1 Q1d

This weighted average is growing asymptotically as a function of F(N) with respect to $O(N^2)$

Given $MAX = \frac{N(N-1)}{2}$

F(N) is the weighted average.

$$F(N) = \frac{(Freq(0) * 0 + Freq(1) * 1 + ... + Freq(MAX) * MAX)}{n!}$$

And notice Freq(0) = Freq(Max), Freq(1) = Freq(Max-1), ...

Case 1:
If MAX is even (e.g n=4),
Notice 0*1 + 1*3 + 2*5 + 3*6 + 4*5 + 5*3 + 6*1
can be simplified to 6*1 + 6*3 + 6*5 + 3*6

$F(N) = \frac{(MAX*(Freq(0)+...+Freq(MAX/2-1))+\frac{MAX}{2}*(Freq(max/2))}{n!}$
$F(N) = \frac{\frac{N(N-1)}{2}*(\frac{N!}{2}-\frac{Freq(N/2)}{2})+(\frac{n(n-1)}{4}*freq(n/2))}{n!}$
$F(N) = \frac{\frac{n(n-1)}{4}[(n!-freq(n/2)+Freq(n/2))}{n!}$
$F(N) = \frac{\frac{N(N-1)}{2}*\frac{n!}{2}}{n!}$
$F(N) = \frac{(N(N-1)*N!)}{4N!}$
$F(N) = \frac{N(N-1)}{4}$
$F(N) = \frac{N^2-N}{4}$

Case 2:
If MAX is odd (e.g n=3),
Notice 0*1 + 1*2 + 2*2 + 3*1
Can be simplified to 3*1 + 3*2
Hence,
$F(N) = \frac{(MAX*(Freq(0)+...+Freq(MAX/2))}{n!}$
$F(N) = \frac{(MAX*(\frac{n!}{2})}{n!}$
$F(N) = \frac{(\frac{N(N-1)}{2}*\frac{n!}{2})}{n!}$
$F(N) = \frac{(N(N-1)*n!)}{4n!}$
$F(N) = \frac{N(N-1)}{4}$
$F(N) = \frac{N^2-N}{4}$

Hence, we can say that F(N) follows $O(N^2)$.