Statistical Methods for Discrete Response, Time Series, and Panel Data (W271): Lab 2

Nathan Martinez, Haile Bizunehe, Hannah George, and Meera Sharma

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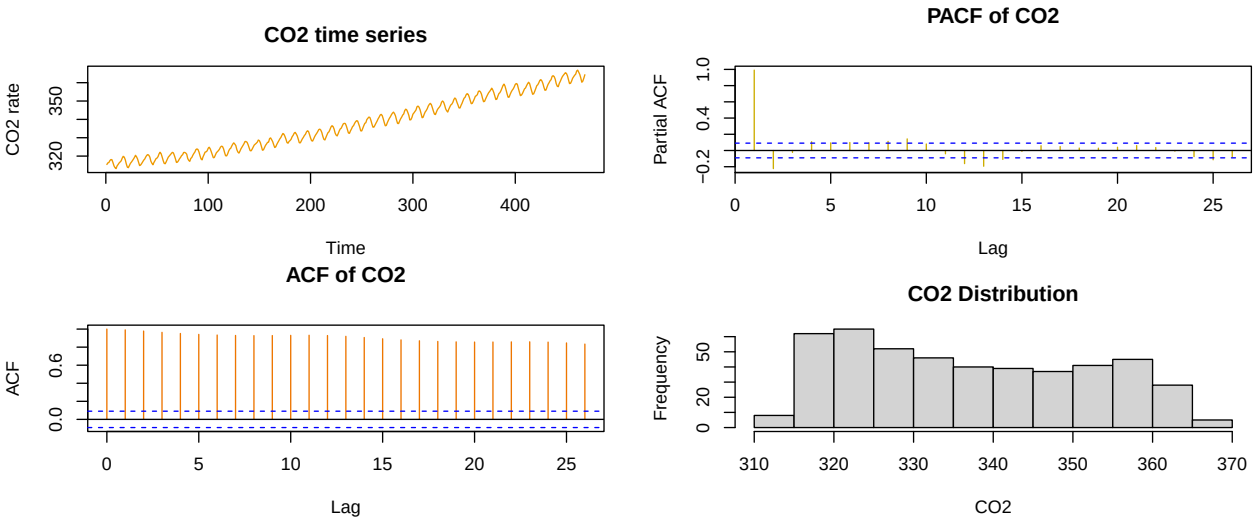
1.1 (3 points) Task 0a: Introduction

1.2 (3 points) Task 1a: CO2 data



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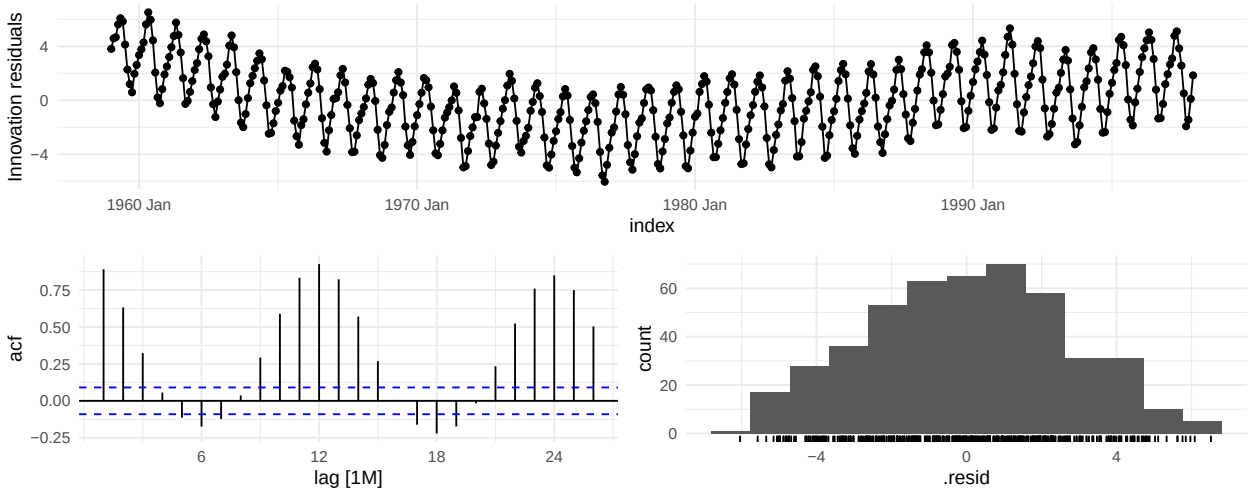
1.3 (3 points) Task 2a: Linear time trend model

```
as_tsibble(co2) -> co2_ts

co2_ts %>%
  model(TSLM(value ~ trend())) -> fit_co2_trend
report(fit_co2_trend)
```

```
## Series: value
## Model: TSLM
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -6.039885 -1.947575 -0.001671  1.911271  6.514852
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.115e+02  2.424e-01  1284.9   <2e-16 ***
## trend()     1.090e-01  8.958e-04   121.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.618 on 466 degrees of freedom
## Multiple R-squared: 0.9695,  Adjusted R-squared: 0.9694
## F-statistic: 1.479e+04 on 1 and 466 DF, p-value: < 2.22e-16
```

'There is an average upward trend of 0.11 CO2 level.'

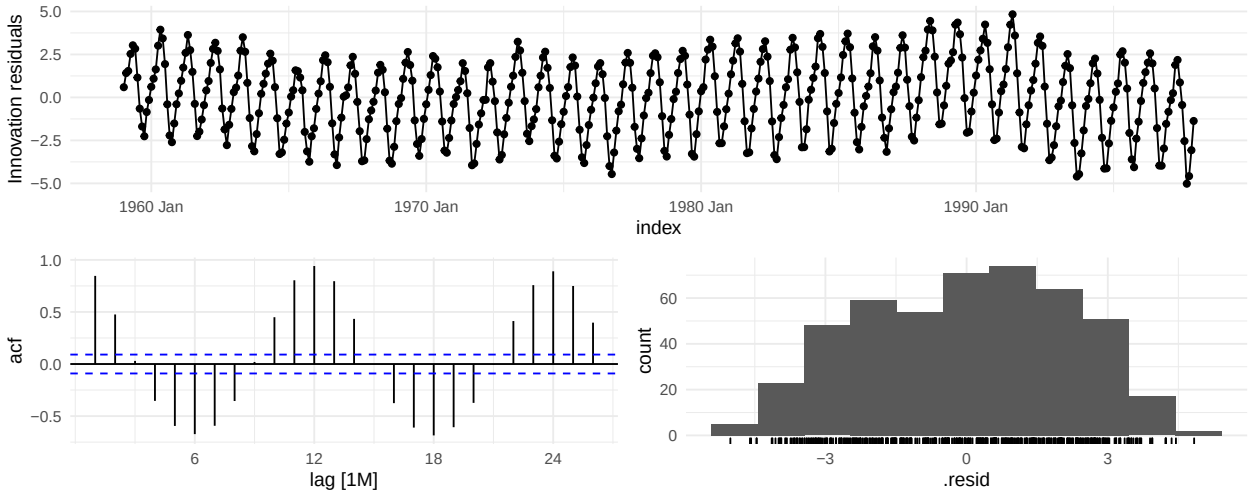


'Both in the scatter plot and the ACF, we see clearly that there is seasonality. There is also a curvy trend that we missed to be captured.'

```
as_tsibble(co2) -> co2_ts

co2_ts %>%
  model(TSLM(value ~ trend()+ I(trend()^2))) -> fit_co2_quad_trend
report(fit_co2_quad_trend)
```

```
## Series: value
## Model: TSLM
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.0195 -1.7120  0.2144  1.7957  4.8345
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.148e+02  3.039e-01 1035.65   <2e-16 ***
## trend()      6.739e-02  2.993e-03   22.52   <2e-16 ***
## I(trend()^2) 8.862e-05  6.179e-06   14.34   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.182 on 465 degrees of freedom
## Multiple R-squared: 0.9788,  Adjusted R-squared: 0.9787
## F-statistic: 1.075e+04 on 2 and 465 DF, p-value: < 2.22e-16
```



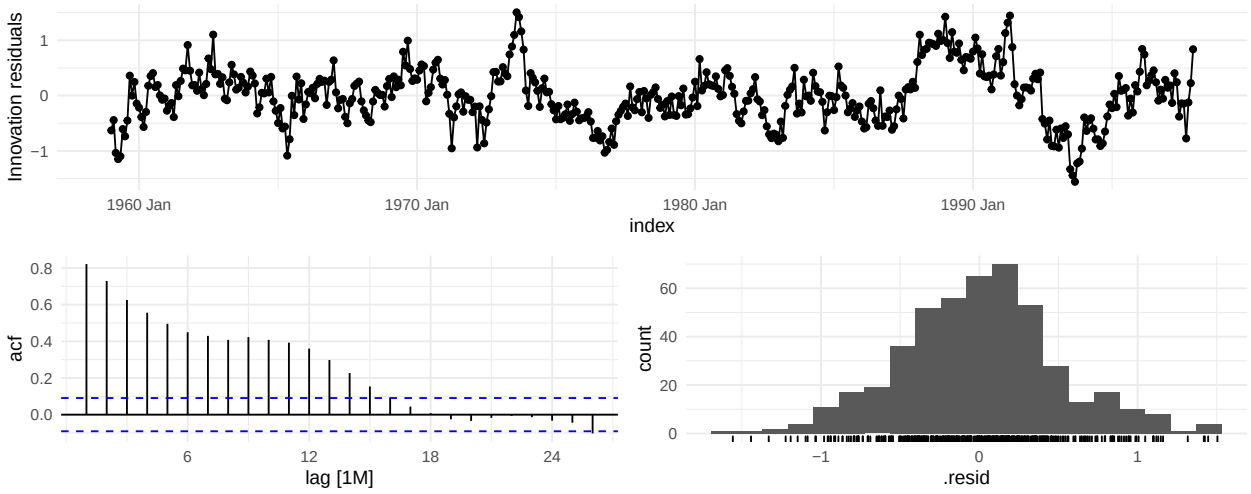
'The trend is better now but there is still an up and down trend that can be modeled using polynomial. We do not see a change on the variance so the log transformation won't be necessary.'

```
as_tsibble(co2) -> co2_ts

co2_ts %>%
  model(TSLM(value ~ trend() + I(trend()^2) + I(trend()^3) + season())) -> fit_co2_full_trend
report(fit_co2_full_trend)
```

```
## Series: value
## Model: TSLM
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.5573094 -0.3312054  0.0008042  0.2880086  1.5039635
##
## Coefficients:
##                  Estimate Std. Error  t value Pr(>|t|)
## (Intercept)     3.160e+02  1.210e-01 2611.629  < 2e-16 ***
## trend()         3.275e-02  1.740e-03   18.827  < 2e-16 ***
## I(trend()^2)    2.744e-04  8.614e-06   31.850  < 2e-16 ***
## I(trend()^3)   -2.640e-07  1.207e-08  -21.863  < 2e-16 ***
## season()year2   6.700e-01  1.145e-01    5.852 9.32e-09 ***
## season()year3   1.419e+00  1.145e-01   12.390  < 2e-16 ***
## season()year4   2.555e+00  1.145e-01   22.319  < 2e-16 ***
## season()year5   3.040e+00  1.145e-01   26.550  < 2e-16 ***
## season()year6   2.383e+00  1.145e-01   20.811  < 2e-16 ***
## season()year7   8.678e-01  1.145e-01    7.578 2.00e-13 ***
## season()year8  -1.194e+00  1.145e-01  -10.429  < 2e-16 ***
## season()year9  -3.013e+00  1.145e-01  -26.311  < 2e-16 ***
## season()year10 -3.191e+00  1.145e-01  -27.860  < 2e-16 ***
## season()year11 -1.996e+00  1.145e-01  -17.428  < 2e-16 ***
## season()year12 -8.738e-01  1.145e-01   -7.628 1.41e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5056 on 453 degrees of freedom
## Multiple R-squared: 0.9989,  Adjusted R-squared: 0.9989
## F-statistic: 2.92e+04 on 14 and 453 DF, p-value: < 2.22e-16
```

,,



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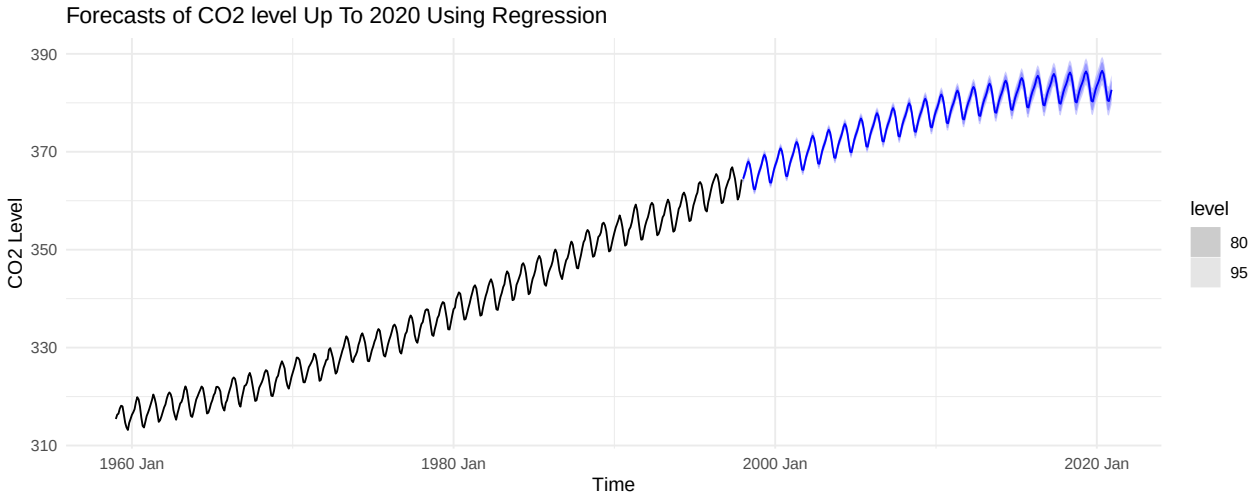

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1.4 (3 points) Task 3a: ARIMA times series model

```
adf.test(co2_ts$value, alternative = "stationary", k = 12)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  co2_ts$value
## Dickey-Fuller = -2.1543, Lag order = 12, p-value = 0.5127
## alternative hypothesis: stationary
```

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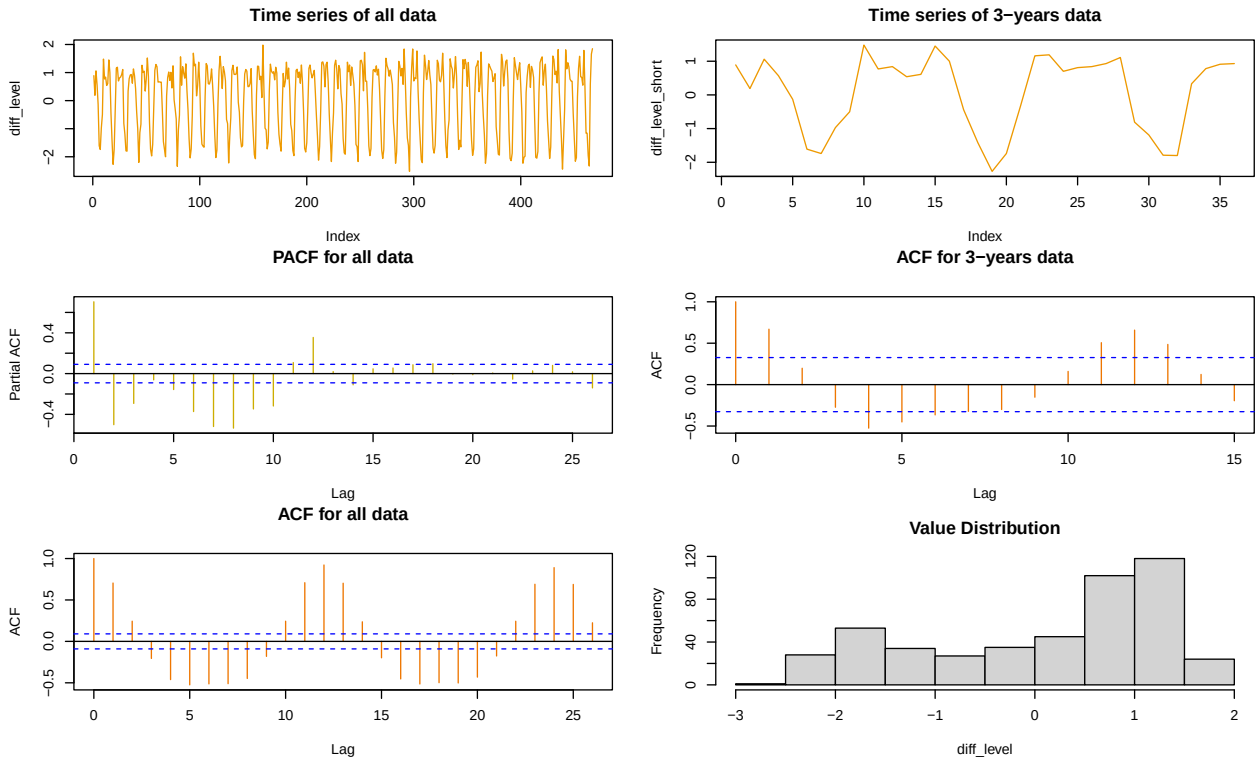

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```
adf.test(diff(co2_ts$value),
         alternative = "stationary", k = 12)
```

```
## Warning in adf.test(diff(co2_ts$value), alternative = "stationary", k = 12): p-
## value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  diff(co2_ts$value)
## Dickey-Fuller = -5.778, Lag order = 12, p-value = 0.01
## alternative hypothesis: stationary
```

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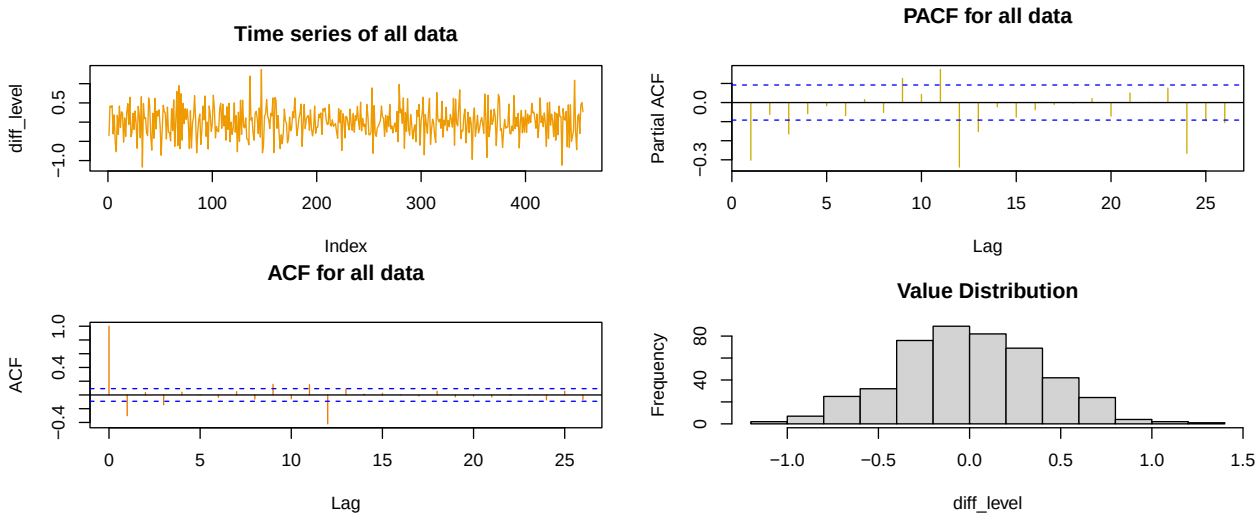

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```
adf.test(diff(diff(co2_ts$value), lag=12),
         alternative = "stationary", k = 3)
```

```
## Warning in adf.test(diff(diff(co2_ts$value), lag = 12), alternative =
## "stationary", : p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  diff(diff(co2_ts$value), lag = 12)
## Dickey-Fuller = -13.69, Lag order = 3, p-value = 0.01
## alternative hypothesis: stationary
```

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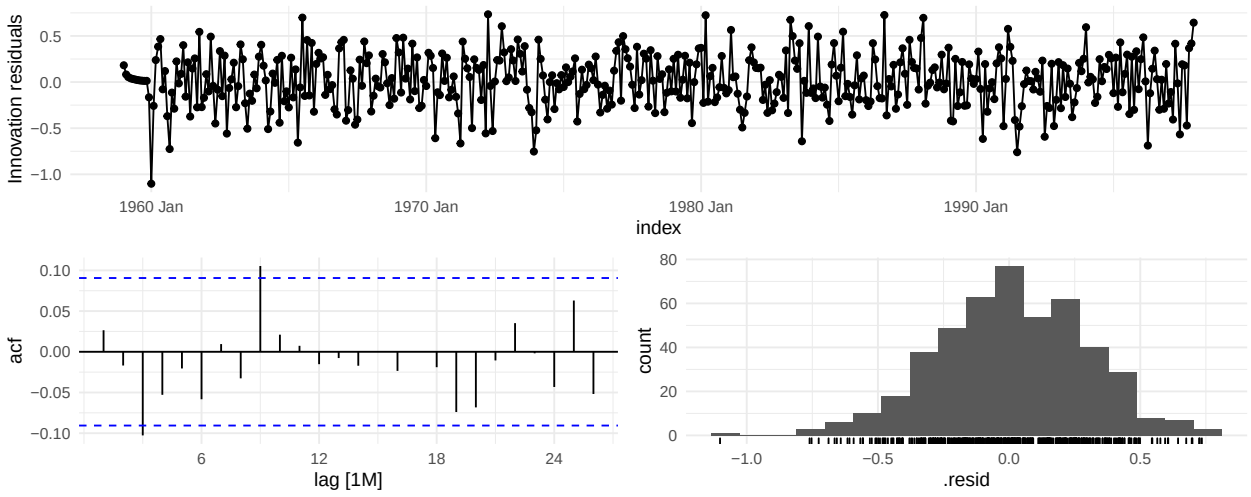
```
model.bic <- co2_ts %>%
  model(ARIMA(value ~ 0 + pdq(0:10, 1, 0:10) + PDQ(0:10, 1, 0:10),
                        ic="bic", stepwise=F, greedy=F))

model.bic %>%
  report()
```

```
## Series: value
## Model: ARIMA(0,1,1)(1,1,2)[12]
##
## Coefficients:
##           ma1     sar1     sma1     sma2
##       -0.3482  -0.4986  -0.3155  -0.4641
## s.e.   0.0499   0.5282   0.5165   0.4367
##
## sigma^2 estimated as 0.08603:  log likelihood=-85.59
## AIC=181.18   AICc=181.32   BIC=201.78
```



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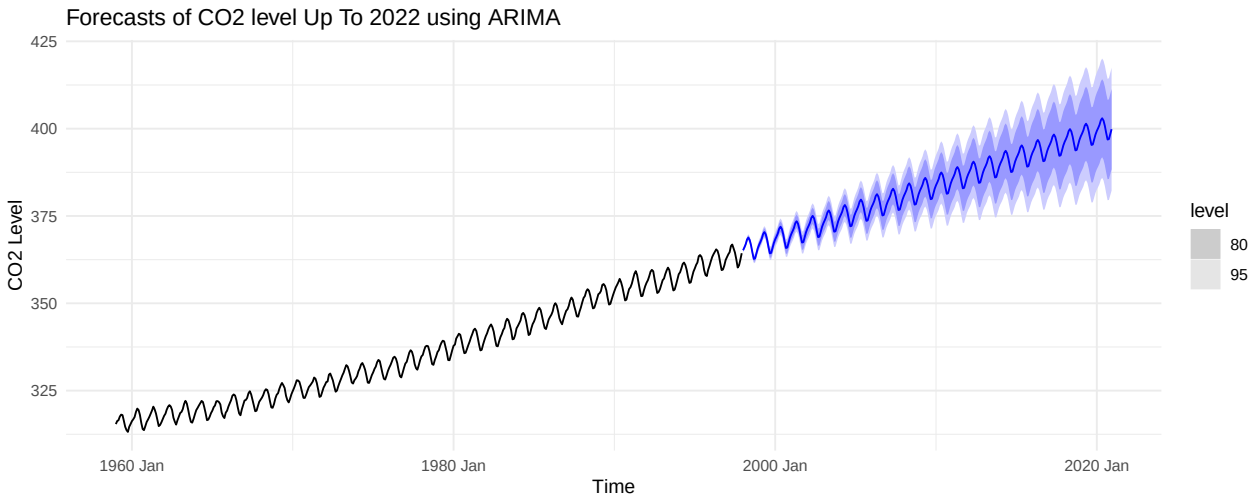

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1.5 (3 points) Task 4a: Forecast atmospheric CO2 growth

```
fc_100_years <- forecast(model.bic, h=1236) %>%
  hilo() %>%
  unpack_hilo(c(`80%`, `95%`))

fc_100_years %>%
  filter(.mean >= 420 & .mean < 421) -> fc_420

fc_100_years %>%
  filter(.mean >= 500 & .mean < 501) -> fc_500

# 420 PPM levels for the first and last time
fc_420_first <- head(fc_420, n =1)
fc_420_last  <- tail(fc_420, n =1)
```

```r
fc_420_first_time  <- fc_420_first$index
fc_420_first_lower <- round(fc_420_first$`95%_lower`, 2)
fc_420_first_upper <- round(fc_420_first$`95%_upper`, 2)
fc_420_first_mean  <- round(fc_420_first$.mean, 2)

fc_420_last_time  <- fc_420_last$index
fc_420_last_lower <- round(fc_420_last$`95%_lower`, 2)
fc_420_last_upper <- round(fc_420_last$`95%_upper`, 2)
fc_420_last_mean  <- round(fc_420_last$.mean, 2)

# 500 PPM levels for the first and last time
fc_500_first <- head(fc_500, n =1)
fc_500_last  <- tail(fc_500, n =1)

fc_500_first_time  <- fc_500_first$index
fc_500_first_lower <- round(fc_500_first$`95%_lower`, 2)
fc_500_first_upper <- round(fc_500_first$`95%_upper`, 2)
fc_500_first_mean  <- round(fc_500_first$.mean, 2)

fc_500_last_time  <- fc_500_last$index
fc_500_last_lower <- round(fc_500_last$`95%_lower`, 2)
fc_500_last_upper <- round(fc_500_last$`95%_upper`, 2)
fc_500_last_mean  <- round(fc_500_last$.mean, 2)

# Atmospheric CO2 levels in the year 2100
fc_2100  <- tail(fc_100_years, n =1)

fc_2100_lower <- round(fc_2100$`95%_lower`, 2)
fc_2100_upper <- round(fc_2100$`95%_upper`, 2)
fc_2100_mean  <- round(fc_2100$.mean, 2)
```

'CO2 is expected to be at 420 ppm level for the first time on 2031 May with expected value of 420.06 with 392.3 - 447.82 95% confidence interval.'
'CO2 is expected to be at 420 ppm level for the last time on 2035 Oct with expected value of 420.3 with 387.79 - 452.8 95% confidence interval.'
'CO2 is expected to be at 500 ppm level for the first time on 2083 Apr with expected value of 500.21 with 403.22 - 597.2 95% confidence interval.'
'CO2 is expected to be at 500 ppm level for the last time on 2087 Sep with expected value of 500.89 with 396.82 - 604.96 95% confidence interval.'
'By the end of 2100 year, the CO2 is expected to be at 524.03 ppm level with 397.75 - 650.31 95% confidence interval.'

2 Report from the Point of View of the Present

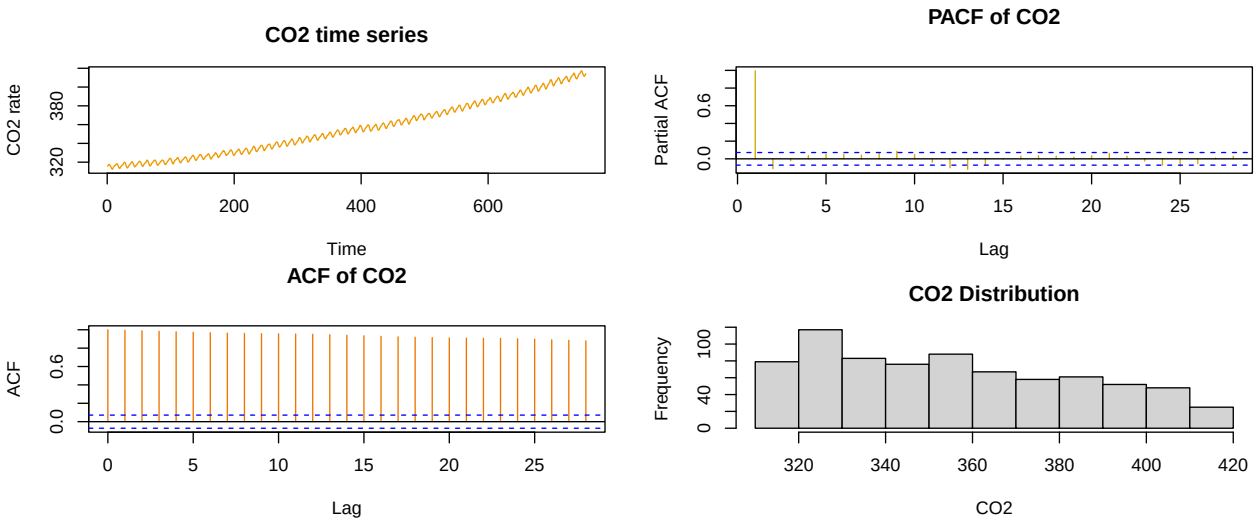
2.1 (1 point) Task 0b: Introduction

2.2 (3 points) Task 1b: Create a modern data pipeline for Mona Loa CO2 data.

Below is our code to read in the data from the appropriate URL and perform minor transformations in order to get the data into a proper time series object.

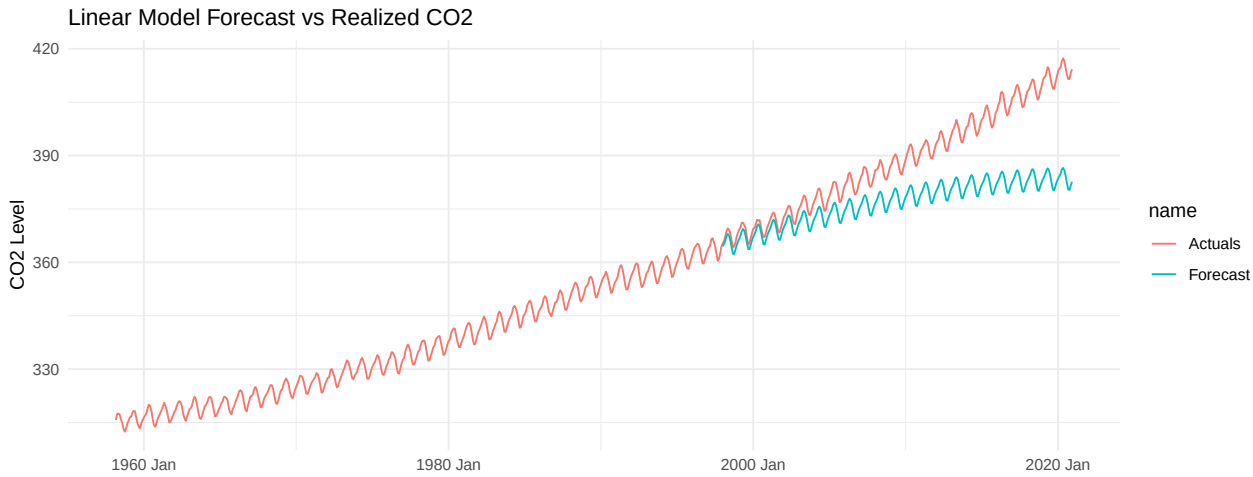
```
read.csv(
  url("https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_mm_mlo.csv"),
  skip = 52) %>%
  mutate(time_index = make_datetime(year, month)) %>%
  mutate(time_index = yearmonth(time_index)) %>%
  filter(year < 2021) %>%
  as_tsibble(index = time_index) -> co2_present
```

Now we will start our EDA of the present data.



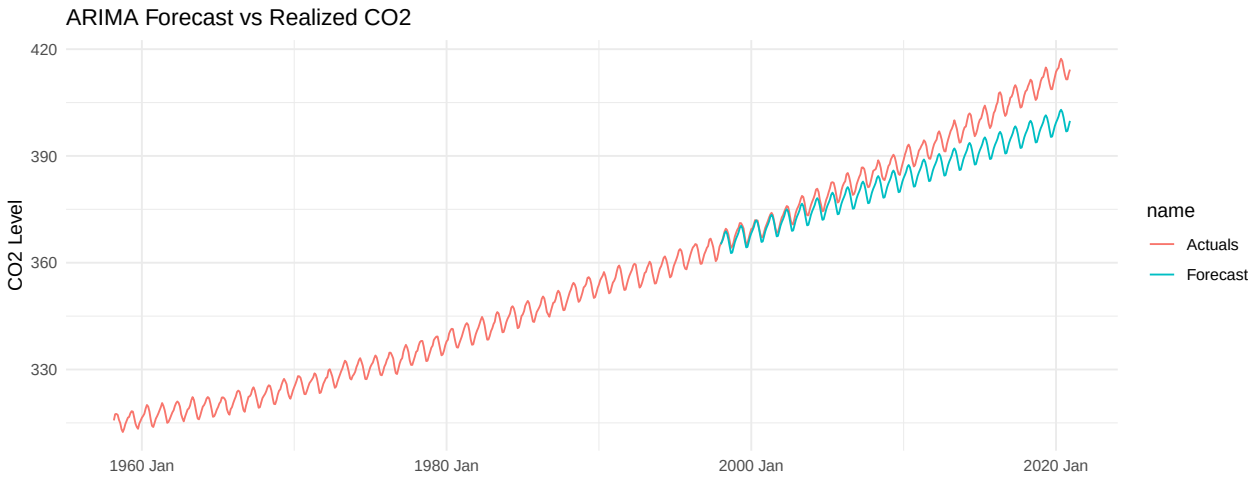
When looking at the present data, there is not a huge difference between how it looks now versus how it looked in 1997. The time series plot shows that the CO2 levels continued to grow - at perhaps a stepper level of growth than before. The most notable difference is the CO2 Distribution histogram. In 1997 the distribution appeared to be almost bimodal, whereas now the distribution looks more heavy-tailed, and extends to much higher levels than it did previously.

2.3 (1 point) Task 2b: Compare linear model forecasts against realized CO2



The linear model forecast did not correctly capture the trend of the realized CO2 levels. The forecast appears to predict a stabilization in the CO2 levels, whereas the actual CO2 level trend increased.

2.4 (1 point) Task 3b: Compare ARIMA models forecasts against realized CO2



The ARIMA forecast is much closer to the realized CO2 levels than the Linear Model forecast. The only difference is that the ARIMA model appears to have forecasted a linear trend, while the realized CO2 levels followed an almost exponential growth.

2.5 (3 points) Task 4b: Evaluate the performance of 1997 linear and ARIMA models

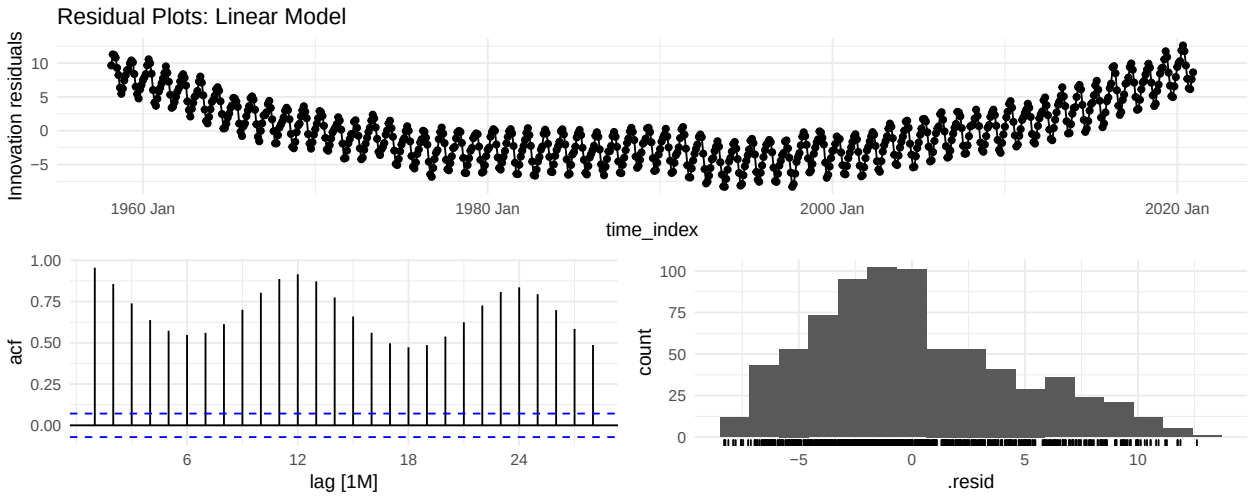
```
# Max actual data from 1958 to 2020
max_actual_row <- co2_present[which.max(co2_present$average),]
max_actual_date <- max_actual_row$time_index
max_actual_value <- max_actual_row$average

# Predicted values and date over the max actual
fc_100_years %>%
  filter(.mean >= max_actual_value) -> fc_417
fc_417_first <- head(fc_417, n =1)
fc_417_first_value <- fc_417_first$.mean
fc_417_first_time  <- fc_417_first$index

# Precited values on actual maximum date
fc_100_years %>%
  filter(index == max_actual_date) -> max_date_pred_values
pred_date_average <- round(max_date_pred_values$.mean, 2)
pred_date_lower <- round(max_date_pred_values$`95%_lower`, 2)
pred_date_upper <- round(max_date_pred_values$`95%_upper`, 2)

# Difference between predicted and actual on max date
pred_act_diff <- max_actual_value - pred_date_average
```

We originally predicted that CO2 would cross the 420 ppm threshold for the first time on 2031 May with an expected value of 420.06 with a 95% confidence interval between 392.3 - 447.82. The maximum average monthly value to date is 417.31 on 2020 May. In our predictions, the nearest date with a value at or over 417.31 is 2030 Apr. Therefore, our predicted average CO2 values were approximately 10-years behind the

actuals. When comparing our predicted values in 2020 May to the actual data, we observe an average predicted value of 402.99 with a 95% confidence interval between 385.92 - 420.05. Although our predicted average value was lower than the actual by value by 14.32 ppm, the actual value of 417.31 is between our 95% confidence interval 385.92 - 420.05.

```
# Build model
co2_present %>%
  model(TSLM(average ~ trend())) -> fit_co2_trend_present

# Print report
report(fit_co2_trend_present)
```

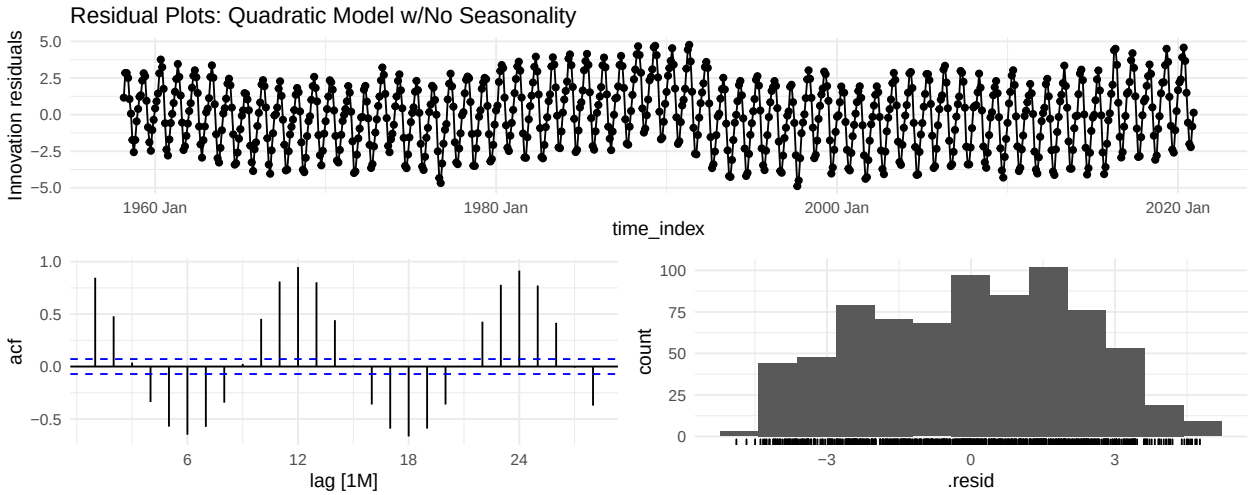
```
## Series: average
## Model: TSLM
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2948 -3.1486 -0.6718  2.7815 12.6017
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.059e+02  3.223e-01   949.2   <2e-16 ***
## trend()     1.323e-01  7.396e-04   178.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.42 on 752 degrees of freedom
## Multiple R-squared: 0.977,   Adjusted R-squared: 0.977
## F-statistic: 3.198e+04 on 1 and 752 DF, p-value: < 2.22e-16
```



Similar to the previous linear model, the linear model exhibits a significant negative curve, meanding the linear prediction is lower than that actual values in the initial and last periods but that it has higher predicted values in the time period in-between the initial and last periods. Therefore, the mean of the residuals are not close to zero and the residual variance does not appear to be constant. The ACF shows a positive relationship between the lag periods, which becomes stronger as the month lag approaches 12. This demonstrates a strong correlation between time periods, which is due to the strong correlation between seasons and the overall positive trend in CO2 emissions. The histogram of the residuals is skewed to the right, indicating that the residuals are not normally distributed.

```
# Build model
co2_present %>%
  model(TSLM(average ~ trend()+ I(trend()^2))) -> fit_co2_quad_trend_present

# Print model
report(fit_co2_quad_trend_present)
```

```
## Series: average
## Model: TSLM
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.8867 -1.8137  0.1124  1.7773  4.7687
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.145e+02  2.441e-01 1288.45   <2e-16 ***
## trend()      6.431e-02  1.493e-03   43.07   <2e-16 ***
## I(trend()^2) 8.999e-05  1.915e-06   47.00   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.228 on 751 degrees of freedom
## Multiple R-squared: 0.9942,  Adjusted R-squared: 0.9942
## F-statistic: 6.404e+04 on 2 and 751 DF, p-value: < 2.22e-16
```

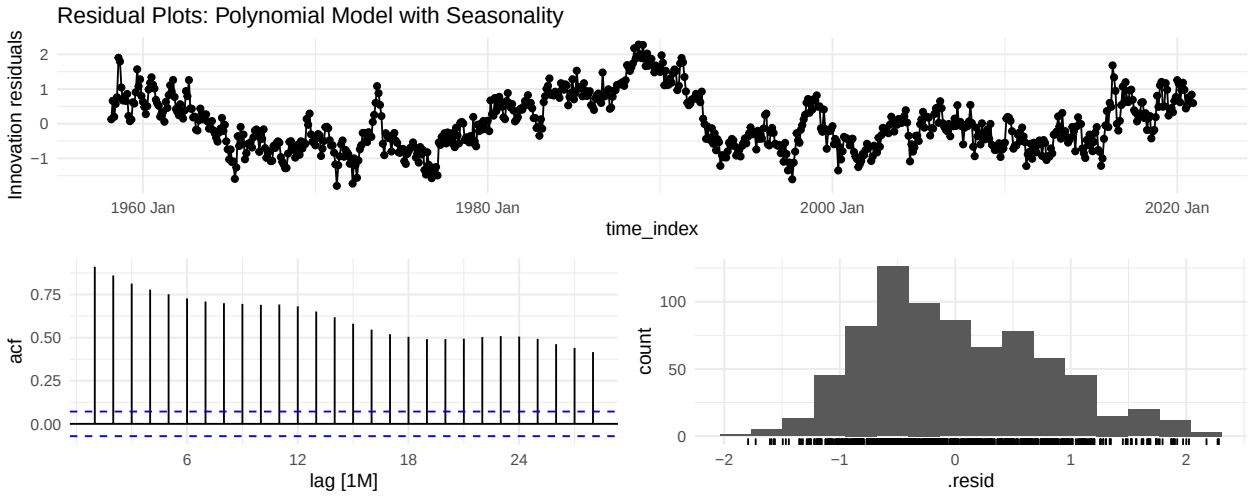


The residual plots for the polynomial model indicate a mean and a variance that is closer to zero. However, there are some clear trends in the residual plots, indicating that the residuals are not constant. The ACF shows a positive relationship between the lag periods that are 12 months apart, while exhibiting a negative correlation at a 6-month lag. These values are beyond the confidence thresholds, meaning the residual are strongly correlated. The histogram of the residusls has a distribution that is significantly wide, representing a cross between a bell-shaped curve and a uniform distriubtion.

```
# Build model
co2_present %>%
  model(TSLM(average ~ trend() + I(trend()^2) + I(trend()^3) + season())) -> fit_co2_full_trend_present

# Print model
```

```
report(fit_co2_full_trend_present)
```

```
## Series: average
## Model: TSLM
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.7936 -0.5823 -0.1251  0.5784  2.2826
##
## Coefficients:
##                   Estimate Std. Error  t value Pr(>|t|)
## (Intercept)     3.141e+02  1.492e-01 2105.620  < 2e-16 ***
## trend()         7.030e-02  1.313e-03   53.551  < 2e-16 ***
## I(trend()^2)    7.040e-05  4.039e-06   17.430  < 2e-16 ***
## I(trend()^3)    1.735e-08  3.517e-09    4.934 9.96e-07 ***
## season()year2   6.277e-01  1.405e-01    4.469 9.11e-06 ***
## season()year3   1.361e+00  1.399e-01    9.727  < 2e-16 ***
## season()year4   2.508e+00  1.399e-01   17.927  < 2e-16 ***
## season()year5   2.960e+00  1.399e-01   21.152  < 2e-16 ***
## season()year6   2.247e+00  1.399e-01   16.060  < 2e-16 ***
## season()year7   6.013e-01  1.399e-01    4.298 1.96e-05 ***
## season()year8  -1.538e+00  1.399e-01  -10.995  < 2e-16 ***
## season()year9  -3.233e+00  1.399e-01  -23.103  < 2e-16 ***
## season()year10 -3.323e+00  1.399e-01  -23.749  < 2e-16 ***
## season()year11 -2.120e+00  1.399e-01  -15.150  < 2e-16 ***
## season()year12 -9.391e-01  1.399e-01   -6.712 3.83e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7821 on 739 degrees of freedom
## Multiple R-squared: 0.9993,  Adjusted R-squared: 0.9993
## F-statistic: 7.462e+04 on 14 and 739 DF, p-value: < 2.22e-16
```

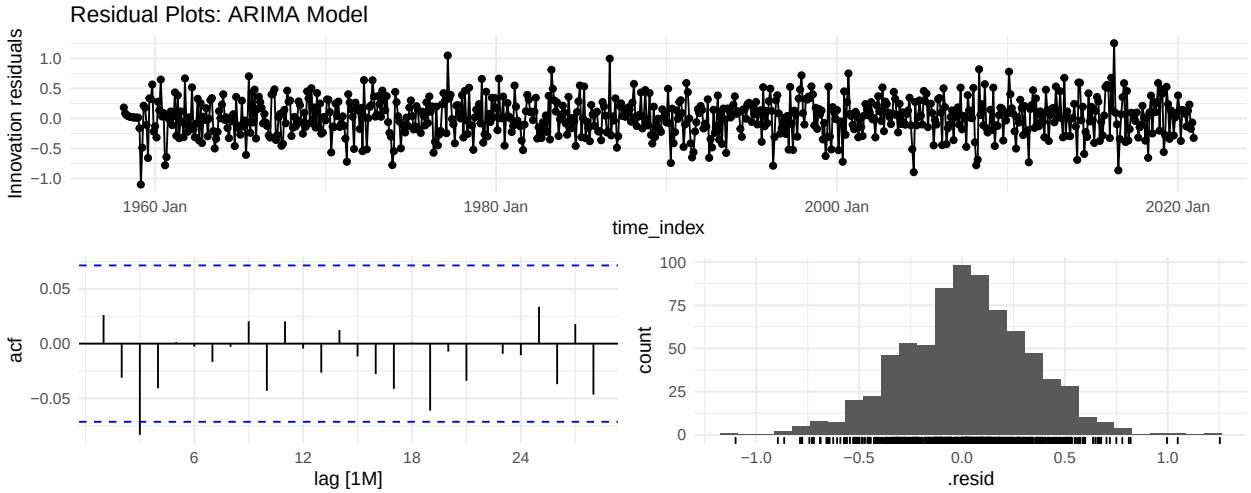


The time series plot of the residuals demonstrates residual values that may have a mean eqaul to zero but both the mean and the variance are not constant. This suggests that the the polynomial model with the seasonal adjustment might not be predicting the seasonal and/or positive trend of the underlying data. The ACF shows a strong correlation between the first period and the proceeding 24 - 30 months. The histogram

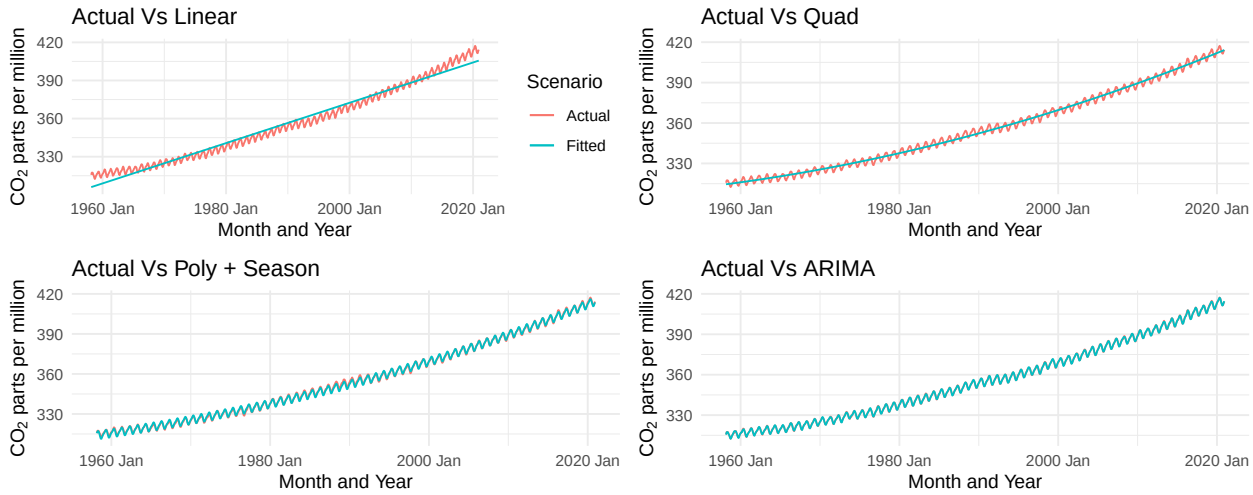
of the residuals is skewed to the right, meaning the residuals are not normally distributed.

```
model.bic2 <- co2_present %>%
  model(ARIMA(average ~ 0 + pdq(0, 1, 1) + PDQ(1, 1, 2),
                        ic="bic"))

model.bic2 %>%
  report()
```

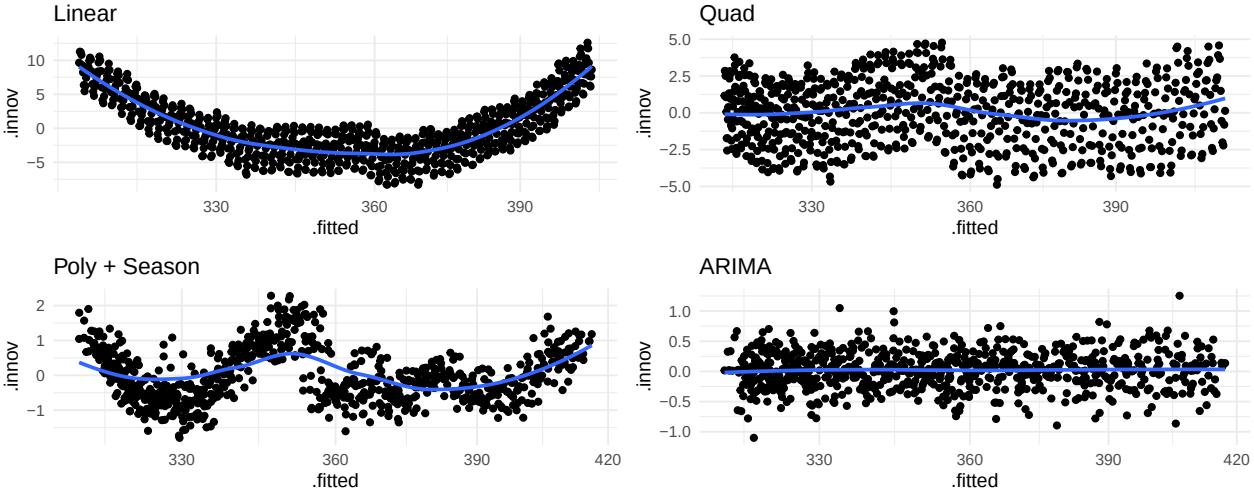
```
## Series: average
## Model: ARIMA(0,1,1)(1,1,2)[12]
##
## Coefficients:
##           ma1     sar1     sma1     sma2
##       -0.3816  -0.3091  -0.5396  -0.2793
## s.e.   0.0381   1.4942   1.4992   1.2918
##
## sigma^2 estimated as 0.09794:  log likelihood=-190.1
## AIC=390.2   AICc=390.29   BIC=413.24
```



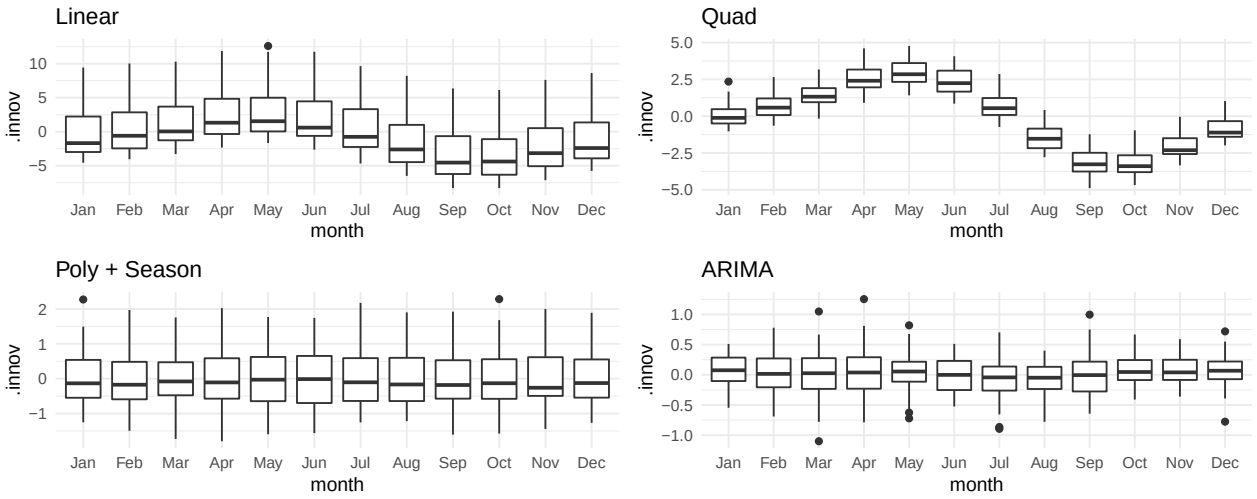
The original ARIMA model we had developed had the following hyper parameters (model.bic2): ARIMA(0,1,1)(1,1,2)[12]. However, when using the same code to identify the best model with the updated data from 1997 to 2020, the following hyper parameters were identified as the best fit: ARIMA(1,1,1)(2,1,1)[12]. This suggests that a different set of hyperparameters should be selected given the updated data. However, based on our research question we decided to use the same model as before in our diagnostic plots. The residual plots for the arima model indicate a mean and a variance that is is constant and closer to zero.The ACF shows what looks like a random trend and there is only one value at time lag 3 that extends beyond the threshold. If should be noted that no ACF values extend beyond this threshold using the new hyper parameters. The histogram of the residuals has a distribution that appears to be normally distributed.



The graphs above shows the plots of the fitted models versus the actuals over the observation period. Both the linear and quadratic with no seasonal adjustment do not adjust for any seasonal variation. Although the polynomial model with seasonal variation appears to fit the data quite well, there are periods where the model under estimates or over estimates. The ARIMA model appears to have the most accurate predictions relative to the other models.



The graphs above shows the plots of the residuals versus the fitted values. With the exception of the ARIMA model, all residual plots show a pattern, meaning there may be heteroscedasticity in the errors and the variance of the residuals may not be constant



The graphs above shows boxplots of the residuals by month. The linear and quadratic models show obvious deviations from mean of zero, while the quadratic plus seasonal model and the ARIMA model have residual values near zero.

```
augment(fit_co2_trend_present) %>%
  features(.innov, ljung_box, dof = 14, lag = 24)
```

```
## # A tibble: 1 x 3
##   .model                   lb_stat lb_pvalue
##   <chr>                      <dbl>     <dbl>
## 1 TSLM(average ~ trend())   9264.         0
```

```
augment(fit_co2_quad_trend_present) %>%
  features(.innov, ljung_box, dof = 14, lag = 24)
```

```
## # A tibble: 1 x 3
##   .model                                lb_stat lb_pvalue
##   <chr>                                   <dbl>     <dbl>
## 1 TSLM(average ~ trend() + I(trend()^2))   6081.         0
```

```
augment(fit_co2_full_trend_present) %>%
  features(.innov, ljung_box, dof = 14, lag = 24)
```

```
## # A tibble: 1 x 3
##   .model                                                      lb_stat lb_pvalue
##   <chr>                                                         <dbl>     <dbl>
## 1 TSLM(average ~ trend() + I(trend()^2) + I(trend()^3) + seas~   7886.         0
```

```
augment(model.bic2) %>%
  features(.innov, ljung_box, dof = 14, lag = 24)
```

```
## # A tibble: 1 x 3
##   .model                                                      lb_stat lb_pvalue
##   <chr>                                                         <dbl>     <dbl>
## 1 "ARIMA(average ~ 0 + pdq(0, 1, 1) + PDQ(1, 1, 2), ic = \"bi~    16.7    0.0802
```

Finally, the p-value for all of the respective ljung_box test statistic is less than 0.05, except the ARIMA model. Therefore, we can conclude that the residual values are independent for the ARIMA model only.

2.6 (4 points) Task 5b: Train best models on present data

Splitting both the SA and NSA time series into training and test sets.

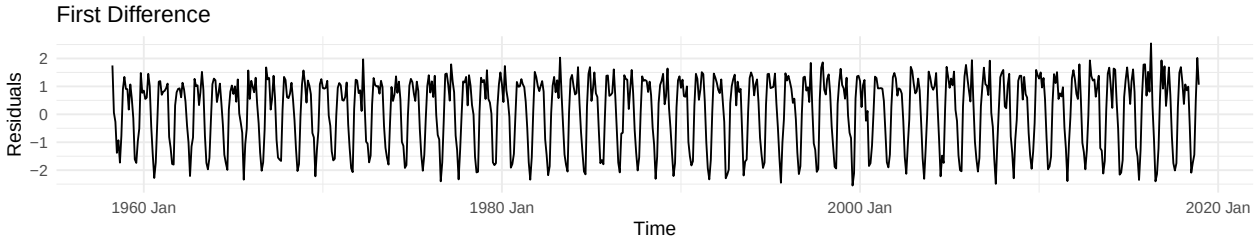
```
# Test size
test.size <- 24 # Months
test.df <- tail(co2_present, test.size) %>%
  mutate(.mean = average) # Used later for ggplot

# Train for non-seasonal (ns)
train_ns <- head(co2_present, nrow(co2_present) -  test.size)

# Train for seasonal (s)
train_s <- head(co2_sa, nrow(co2_sa) -  test.size) %>%
  subset(select=-c(.model))
```

2.6.1 Training the ARIMA Model on the NSA Data

```
adf.test(train_ns$average, alternative = "stationary")
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  train_ns$average
## Dickey-Fuller = -0.72341, Lag order = 8, p-value = 0.9684
## alternative hypothesis: stationary
```

The ADF test statistic and p-value come out to be equal to -0.72 and 0.96 respectively. Since the p-value is greater than 0.05, we would fail to reject the null hypothesis that the time series is non-stationary. Thus, we will apply differencing in order to attempt to make the time series stationary.



Based on the above plot, it would appear that the first difference appears to be stationary. We will verify with an ADF test.

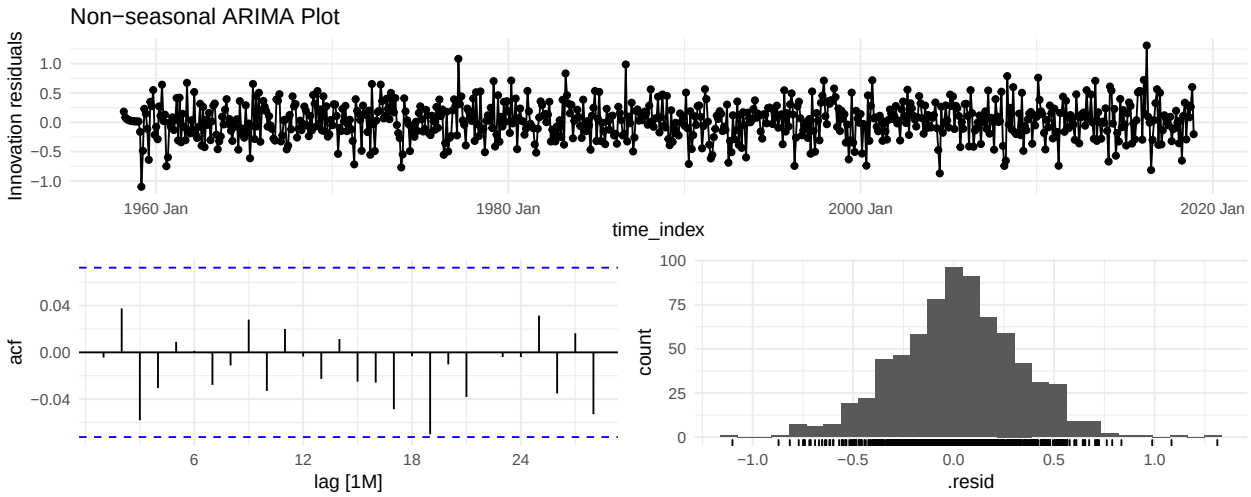
```
ns_diff_data <- train_ns %>%
  mutate(second_diff = difference(average, differences = 1)) %>%
  filter(!is.na(second_diff))

adf.test(ns_diff_data$second_diff, alternative = "stationary")
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ns_diff_data$second_diff
## Dickey-Fuller = -34.717, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
```

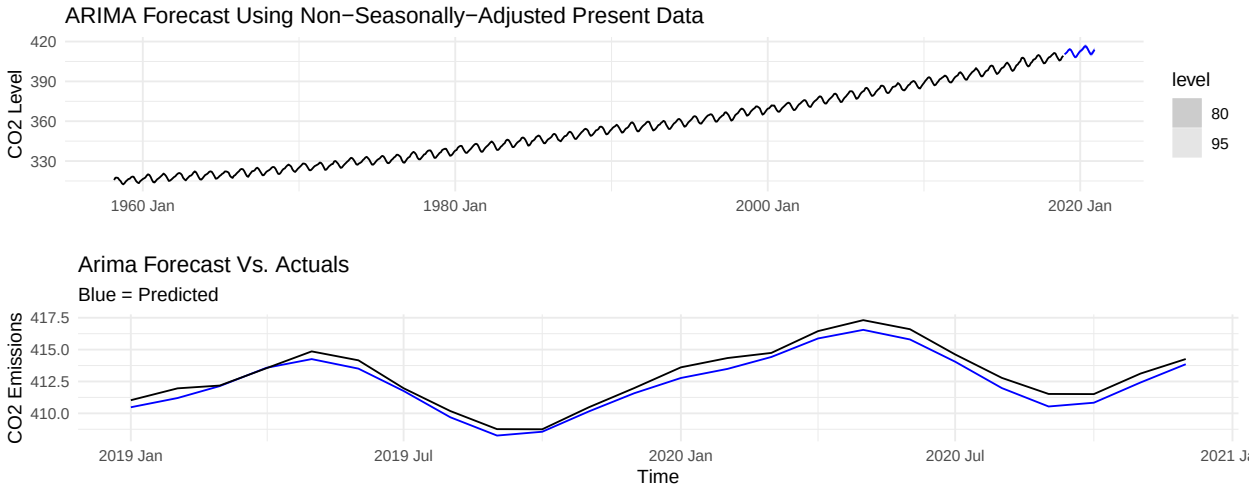
The ADF test statistic and p-value come out to be equal to -34.717 and 0.01, respectively. Since the p-value is less than 0.05, we would reject the null hypothesis that the time series is non-stationary. Because we know we should use first differencing, we will set these hyper parameters and then select a model with the lowest BIC.

```
model.ns <- train_ns %>%
  model(ARIMA(average ~ 0 + pdq(0:10, 1, 0:10) + PDQ(0:10, 1, 0:10),
                        ic="bic", stepwise=F, greedy=F))
model.ns %>%
  report()
```

```
## Series: average
## Model: ARIMA(1,1,1)(2,1,1)[12]
##
## Coefficients:
##          ar1      ma1     sar1     sar2     sma1
##       0.1961  -0.5508  -0.0052  -0.0252  -0.8596
## s.e.  0.1019   0.0880   0.0443   0.0427   0.0244
##
## sigma^2 estimated as 0.0981:  log likelihood=-184.29
## AIC=380.57   AICc=380.69   BIC=408.02
```



The residual plots for the ARIMA model appear to have constant variance and a mean near zero. The histogram also appears to be normally distributed.



The forecast plots seem to track nicely with the actuals for the 24 month period.

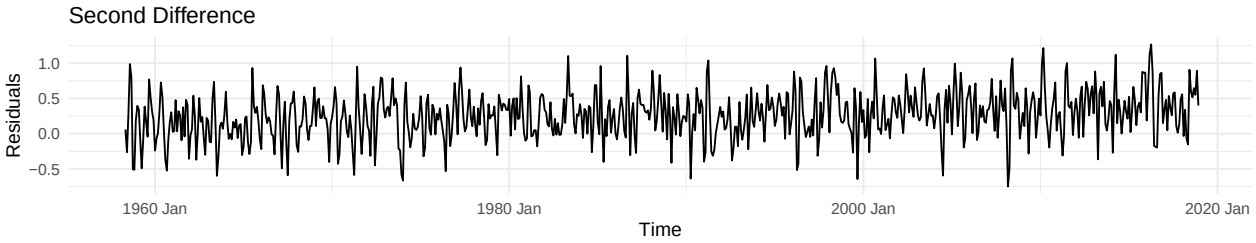
2.6.2 Training the ARIMA Model on the SA Data

We will perform an ADF test to determine whether the seasonally-adjusted data is stationary.

```
adf.test(train_s$season_adjust, alternative = "stationary")
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  train_s$season_adjust
## Dickey-Fuller = -0.33025, Lag order = 8, p-value = 0.9895
## alternative hypothesis: stationary
```

The ADF test statistic and p-value come out to be equal to -0.33 and 0.9895 respectively. Since the p-value is greater than 0.05, we would fail to reject the null hypothesis that the time series is non-stationary. Thus, we will apply differencing in order to attempt to make the time series stationary.



Based on the above plot, it would appear that the first difference appears to be stationary. We will verify with an ADF test.

```
s_diff_data <- train_s %>%
  mutate(first_diff = difference(season_adjust, 2)) %>%
  filter(!is.na(first_diff))

adf.test(s_diff_data$first_diff, alternative = "stationary")
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  s_diff_data$first_diff
## Dickey-Fuller = -7.8626, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
```

The ADF test statistic and p-value come out to be equal to -7.86 and 0.01 respectively. Since the p-value is less than 0.05, we could reject the null hypothesis that the time series is non-stationary.

Due to these results, we decided to use a second difference and a seasonal difference of 0, and we let the ARIMA function choose the remaining hyper parameters.

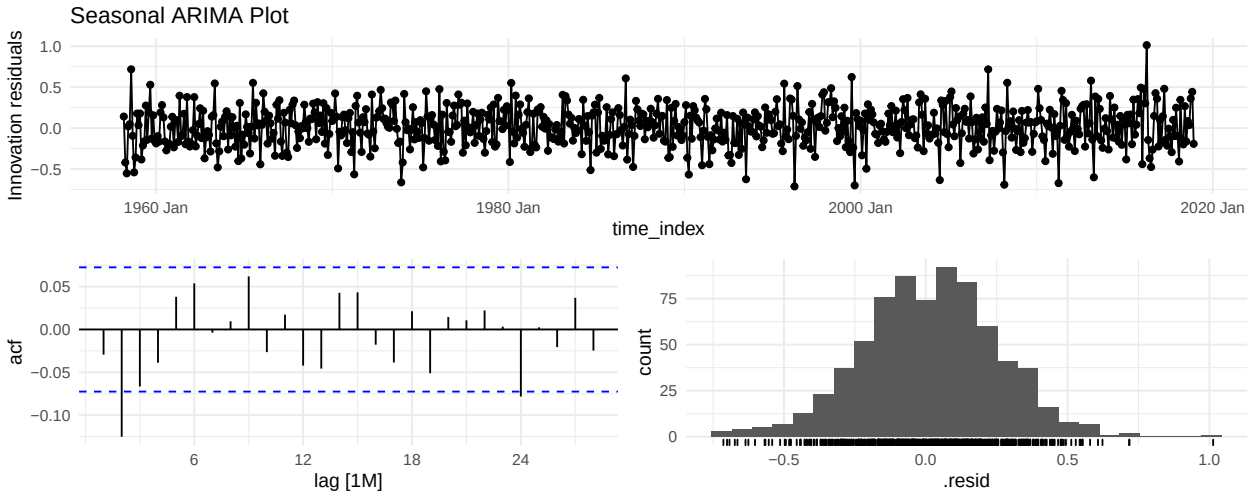
```
model.s <- train_s %>%
  model(ARIMA(season_adjust ~ 0 + pdq(0:10, 2, 0:10) + PDQ(0:10, 0, 0:10),
                        ic="bic", stepwise=F, greedy=F))
```

```
## Warning in sqrt(diag(best$var.coef)): NaNs produced
```

```
model.s %>%
  report()
```

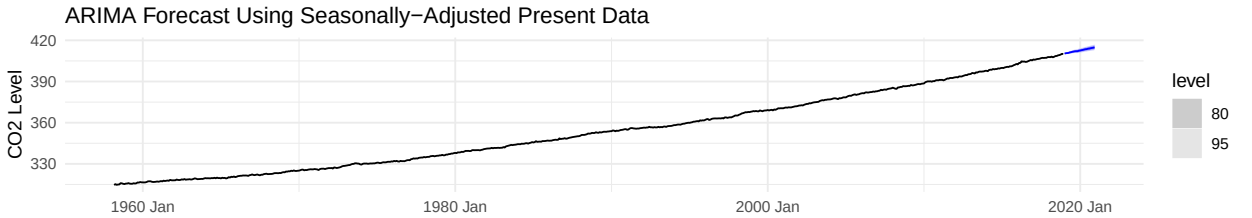
```
## Series: season_adjust
## Model: ARIMA(1,2,1)(4,0,0)[12]
##
```

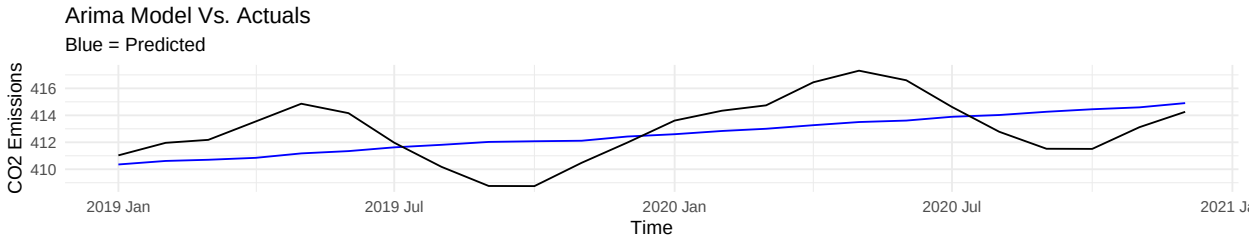
```
## Coefficients:
##           ar1      ma1     sar1     sar2     sar3     sar4
##       -0.3249  -0.9694  -0.3854  -0.3802  -0.3001  -0.2129
## s.e.      NaN   0.0078   0.0037      NaN      NaN      NaN
##
## sigma^2 estimated as 0.05821:  log likelihood=1.5
## AIC=11   AICc=11.16   BIC=43.14
```




```
augment(model.s) %>%
  features(.innov, ljung_box, dof = 14, lag = 24)
```

```
## # A tibble: 1 x 3
##   .model                                                lb_stat lb_pvalue
##   <chr>                                                   <dbl>     <dbl>
## 1 "ARIMA(season_adjust ~ 0 + pdq(0:10, 2, 0:10) + PDQ(0:10, 0~    38.0 0.0000385
```



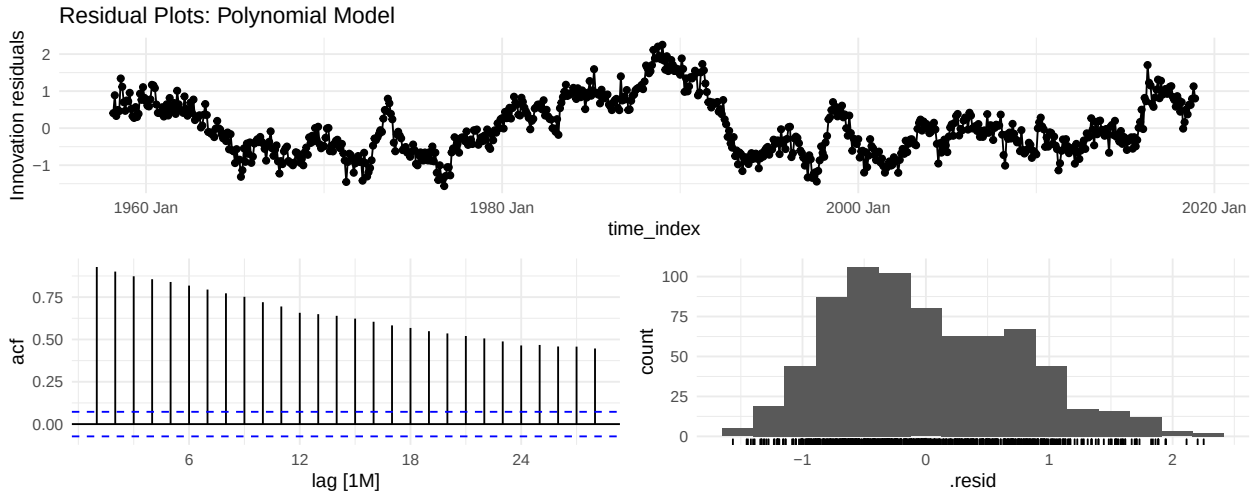


The ARIMA hyper parameters that were selected include the following: ARIMA(1,2,1)(4,0,0)[12]. Using these values, the residual plots appear to meet the condition of constant variance and a mean near zero. The test results also past the ljung_box statistical test. The ACF had some extreme values at the 2nd and 24th time lag, which seems a bit strange. The forecast estimates appear strange when juxtaposed next to the monthly data inclusive of the seasonal trends but that is because the underlying seasonality has been removed.

2.6.3 Training the Polynomial Time-Trend Model on the SA Data

```
# Build model
train_s %>%
  model(TSLM(season_adjust ~ trend() + I(trend()^2) + I(trend()^3))) -> fit_poly

# Print model
report(fit_poly)
```

```
## Series: season_adjust
## Model: TSLM
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5643 -0.5742 -0.1199  0.5283  2.2516
##
## Coefficients:
##               Estimate Std. Error  t value Pr(>|t|)
## (Intercept)  3.142e+02  1.103e-01 2849.568   <2e-16 ***
## trend()      6.772e-02  1.305e-03   51.875   <2e-16 ***
## I(trend()^2) 8.074e-05  4.148e-06   19.463   <2e-16 ***
## I(trend()^3) 6.554e-09  3.730e-09    1.757   0.0793 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.741 on 726 degrees of freedom
## Multiple R-squared: 0.9993,  Adjusted R-squared: 0.9993
## F-statistic: 3.375e+05 on 3 and 726 DF, p-value: < 2.22e-16
```



The residual diagnostic plots does not produce promising results. The residual time series does not have a constant variance or mean, and it is not clear that the mean is near zero. All values within the ACF exceed the threshold values and the histogram of the residuals appears to be skewed to the right.

As a result, we have decided to go with the `ARIMA(1,1,1)(2,1,1)[12]` model because it adjusts for the underlying seasonality, which we may need to adjust going forward, all of its residual values are are within the acceptable bands of the ACF, and it is easier to explain the model results to a non-technical audience.

2.7 (3 points) Task Part 6b: How bad could it get?

We will now retrain the selected model on all of the available data.

```
model.bic3 <- co2_present %>%
  model(ARIMA(average ~ 0 + pdq(1, 1, 1) + PDQ(2, 1, 1), ic="bic"))
```

Now we will get predictions from the model.

```
# Prediction from 2021 to 2121
fc2_100 <- forecast(model.bic3, h=1224) %>%
  hilo() %>%
  unpack_hilo(c(`80%`, `95%`))

fc2_100 %>%
  filter(.mean >= 420 & .mean < 421) -> fc2_420

# 420 PPM levels for the first and last time
fc2_420_first <- head(fc2_420, n =1)
fc2_420_last  <- tail(fc2_420, n =1)

# Get values for 420 ppm predictions
fc2_420_first_time  <- fc2_420_first$time_index
fc2_420_first_lower <- round(fc2_420_first$`95%_lower`, 2)
fc2_420_first_upper <- round(fc2_420_first$`95%_upper`, 2)
fc2_420_first_mean  <- round(fc2_420_first$.mean, 2)

fc2_420_last_time  <- fc2_420_last$time_index
fc2_420_last_lower <- round(fc2_420_last$`95%_lower`, 2)
fc2_420_last_upper <- round(fc2_420_last$`95%_upper`, 2)
fc2_420_last_mean  <- round(fc2_420_last$.mean, 2)
```

Based on the findings from our model, we predict that CO2 will cross the 420 ppm threshold for the first time on 2023 Jan with an expected value of 420.42 with a 95% confidence interval between 418.48 - 422.36. Our model also predicts that the last time CO2 will be between the 420 and 421 ppm threshold will be on 2024 Oct with an expected value of 420.77 with a 95% confidence interval between 417.92 - 423.62.

```
fc2_100 %>%
  filter(.mean >= 500 & .mean < 501) -> fc2_500

# 420 PPM levels for the first and last time
fc2_500_first <- head(fc2_500, n =1)
fc2_500_last  <- tail(fc2_500, n =1)

# Get values for 420 ppm predictions
fc2_500_first_time  <- fc2_500_first$time_index
fc2_500_first_lower <- round(fc2_500_first$`95%_lower`, 2)
fc2_500_first_upper <- round(fc2_500_first$`95%_upper`, 2)
fc2_500_first_mean  <- round(fc2_500_first$.mean, 2)

fc2_500_last_time  <- fc2_420_last$time_index
fc2_500_last_lower <- round(fc2_500_last$`95%_lower`, 2)
fc2_500_last_upper <- round(fc2_500_last$`95%_upper`, 2)
fc2_500_last_mean  <- round(fc2_500_last$.mean, 2)
```

Based on the findings from our model, we predict that CO2 will cross the 500 ppm threshold for the first time on 2056 Apr with an expected value of 500.96 with a 95% confidence interval between 475.25 - 526.67. Our model also predicts that the last time CO2 will be between the 500 and 501 ppm threshold will be on 2024 Oct with an expected value of 500.68 with a 95% confidence interval between 472.63 - 528.72.

```
# Atmospheric CO2 levels in the year 2122
fc_2122  <- tail(fc2_100, n =1)

# Carbon values in 2122
fc_2122_lower <- round(fc_2122$`95%_lower`, 2)
fc_2122_upper <- round(fc_2122$`95%_upper`, 2)
fc_2122_mean  <- round(fc_2122$.mean, 2)
```

'By the end of 2122, our model predicts that CO2 will be 654.1 ppm with a 95% confidence interval between 546.98 - 761.22. Although we are not very confident in the point estimate for the average CO2 emissions at the end of 2022, we are fairly confident (95%!) that average CO2 emissions will be between our confidence interval of 546.98 - 761.22.