances known

pop 1	pop 2
$\bar{x}_1 = 81$	$\bar{x}_2 = 76$
$\sigma_1 = 5.2$	$\sigma_2 = 3.4$
$n_1 = 25$	$n_2 = 36$

$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 \neq 0$

$\alpha = 0.01 \rightarrow z_{1-\frac{\alpha}{2}} = z_{\frac{\alpha}{2}} = \pm 2.58$

$$z = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{81 - 76 - 0}{\sqrt{\frac{5.2^2}{25} + \frac{3.4^2}{36}}} = 4.2217$$

since $4.2217 > 2.58 \rightarrow$ reject H_0

or using p-value: $2P(z < -4.2217)$
$= 0.00002 < \alpha = 0.05$
reject H_0

case 2: unknown but equal population variances

	lab	no lab
	$\bar{X}_1 = 85$	$\bar{X}_2 = 79$
	$S_1 = 4.7$	$S_2 = 6.1$
	$n_1 = 11$	$n_2 = 17$

① $H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 > 0$

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$$

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 1} = \frac{(4.7)^2(10) + (6.1)^2(16)}{26} = 31.395$$

$$T = \frac{85 - 79 - 0}{\sqrt{\frac{31.395}{11} + \frac{31.395}{17}}} = 2.767$$

$\alpha = 0.05 \rightarrow \cancel{t_{\alpha/2}} \; t_\alpha = t_{0.05,\, n_1 + n_2 - 2} = t_{0.05,\, 26} = 1.706$

since $2.767 > 1.706 \rightarrow$ reject H_0

or using p-value: $P(T_{26} > 2.767) = 0.00514 < \alpha = 0.05 \rightarrow$ reject H_0

by atleast 5 points:

$H_0: \mu_1 - \mu_2 = 5$

$H_a: \mu_1 - \mu_2 > 5$

$$T = \frac{85-79-5}{\sqrt{\frac{31.395}{11} + \frac{31.395}{17}}} = 0.4612 < 1.706 \rightarrow \text{fail to reject}$$

p-value: $P(T_{26, 0.05} > 0.4612) = 0.32425 > \alpha = 0.05$

$\rightarrow$ fail to reject

Case 3: unequal variances - Right tailed

F M

$\bar{X}_1 = 54.05$ $\bar{X}_2 = 22.42$

$S_1 = 41.61$ $S_2 = 23.02$

$n_1 = 37$ $n_2 = 12$

$H_0: \mu_F - \mu_M = 0$

$H_a: \mu_F - \mu_M > 0$

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{54.05 - 22.42 - 0}{\sqrt{\frac{41.61^2}{37} + \frac{23.02^2}{12}}} = 3.3166$$

$\alpha = 0.10 \rightarrow t_{\alpha = 0.10,\, V = 49} = 1.299$

$$V = dof = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\frac{41.61^2}{37} + \frac{23.02^2}{12}\right)^2}{\frac{\left(\frac{41.61^2}{37}\right)^2}{36} + \frac{\left(\frac{23.02^2}{12}\right)^2}{11}} = \underline{49.246}$$

round down to nearest integer $= 49$

Since $3.3166 > 1.299 \rightarrow$ reject H_0

or p-value: $P(T_{49} > 3.3166) = 0.000861 < \alpha \rightarrow$ reject H_0