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**RISK ANALYSIS OF A PORTFOLIO**

**Master's thesis written under the supervision of**  
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**Warsaw, 2022**

## **CERTIFICATION**

I, the undersigned, attest that I have read and thus suggest to the University of Economic and Human science at Warsaw a dissertation titled; “RISK ANALYSIS OF PORTFOLIO.

In partial fulfilment for the degree of Masters of INTERNATIONAL FINANCE AND TRADE offered by the University of Economic and Human Science at Warsaw.

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**Dr. Olga Palczewska-Wielińska**

**Supervisor**

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I want to express my gratitude to everyone who helped with this study in any form, including my fellow students and those who helped with the writing of this paper.

## **DEDICATION**

I lovingly dedicate this project to my respective without my mother's love and support, this project would not have been completed. She has been a continual source of inspiration for me and has given me the motivation and discipline to approach every assignment with excitement and dedication. My sisters and brothers, and friends have never left my side and are very special.

## ABSTRACT

The purpose of the project was to estimate the Value-at-Risk (VaR) of a portfolio consisting of four financial instruments based on two GARCH-family models. We first created a portfolio and checked the basic properties of a time series of this portfolio. Then we focused on finding the best model from the GARCH family. Finally, we estimated the VaR of the portfolio. We started by downloading a dataset of daily observations of SP&500, DAX, WIG20 and BTC. The chosen period was 2009/01/01 – 2022/05/31. We built a portfolio consisting of the above four instruments. We also calculated the log returns of every instrument as well as the returns for equally weighted portfolios (EWP). We estimated Value-at-Risk for both the in-sample and out-of-sample periods for the equally weighted portfolio we had set. The forecasts seem to be quite good. Apparently, the model AR(1)GARCH(2,1) that we chose was the correct one. The results we got – the losses of the portfolio higher than VaR at 1,17% in the in-sample period and 1,36% in the out-of-sample period – confirm that the estimation was accurate. The conclusion we have reached is that the peaks-over-threshold model that is used for measuring VaR is not suitable for capturing the volatility clustering that occurs on the Montenegrin stock market and that it should be improved, despite the fact that it is accurate in terms of correctly exceeding its thresholds. In this article, we discovered that ARMA(1,2)-TS GARCH(1,1) model with Student-t residual distribution, ARMA(1,2)-T GARCH(1,1) model with Student-t residual distribution, and the ARMA(1,2)-EGARCH(1,1) model with a Student-t distribution of residuals all fit the data better than the other models. Parameterised JSU distribution of residuals passed the joint Christoffersen test with a 95% confidence level.

## KEYWORDS

VaR: Value at Risk, GARCH MODEL: Generalized AutoRegressive Conditional Heteroskedasticity, fat tails; Kupiec test; Christoffersen test; Pearson's Q test

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## INTRODUCTION

As a consequence of the search for more appropriate approaches to risk assessment, there has been a general rise in monetary unpredictability all over the world. In reaction to the financial crisis, as well as the subsequent increase in the volatility of the market, more sophisticated tools for analysing and predicting risk have been developed. Value at Risk, or VaR, is the most well-known risk indicator. Rebecca Lake (2022 ) A "greatest loss" is defined as the greatest loss for a given level of confidence over a particular time horizon. VaR is calculated using a standard deviation formula. It is feasible to make an estimate of the most extreme points of the empirical distribution of financial losses. Aswath Damodaran (2007) It is applicable to each and every type of financial risk measurement. In this section, we discuss its application in the assessment of market risk. Pilar Grau Carles (2015) In 1994, J.P. Morgan introduced Value at Risk (VaR) as the risk management technique that would eventually become the foundation of its Risk Metrics system. all contribute to the conceptual groundwork that underpins the VaR method; despite the numerous theoretical studies that have been conducted about the deficiencies of VaR brought on by Neither sub-additivity nor convexity can be found in it, there is still no better way to estimate the risk. Cheng, Liu, and Wang (2004). With the assistance of empirical data obtained from measuring VaR in the stock market, which is still expanding, the purpose of this study is to estimate the Value-at-Risk (VaR) of a portfolio that is comprised of four financial instruments based on two GARCH-family models that are based on the econometric approach. The empirical data was obtained by measuring VaR in the stock market. Vesna Bucevska(2013)

This technique makes use of generalized autoregressive conditional heteroskedasticity (GARCH) models in order to develop a reliable model that can estimate the Value at Risk and make predictions both within the sample and outside of the sample that was taken. We have opted to employ both asymmetric and symmetric GARCH-type models despite the fact that there are four unique residual distributions, including normal, Student-t (all of which are applied more commonly), skewed Student-t, and a reparametrized Johnson distribution. These distributions are as follows: normal, Student-t (both of which are utilized more frequently), and skewed Student-t Then, we determined how precise the estimation of VaR that the various GARCH models generated was by using backtesting to evaluate the models' output. Christofferson's method was utilized in the computation of the Kupiec and Christofferson tests, which are also referred to as likelihood ratio coverage tests. Pearson's Q test to determine

whether or not a model is a good match for the data was also one of the tests that were carried out. Julija, Milena & Saša (2017)

Our objective is to evaluate the performance of the two GARCH family models in determining and predicting the VaR of the portfolio over a significant sample period, beginning after the global financial crisis of 2008 and continuing until the present (1 January 2009 to 31 May 2022, the time frame covered). Backtesting was then utilized to determine how precisely the various GARCH models' estimates of VaR compared to one another in terms of precision. The method developed by Christoffersen was utilized in the computation of the Kupiec and Christoffersen tests, which are also referred to as likelihood ratio coverage tests, in addition to Pearson's Q test for determining whether or not a model is a good fit. Even though it has been improved, It is not possible to use the peaks-over-threshold approach at this time considered an accurate Model of value at risk for the portfolio. With this in mind, the primary contribution of this thesis is to expand on the limited empirical research on VaR estimation and forecasting insample, and out-of-sample of that portfolio, as well as to can get beyond the limitations of empirical investigations by comparing how well GARCH models perform when it comes to the estimate of stock market volatility. Julija, Milena and Saša (2017)

The modern version of portfolio theory is based on an analysis that was established by Markowitz (1952). "Analysis of Portfolio Risk Using the ARCH and GARCH Models in the Context of the Worldwide Financial Crisis 77," Rubinstein (2002) highlighted the relevance of Markowitz's work as the first mathematical formalization of the notion of diversifying one's investment portfolio. Niki Tehno(2011). He emphasized the idea that even though diversification can lessen the risk of an investment portfolio, it is not possible for it to eliminate all of the risks entirely. Therefore, the risk may be mitigated by diversity in a manner that has no impact whatsoever on the predicted return of the portfolio. Therefore, stocks with a low degree of perfect correlation are not the "ideal candidates" to be included in a portfolio. In 1974, amongst other people, Solnik expanded the basic Capital Asset Pricing Model (CAPM) and claimed that international diversification leads to better outcomes than local diversification. This was included in the paper by Solnik (1974). However, because of the substantial connection that financial integration causes between the returns on different securities, the advantages of international diversification are significantly diminished. Alouis (2010).

The purpose of this study, which takes into account all of these different factors, is to investigate how the level of risk associated with a globally diversified portfolio has changed in the context of the recent global financial crisis.

In addition to this, we plan to evaluate the significance of our findings in light of those obtained by Cerovic and Karadzic (2015). The thesis is organized as follows: The subsequent section offers a concise review of the existing research in the field. In the third section, the methodology for calculating VaR using an econometric approach is discussed in detail. In the fourth chapter, the backtesting procedure, data, and descriptive statistics are presented for the purpose of examining whether or There is an error in the assessment of the value at risk being utilized. Our findings based on the empirical research are presented in the fourth chapter, along with our conclusions.

## CHAPTER 1. LITERATURE REVIEWER

It would appear that accurately measuring and projecting market losses is of the utmost importance in both developed and emerging financial markets. The increasing interest shown by foreign financial investors in investing in emerging financial markets highlights the significance of accurately quantifying and predicting market risk. Zikovic & Aktan (2009) . Lower liquidity, frequent internal and external shocks, and a higher degree of insider trading are the fundamental characteristics of emerging markets, all of which contribute to the market's increased volatility. Miletic (2013).

Emerging markets are distinguished from developed countries by a greater influence of internal trade, as well as by high volatility, illiquidity, and market shallowness. These characteristics combine to make the assessment of VaR using typical techniques that assume a normal distribution substantially more difficult, and they are only a few of the numerous studies that deal with the proper estimate and forecasting. Alexander and Leigh (1997). Despite this, there is no such thing as a one-size-fits-all model for risk mapping. Within the United States Federal Crop Insurance Program context, investigated VaR and dependence models. Ramsey and Goodwin (2019)

Due to the fact that the program encompasses policies for such a diverse assortment of crops, plans, and locations, this matter is quite complicated. The outcomes are influenced by the weather as well as other latent variables such as pests. Because of this, the dependency structure is quite complicated and a standard reinsurance agreement (SRA) makes it possible for a portion of the risk to be transferred to the federal government, the calculation of value at risk (VaR) is extremely important. Christoffersen, Hahn, and Inoue (2001), It is common practice to express VaR in terms of a probable maximum loss, also known as PML, or as a return in the form of a loss over a specified time period. The body of work on VaR models in emerging financial markets, as well as the empirical research into these models, is not nearly as extensive as the body of work that addresses the calculation of VaR in developed financial markets some examples of this type of work include. De Melo Mendes (2003).

This is in contrast to the body of work that addresses the calculation of VaR in developed financial markets. Using, but not limited to, historical time series data did not make it possible to carry out an econometric analysis that could be relied upon. States that this is due to the elliptical shape of the data. (the stock markets of the vast majority of these countries didn't even come into existence until the early 1990s) .Zikovic and Aktan (2009) To determine whether there was or wasn't Connection between the start of the global financial crisis and the relative performance of VaR models of the returns of indices on the Turkish and Croatian stock exchanges. Through the use of this study, they were able to show that theoretical frameworks such as extreme value theory and The models that are

the most accurate are hybrid historical simulations, whereas other models grossly underestimate the level of risk. Djakovic and Radisic (2009).

The research conducted an analysis of the stock markets of four developing nations Slovenia, Croatia, Serbia, and Hungary; primary takeaways from their research indicated that, given stable market conditions, the analyzed stock markets had the potential to outperform the developed nations' stock markets, while the market volatility was being analyzed, the models which included parametric and historical simulation models provided accurate forecasts of VaR estimations with a confidence level of 95%. Cheng, and Wong (2002), Andjelic (2010). Estimates of VaR parameters can be obtained from models, and the level of confidence in these estimates is typically at least 99% of the time method, in addition to Forecasting the value at risk of an index on a stock exchange in the Serbian financial market using the GARCH and integrated GARCH (IGARCH) models.

Mladenovic, Andjelic and Radisic (2010) .

An investigation was conducted into the efficacy of the RiskMetrics method, and They arrived at the conclusion that GARCH models, when used in conjunction with the theory of extreme values (the peaks-over-threshold method), decrease the overall mean value of the value at risk, and that these models are superior to the RiskMetrics method, which is used in conjunction with the IGARCH model. Miletic and Miletic (2012)

Despite the fact that the phrase "Value at Risk" did not become common parlance until the middle of the 1990s, the measurement's roots go far further back in time. The mathematical foundations of VaR were principally conceived of and developed within the framework of portfolio theory . Harry Markowitz (1990). Despite the fact that they were working towards a distinct goal, which was the development of optimum investment portfolios for equity investors. VaR is calculated with a primary emphasis on market risks and the consequences of the comovements in these risks. In particular, the focus on market risks is key to the process. Karen K. Lewis (1999)

However, the crises that have befallen financial service corporations over the course of time and the regulatory reactions to these crises served as the motivation for the application of VaR measures in these organizations. Following the Great Depression and the many bank failures that took place during this era, there was a period of economic instability. Aswath Damodaran (2007), the Securities Exchange Act was passed. Zikovic and Aktan (2008) This act established the Securities and Exchange Commission (SEC) and mandated that banks keep their borrowings at a level that was

less than 2,000 per cent of their equity capital. This marked the start of the regulatory capital requirements that banks have to comply with. In the decades that followed, financial institutions developed various risk mitigation strategies and management mechanisms in order to guarantee that they would satisfy these capital needs. Randall Guynn, Davis Polk (2010)

The Uniform Net Capital Rule (UNCR), which was enacted by the SEC in 1975, was responsible for the refinement and expansion of capital requirements. In accordance with this rule, the financial assets that banks held were divided into twelve distinct depending on the degree of risk connected with each, and the banks were required to maintain varying levels of capital for each class, ranging from 0% for short-term treasuries to 30% for equities. <https://www.sec.gov/enforce/34-82951>

The heightened risk that was produced in the early 1970s due to the emergence of derivative markets and floating-rate currency rates led to the rise of capital needs in many different industries. Aswath Damodaran (2007). This was done in response to In quarterly statements that were termed " In its "Financial and Operating Combined Uniform Single" (FOCUS) reports, financial institutions were required to report on the calculations that went into their capital reserves". Roberto Bustreo (2013)

In 1980, the SEC instituted the first regulatory measures that evoke Value at Risk by tying comparing the capital needs of financial service companies to the damages that might result from a 95% confidence level over a 30-day interval in various security classes; these potential losses were computed using historical returns. Aswath Damodaron (2007) Value at Risk is a risk management framework that was developed by J. Markowitz and first introduced in the 1970s. The SEC clearly intended for financial services businesses to start the process of assessing one-month 95% Value at Risks and keep sufficient capital to cover the probable losses, despite the fact that the metrics were labeled as haircuts rather than Value or Capital at Risk. Aswath Damodaran (2007) This was the case despite the fact that in place of Value or Capital at Risk, metrics were referred to as haircuts.

VALUE AT RISK (VAR) - New York

University. <https://pages.stern.nyu.edu/~adamodar/pdfiles/papers/VAR.pdf>

Around the same time, the trading portfolios of investment and commercial banks were getting bigger and more volatile, which resulted in the need for risk management techniques that were both more complex and more timely. In 1986, Ken Garbade at Banker's Trust developed sophisticated metrics of Value at Exposure of the company's fixed-income holdings to danger. Bond interest rates of varying maturities were used to derive these measures. All save the most senior workers of Banker's Trust had access to these internal documents. [https://documentsn.com/document/c5b0\\_value-at-risk-var.html](https://documentsn.com/document/c5b0_value-at-risk-var.html) By the early 1990s, a large

number of companies that provide financial services had created basic measurements of Value at Risk, with a large variety of approaches to its measurement used. Ken Garbade (1990)

After a series of disastrous losses associated with the use of derivatives and influence around 1993 and 1995, firms were ready for more comprehensive risk measures. This was made possible by the failure of Barings, the British investment bank, due to the unlicensed dealing in Nikkei futures and options by Singaporean trader Nick Leeson. <https://pages.stern.nyu.edu/~adamodar/pdfiles/papers/VAR>. The collapse of Barings was the climax of these losses, and it signalled that businesses were ready to implement broader risk controls. J.P. Morgan (1995) The data on the variances of and covariances across various security and asset classes, which J.P. Morgan had been using internally for almost a decade to manage risk, were made available to the public. J.P. Morgan (1995). This enabled software manufacturers to develop software that could measure risk. The company decided to call the service "Risk Metrics," and it described the risk measure that was derived from the data using the phrase "Value at Risk." The commercial and investment banks as well as the regulatory bodies that oversaw them, quickly warmed up to the proposal because of its intuitive appeal, which found a ready audience with them the previous 10 years have seen a number of changes, value at risk (VaR) has emerged as the preeminent method for determining the level of risk exposure in financial service corporations and has even started to gain popularity in other types of businesses. J.P. Morgan (1995)

The approach that is taken by extreme value theory is marginally superior to that which is taken by the GARCH model was the conclusion that was reached. Mladenovic et al. (2012). This research was based on the authors' findings. In addition to the nations located in Eastern Europe (Bulgaria, the Czech Republic, Hungary, Croatia, and Romania). And Serbia), but in general, they recommend utilizing both strategies for improved market risk management. This is due to the fact that markets are becoming increasingly volatile. Measurement. Bucevska (2012) Some evidence that the most accurate models of the GARCH family make use of the following formula: Estimating the level of volatility that exists on the Macedonian stock market using the asymmetric exponential moving average is the model that has proven to be the most accurate. Julija, Milena & Saša (2017) GARCH (EGARCH) model with Student's t distribution, the EGARCH model with normal, GARCH (EGARCH) model with GARCH (EGARCH) model with, distribution in addition to the GARCH-GJR model that was developed by Glosten, Jagannathan, and Runkle. GARCH (EGARCH) model with normal distribution. This study's conclusions have substantial implications for the Methodology that should be used to estimate VaR in volatile environments. Julija, Milena & Saša (2017) The findings also shed light on the conditions that investors in emerging capital markets will

need to address in order to be successful. In addition to Miletic, Miletic was also responsible for the implementation of the GARCH models (2015). The study's objective has been to develop estimate methods that are more accurate and provide more trustworthy values for variance and covariance, which can then be used in VaR calculations. Some people have proposed making improvements to sampling techniques and developing new data innovations, both of which would make it possible to get more accurate estimates of variances and covariances going ahead. Some people believe that improvements in statistical methods may provide more accurate estimates from the same data. Specifically, the standard deviation of returns is assumed not to fluctuate over time in order to calculate VaR using traditional methods (homoskedasticity). When the standard deviation is allowed to fluctuate over time, Engle contends, we receive considerably more accurate predictions. This is due to the fact that these models include time-varying standard deviations (heteroskedasticity). Engle, R., (2001), Aswath Damodaran ( 2007)

In fact, he suggests two variants—Autoregressive Conditional Heteroskedasticity (ARCH) and Autoregressive Conditional Heteroskedasticity (GARCH)—that provide more accurate variance predictions and, thus, more reliable valuations of risk. These forms of heteroskedasticity are called as Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH), respectively. However, the variance-covariance VaR estimate was designed for portfolios in which the relationship between risk and portfolio positions is linear, which is another criticism that may be levelled against it. Aswath Damodaran(2007) This is a criticism that may be thrown against the variance-covariance estimate of VaR. Since of this, it is possible for it to fail when the portfolio contains options because the payoffs on options do not follow a linear pattern. Researchers have created quadratic Value at-risk metrics to manage the implications of options and other non-linear instruments that might be used in portfolios. Britten-Jones, M. and Schaefer, S.M (1999)

Researchers are now able to estimate the Value at Risk for complex portfolios that incorporate options and option-like instruments such as convertible bonds. This contrasts with the more conventional linear models, which are known as delta-normal models. The price to pay, however, is that the mathematics involved in determining the VaR will become far more sophisticated, and a portion of the intuitive understanding will be lost along the way. Julija,



Milena & Saša (2017) The conditional standard deviation used herein serves as the first theoretical model of volatility, and it was proposed by Engle (1982) as a solution to the problem of the volatility of inflation rate time series. The Autoregressive Conditionally Heteroskedastic (ARCH) model is an example of this type of model. The Generalized Autoregressive Conditionally Heteroskedastic (GARCH) model was introduced by Bollerslev (1986) primarily for financial data. This model is particularly well-suited for the investigation and prediction of volatility. Nelson's (1991)

Exponential GARCH (EGARCH) model enables both positive and negative returns on assets to have an asymmetric influence on the model's output of volatility. Another volatility model that takes into account leverage is called the Threshold-GARCH (TGARCH) model, which was developed by Zakoian (1994). The univariate GARCH model is converted into the multivariate GARCH model so that the portfolio of assets may be analyzed. The Constant Conditional Correlation (CCC-) GARCH model was initially suggested by Bollerslev (1990). Engle and Sheppard (2001) enhanced the CCC-GARCH model by configuring the correlation matrix to vary over time. As a result, the Dynamic Conditional Correlation (DCC-) GARCH model was established. Julija, Milena & Saša (2017)

In recent years, some accomplishments have been accomplished using the ARMA model, ARCH model, and SGARCH model in the research of volatility in the Chinese mainland stock market. Wang, Yu, and Zhou (2014) conducted an empirical investigation of the link that exists between the Shanghai Composite Index's return rate and the threat it presents. <https://en.frontsci.com/index.php/memf/article/view/327/431>

According to the findings, there is a satisfactory fitting effect, which illustrates the connection that exists between the rate of return and the level of risk. This helps me not only study the return rate but also have volatility to gauge the danger while researching the Chinese mainland stock market. Lu (2006) Developed a non-parametric GARCH model to forecast the volatility of the Chinese stock market better. Both the ARIMA and ARCH models were employed by him (2008) to forecast stock prices, and the findings demonstrate that the ARCH model is more satisfying. For this reason, I combine the GARCH model with the ARMA model.

The exchange rate risk and interest rate risk were both integrated into the capital market-based asset-pricing model that Solnik (1972) developed. After conducting research on the stock markets of

seven different countries, the researchers came to the conclusion that although domestic factors are still the most significant contributors to a country's capital price, factors from other countries do have some impact on the capital price of a country. On the basis of this, King and Wadhvani (1990) applied the model to the markets of many different countries, thereby providing further proof that the stock price of a country is influenced not only by the information of its own public information but also by the stock price information of other countries. Karolyi (1996) discovered that there is a transmission in both directions between the stock markets of the United States and Japan. According to the findings of Hu and Xu (2003), the volatility of the Hong Kong stock market is more strongly correlated with that of the Shanghai composite index than that of the United States stock market. Additionally, they learned that the Shanghai Composite Index's and the S&P Index's regression coefficients' degree of fitting is very low. Dongya Zhou (2020)

This finding indicates that the connection between the United States and the Chinese mainland is still extremely poor. After the Asian financial crisis, Chen, Wu, and Liu (2006) observed that the interconnectedness of stock markets in 11 nations and regions in the Asia-Pacific area has greatly improved. This was the conclusion reached by the researchers.

Although there may have been a short-term guiding relationship between the U.S. stock market and the Chinese mainland stock market, there is no stable cointegration relationship between the Chinese mainland stock market and any other stock markets. The Chinese mainland stock market has a weak exogenous. Zhang (2005)

further shown via the use of the E-GARCH model that there is no long-term cointegration link between the international stock market and the stock market on the Chinese mainland. However, the Hong Kong, London, and New York stock markets had one-way short-term spillovers on the stock market of the Chinese mainland mostly before the financial crisis that occurred in 1997. Zhang (2005)

## CHAP 2 METHODOLOGY

In percentage terms, we may write a log return series at time  $t$  as  $r_t$ . The loss over time  $[t, t+h]$  is denoted by the random variable  $L_{t+h} = (r_t + L_h r_t) = r_t(h)$ . As a result,  $F(x) = P(L \leq x)$  is the loss distribution's cumulative function, and  $F_L$  is the loss distribution's cumulative function. The value at risk (VaR) at the significance level (often 1% or 5%) is a quantile distribution function  $F_L$ , or, alternatively, the smallest real number meeting the inequation  $F_L(x)$ .

$$VaR_\alpha = \inf\{x | F_L(x) \geq \alpha\}. \quad (1)$$

Using time series econometric models, the econometric method calculates VaR. To estimate a log return series, we employ an autoregressive moving-average model (ARMA) with  $p$  and  $q$  orders, denoted ARMA ( $p, q$ ):

$$r_t = \phi_0 + \sum_{i=1}^p \phi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j} \quad (2)$$

$$a_t = \sigma \epsilon_t$$

The parameters of equation (2) that reflect the ARMA ( $p, q$ ) model are denoted by the symbols  $\phi_0, \phi_1, \dots, \phi_p$ , and  $\theta_1, \dots, \theta_q$ , respectively. The model's error term, denoted by the letter  $a_t$ , is a function of  $t$ , which is composed of a number of independent random variables that are uniformly distributed and have the characteristics of white noise.

The following volatility models will be utilized to simulate the conditional variance of returns  $r_t$ : the Asymmetric Power ARCH (APARCH), the Generalized ARCH (GARCH), the Taylor-Schwert GARCH (TS GARCH), the Threshold GARCH (TGARCH), the Nonlinear GARCH (NARCH), the Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH), the Exponential GARCH (EGARCH) (IGARCH).

Recently, Ding, Granger, and Engle presented the APARCH model (1993). It is an asymmetrical variant of the traditional GARCH model for simulating volatility. It assumes that shocks, both positive and negative, do not have the same impact on volatility. This is the basic outline:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i (|a_{t-i}| - \gamma_i a_{t-i}) \delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad (3)$$

The parameter  $\alpha$  denote the leverage effect,  $\gamma$  and  $\alpha$  is the Box-Cox transformation of the conditional standard deviation. Shocks, both positive and negative, may be differentiated by the leverage effect. That's why positive innovations multiply by  $\alpha + \gamma$ , whereas negative ones multiply by  $\alpha - \gamma$ .

The following prerequisites should be satisfied by the model's parameters.:

$$\alpha_0 > 0, \delta \geq 0,$$

$$\alpha_i \geq 0, i = 1, 2, \dots, u,$$

$$\beta_j \geq 0, j = 1, 2, \dots, v, \quad (4)$$

$$-1 \leq \gamma_i \leq 1, i = 1, 2, \dots, u.$$

This model comes in a variety of customized variants, including:

When all of the following conditions are met, the APARCH model is an ARCH model: when  $u = 2, I = 0, I = 1, 2, \dots, u$ , and  $j = 0, I = 1, 2, \dots, v$ ; when all of these conditions are met, the APARCH model is a GARCH model: when all of these conditions are met, the APARCH model is a GJR GARCH model: when all of these conditions are met (Geweke, 1986; and Pantula, 1986)

Bollerslev (1986) introduced the GARCH model as a generalization of the autoregressive conditional heteroscedasticity (ARCH) model (Engle, 1982). This model may be described as follows (Tsay, 2010):

$$\sigma_{t2}^2 = \alpha_0 + \sum \alpha_i a_{t-i}^2 + \sum \beta_j \sigma_{t-j}^2 \quad (5)$$

$$\alpha_1, \dots, \alpha_u \geq 0, \beta_1, \dots, \beta_v \geq 0, \sum_{i=1}^{\max(u,v)} (\alpha_i + \beta_i) < 1.$$

$$i=1$$

It was Taylor (1986) who modelled the conditional standard deviation function rather than the conditional variance. In addition, Schwert (1989) developed a linear model for the conditional standard deviation that relies on lagging absolute residuals. And hence, we have the following definition for the TS GARCH(p, q) model:

$$\sigma_t = \alpha_0 + \sum \alpha_i |a_{t-i}| + \sum \beta_j \sigma_{t-j}. \quad (6)$$

Zakoian (1994) proposed the T GARCH model, which uses linear combinations of historical error terms and standard deviations to predict future volatility. It's a lot like the TS GARCH, and you can get it from:

$$\sigma_t = \alpha_0 + \sum \alpha_i^+ a_{t-i}^+ - \alpha_i^- a_{t-i}^- + \sum \beta_j \sigma_{t-j}. \quad (7)$$

The words that are positive and negative of  $a_t$  are  $a_t^+ = \max(a_t, 0)$  and  $a_t^- = \min(a_t, 0)$ .

It is possible to interpret the NARCH model developed by Higgins and Bera (1992) as a Box-Cox power transformation of the variables underlying the linear ARCH model. In order to integrate the GARCH model, the model may be rapidly and simply modified by adding Box-Cox transformations of lagged variance values to the right-hand side of its general form. Which can be stated as:

$$\sigma_t = \left[ \alpha_0 (\sigma^2)^\delta + \sum_{i=1}^u \alpha_i (a_{t-i}^2)^\delta - k_i \right]^{1/2} \quad (8)$$

IN (1993) saw the launch of the GJR-GARCH model by Glosten, Jagannathan, and Runkle. It is provided in the broad form by:

$$\sigma_t^2 = \omega + \sum \alpha_i a_{t-i}^2 + \sum \beta_j \sigma_{t-j}^2 + \gamma_i I_{t-i} a_{t-i}^2, \quad (10)$$

Where  $\alpha$ ,  $\beta$  and  $\gamma$  are three constant parameters and an indicator function that returns 1 0 if  $a_{t-i}$  is zero and negative otherwise. Since the value explains the asymmetry, this dummy variable categorizes shocks as either positive or negative. According to Nelson (1991), the EGARCH model is as follows:

$$\ln(\sigma_t^2) = (\alpha_0 + \sum \zeta_i v_{it}) + \sum (\alpha_i a_{t-i} + \gamma_i (a_{t-i} - E_i a_{t-i})) + \sum \beta_j \ln(\sigma_{t-j}^2), \quad (11)$$

In which the significant impact is represented by the coefficient  $\alpha_i$  and the size effect is represented by  $\gamma_i$ . In other words, EGARCH With the parameters  $\alpha$  and  $\gamma$ , this equation illustrates a process with zero mean. If the error term  $\epsilon_t$  is positive, then  $g$  has a positive slope coefficient of  $\alpha + \gamma$ , and if it is negative, then  $g$  has a negative slope coefficient of  $\gamma - \alpha$ .

The GARCH model (1,1) illustrates a process of limitless expansion of the conditional variability if the parameters add  $1 + 1 = 1$ . The name of such a model is IGARCH (1,1). It serves as the foundation for VaR estimate and represents RiskMetrics, the accepted method of quantifying risk. It may be expressed generally as follows: Orhan & Köksal (2012):

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \sum \alpha_i (a_{t-i}^2 - a_{t-1}^2) + \sum \beta_j (\sigma_{t-j}^2 - a_{t-1}^2) \quad (13)$$

The business J.P. Morgan. Dongersey & More established this approach (1995), and it implies that the conditional distribution of the series of log daily returns is  $r_t | \Omega_{t-1} \sim N(\mu_t, \sigma_t^2)$ , where  $\mu_t$  is the conditional mean, and  $\sigma_t^2$  is the conditional variance of a series  $|r_t$ . The following relations are valid for them:

$$\mu_t = 0, \sigma_t = \alpha \sigma_{t-1} + (1 - \alpha) r_{t-1}, \quad 0 < \alpha < 1. \quad (14)$$

A loss risk emerges in long trading positions when the price of the traded asset falls and in short trading positions when the price of the asset increases. As a result, the long and short ends of the distribution are represented by the corresponding left and right tails.

For the information known at, the conditional distribution of a random variable  $r_{h+1}$  The instant  $h$  inclusive has the same mean  $RH(1)$  and variance  $RH(2)$  as the standard normal distribution if the  $t$  series is a random variable with the standard normal distribution (1). Next, we estimate VaR with 95% confidence and a 1-step forward prediction horizon, using the 5% quantile of the conditional distribution, which looks like this:

$$VaR_{long} = r_h(1) - 1, 65\hat{\sigma}_h(1)$$

$$VaR_{short} = r_h(1) + 1, 65\hat{\sigma}_h(1).$$

(15)

The conditional distribution's fifth percentile or 5% is the following if the random variable  $t$  has a Student-t distribution and has degrees of freedom:  $\text{VaR}_{\text{long}} = \hat{r}_h(1) + \sqrt{\frac{v}{v-2}} t_v(1 - \alpha)$

$$\sqrt{\frac{v}{v-2}} t_v(1 - \alpha) \quad (16)$$

$$\text{VaR}_{\text{short}} = \hat{r}_h(1) + \sqrt{\frac{v}{v-2}} t_v(\alpha),$$

$$\sqrt{\frac{v}{v-2}}$$

where the critical value of the  $(1-\alpha)$  quantile from the  $t$  distribution with degrees of freedom is  $t(1 - \alpha)$ . To account for both skewness and fat tails, Lambert and Laurent (2001) developed a skewed Student-t distribution. The skewed Student-t distribution's log-likelihood function has the following form:

$$L^{skst} = T \left( \ln \Gamma\left(\frac{v+1}{2}\right) - \ln \Gamma\left(\frac{v}{2}\right) - \frac{\ln(\pi(v-2)) + \ln\left(\frac{(sz_t + m)^2}{v-2}\right) + \ln s}{2} \right) - \frac{1}{2} \sum_{t=1}^T \ln(\sigma_t^2) + (1+v) \ln \left( 1 + \frac{(sz_t + m)^2}{v-2} \xi^{-2I_t} \right) \quad (18)$$

If  $z_t$  is a random variable with a skew Student-t distribution,  $I_t = 1$  when  $z_t \geq -m/s$ , where  $m$  is the mean,  $s$  is the standard deviation and is the asymmetry (skewness) parameter.  $z_t \geq -m/s$ , while  $I_t = -1$  when  $z_t < -m/s$ .

The conditional distribution's 5% quantile is if a random variable  $t$  has a skewed Student-t distribution and degrees of freedom:

$$\text{VaR}_{\text{long}} = \hat{r}_h(1) + \text{skst} \alpha, v, \varepsilon \hat{\sigma}_h(1). \quad (19)$$

$$\text{VaR}_{\text{short}} = \hat{r}_h(1) + \text{skst} 1-\alpha, v, \varepsilon \hat{\sigma}_h(1),$$

Where the left and right quantiles of the per cent from the skewed  $t$  distribution with degrees of freedom are denoted by  $\text{skst } \alpha, v, \varepsilon$  and  $\text{skst} 1-\alpha, v, \varepsilon$ , respectively.

The parameterised Johnson SU distribution is a four-parameter all values of the skew and shape parameters in distribution with the mean and standard deviation, respectively, as detailed in Rigby and Stasinopoulos (2005). It is represented as  $\text{JSU}(\alpha, v, \varepsilon)$ .

The log-likelihood function is as follows when using the generic form of Johnson densities:

$$L_{jsu} = n \ln \tau - n \ln \sigma - \frac{n}{2} \ln(2\pi) + \sum_{i=1}^n g\left(\frac{x-\mu}{\sigma}\right) - \frac{1}{2} \sum_{i=1}^n \left(v + \tau g\left(\frac{x-\mu}{\sigma}\right)\right)^2. \quad (20)$$

As a result, the 5% quantile of the conditional distribution is as follows if a random variable  $t$  has a reparameterized JSU distribution:

$$\text{VaR}_{\text{long}} = \hat{r}h(1) + jsu_{\alpha, v, \varepsilon} \hat{\sigma}h(1)$$

$$\text{VaR}_{\text{short}} = \hat{r}h(1) + jsu_{1-\alpha, v, \varepsilon} \hat{\sigma}h(1), \quad (21)$$

Where  $jsu_{(\alpha, v, \varepsilon)}$  and  $jsu_{(1-\alpha, v, \varepsilon)}$  are the left and right quantiles of  $\alpha\%$  from the parameterised JSU distribution.

## 2.1. BACKTESTING

VaR models must be tested for reliability and accuracy before being put into use. Backtesting is a test statistic for verifying the precise estimate of VaR. Backtesting is used to determine if the amount of losses projected by VaR is accurate. Julija, Milena & Saša (2017). This method employs unconditional or conditional coverage checks to determine the proper number of exceedances. The unconditional tests determine if the frequency exceptions for the given time span are consistent with the confidence level. The Kupiec test is the most widely utilized in this category. Conditional coverage tests, on the other hand, investigate conditionality and changes in data over time, with the Christoffersen independence test being the most well-known example. It has been proven that tests that assess a large number of quantiles are the most successful for detecting incorrect Var models. Hence we will include Pearson's Q test. Campbell (2005)



### 2.1.1 THE KUPIEC TEST

Let  $N$  represent the observed number of exceedances in the sample, or, in other words,  $N =$  It is the number of days over a  $T$  period of time when the portfolio loss during a set interval is greater than a certain threshold.  $t \sum r_{t,t+1}$  was larger than the VaR estimate, where (Campbell, 2005) As its name suggests, the Kupiec test has one degree of freedom and follows an asymptotic chi-square distribution. The Christoffersen test is an example of a conditional coverage test that may be used to further evaluate the reliability of a VaR model since it can reject a model for both high and low failures, although its power is usually insufficient, as noted by Kupiec (Kupiec, 1995).

The failure rate exhibits a binomial distribution, with  $p = N/T$  representing the anticipated exception frequency. Under the null hypothesis,  $p$  should be the ratio of failed trials ( $N$ ) to successful ones ( $T$ ). The proper likelihood ratio measurement is:

$$L^R_c = 2 \ln[(1 - N/T)^{T-N} (N/T)^N] - 2 \ln[(1 - p)^{T-N} p^N]. \quad (22)$$

### 2.1.2 THE CHRISTOFFERSEN TEST

In the Christoffersen methodology, the conditional coverage test plays a vital role since it determines whether or not exceptions occur in clusters. This is a crucial piece of information. In other words, it is possible for it to account for the clustering of volatility. If it is possible to demonstrate that clustering does, in fact, occur, then the model has been misspecified and has to be readjusted. Julija, Milena & Saša (2017) From a purely practical standpoint, it is essential to conduct tests to look for volatility clustering. For instance, if a bank plans to set aside capital for a total of 50 exceedances During the Four Years, although the bulk of those exceedances occurs over the course of just two months, the bank may not be able to maintain its liquidity. The previous exception indicator is used to construct a new indicator called  $n_{ij}$ , which is defined as the number of days that  $j$  (exception) happened while it was  $I$  (no exception) the day before. This new indication builds on the previous exception indicator. The notation  $ij$  represents the likelihood that state  $j$  would be noticed provided that state  $I$  was seen the day before; this is because of the state  $I$  was seen first (Jorion, 2007). According to Dowd (2005), The following statistic may be used to test for independence:

$$LR_{cc} = -2 \ln[(1 - p)^{T-NpN}] + 2 \ln[(1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}], (24)$$

where the associated probabilities, which means that 01 represents the likelihood that a nonexception will be followed by an exception, and 11 represents the probability that an exception will be followed by another exception.  $p$  is the symbol used to represent the absolute likelihood that a non-exception or exception will be followed by another exception. The test statistic has two degrees of freedom, and it is distributed as a  $\chi^2$  distribution. Julija, Milena Lipovina & Saša (2017) The Christoffersen test requires several hundred observations to be reliable, and the key benefit of this approach is that it may reject a VaR model that yields either too many or too few clustered violations. This two likelihood ratio (LR) tests may be combined to provide a comprehensive test for independence and coverage, which likewise has a distribution of  $\chi^2$  (2):

$$LR_{cc} = LR_{uc} + LR_{ind}. \quad (25)$$

This is the Christoffersen method for testing the prediction capacity and accuracy of a VaR

	LR	LR	LR
$h_0$	the appropriate conditional coverage	Exceedances stand alone	the appropriate conditional coverage
$h_1$	Incorrect unconditional coverage	Exceedances are not independent	Incorrect conditional coverage

model. The benefit of this test is the combination of two tests that may be evaluated individually to reverse course if the model fails because of incorrect coverage or exception clustering. Julija, Milena & Saša (2017) Overall, these tests give the tools needed to analyze and compare the VaR

models discussed above. Table 1 contains a detailed description of the null and alternative hypotheses for these tests.

**Table 1. For the three-step Christoffersen test, there are null and alternate hypotheses.**

### 2.1.3 THE PEARSON'S Q TEST

As Campbell (2005) has pointed out, Pearson's Q test may be used for more thorough backtesting. It is determined by the number of infractions recorded across various quantiles. The approach necessitates dividing the unit interval into  $k$  distinct sub-intervals (bins). The partition chosen is determined by the collection of quantiles of interest. In circumstances when quantiles more extreme than the 10th are of importance, the partitions  $[0.00, 0.01]$ ,  $[0.01, 0.05]$ ,  $[0.05, 0.1]$ , and  $[0.1, 1]$  are utilized. 2 A Chi-square test statistic is computed using the number of VaR violations that occur inside each bin.:

$$Q = \sum_{i=1}^k \frac{(N_{(l_i, u_i)} - N(u_i - l_i))^2}{N(u_i - l_i)}.$$

The amount of VaR infractions that occurred in the  $i$ th bin is denoted by  $N(l_i, u_i)$ , where  $N$  is the total number of days utilized to perform the test and  $l_i$  and  $u_i$  are the minimum and maximum values of the. Pearson's Q test has  $k-1$  degrees of freedom and is distributed according to the chi-squared distribution.

## CHAP 3 STATISTICAL FINDINGS

Our purpose is to estimate the Value-at-Risk (VaR) of a portfolio consisting of four financial instruments based on two GARCH-family models. We first created a portfolio and checked the basic properties of a time series of this portfolio. Then we focused on finding the best model

from the GARCH family. Finally, we estimated the VaR of the portfolio. Our portfolio is composed of 3 financial instruments and 1 cryptocurrency S&P 500, DAX, WAG20, and BIT our consideration period of 13 years, just after the 2008 world financial crisis, which means that our sample period starts from 01/01/ 2009 to 31/05/2022, which is the last day I started the research

### **3.1 PORTFOLIO ANALYSIS**

We started by downloading a dataset of daily observations of SP&500, DAX, WIG20 and BTC. The chosen period was 2009/01/01 – 2022/05/31. We built a portfolio consisting of the above four instruments. We also calculated the log returns of every instrument as well as the returns for an equally weighted portfolio (EWP). Table 2 Alpha1 is negative and significant, so the asymmetry is found According to the weighted Ljung-box test, the initial lags are NOT serially correlated P-values on the weighted ljung-box test on squared residuals are fairly high, and first, lags are significant, indicating we need not add anything to the covariance equation. P-values on the weighted ARCH LM test are fairly high, indicating we need not add anything to the variance equation.

The test results are higher than the crucial value. Therefore the null is rejected for all the parameters except gamma1. The Nyblom stability test verifies that each parameter separately is stable over time (no structural breaks in the model). We also accept the stability jointly ( $1.9723 < 2.11$ ). The model as a whole is stable over time The Sign Bias Test verifies that the asymmetry has been captured correctly in the estimated model. The model does not require higher order. The statistics are higher than the significance level.

Finally, we plotted the returns of EWP:

**Fig 1 PORTIFOLIO RETURN**

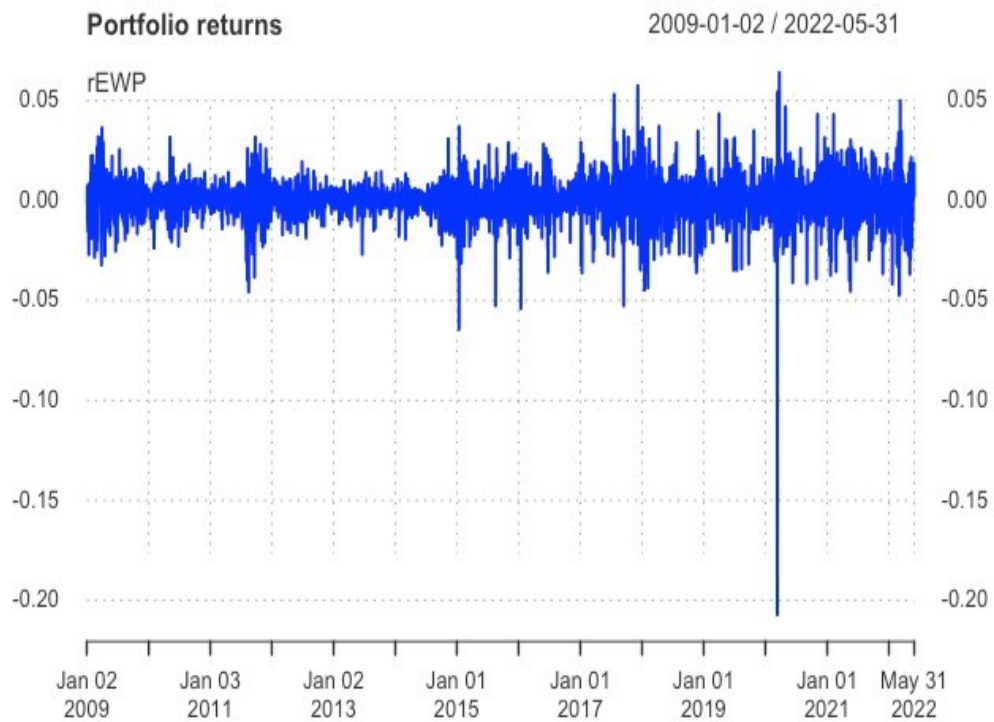
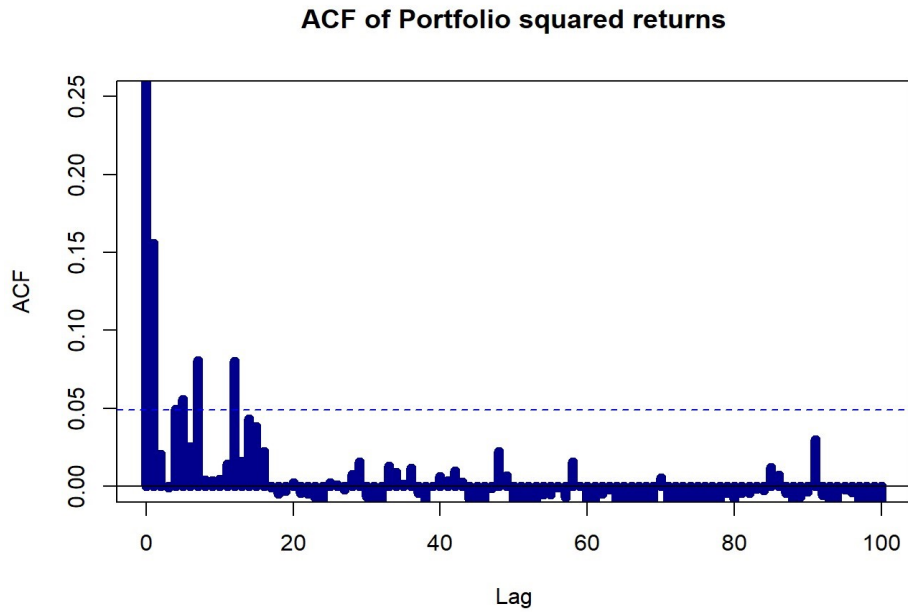


Fig. 1, Portfolio returns of the equally weighted portfolio, 2009/01/01 – 2022/05/31.

There is the volatility clustering effect quite visible (for example, on 03/2020 – large changes). After plotting Fig 2 the ACF function of log-returns of the portfolio as well, we noticed that the values of the ACF function indicate some autoregression and/or moving average correlations among returns. They also indicated several significant lags. We tested for ARCH effects (the null hypothesis about the lack of ARCH effects was strongly rejected) and the normality of returns (also rejected). Then we focused on GARCH models. Values of the ACF function indicate some autoregression and/or moving average correlations among returns. As well as a number of significant lags.



Values of the ACF function indicate some autoregression with lags exponentially decreasing after 1 lag, possibly suggesting an MA(1) model Testing for ARCH effects based on the autocorrelation of squared returns and computing for the autocorrelation of the regression model.

**TABLE 2 ARCH LM -TEST**

ARCH LM-test; Null hypothesis: no ARCH effects

data: data\$rEWP

Chi-squared = 45.856, df = 5, p-value = 0.000000009718 lag

Autocorrelation D-W Statistic p-value

1	0.9999466	0	0
2	0.9998328	0	0
3	0.9997904	0	0
4	0.9997455	0	0
5	0.9997116	0	0

Alternative hypothesis:  $\rho[\text{lag}] \neq 0$

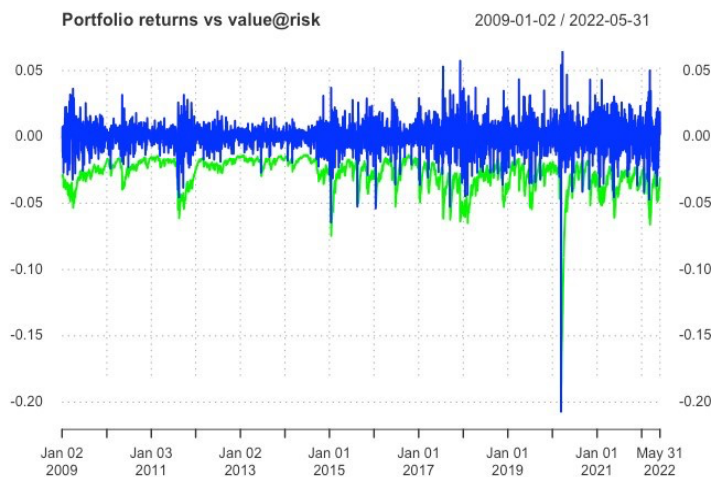
The null hypothesis about lack of ARCH effects strongly rejected Checking if returns come from normal distribution?

### 3.2 GARCH MODELS

None of the lags is significant. Our model is mostly affected by decreasing prices with less shock from increasing prices though the volatility is slightly increasing for increasing prices. Now we will use the `compare_ICs_GARCH` function to compare the different models. Now, let's compare information criteria for all models: We checked both GARCH(1,1) and GARCH(2,1). In both models, there were some insignificant parameters as well as insignificant lags of ACF of standardized residuals and ACF of squared standardized residuals. We tried to improve the model by adding AR(1) components in the mean equation. The AR lag was insignificant, though.

We decided to try the extension of GARCH - eGARCH. After examining the results, we found that there was no need to add anything to the covariance and to the variance equation. Also, the model was stable and did not require a higher order. We used information criteria to help us choose the best model. All of them (AIC, BIC, SIC, HQIC) recommended GARCH(2,1) over GARCH(1,1). Also, all of them preferred GARCH(2,1) with AR1 over eGARCH with the same parameters. We decided to stay with AR(1) and GARCH(2,1) while estimating VaR.

**Fig 2 PORTIFOLIO RETURN VS VALUE AT RISK**



### 3.3 VALUE-AT-RISK

First, we calculated Value-at-risk in the in-sample period and plotted it with the returns of the portfolio: The losses of the portfolio were higher than the assumed VaR in 1,17% of days. Then we made a forecast of a conditional standard deviation – one-day ahead and longrun prediction. We also forecasted conditional variance.



**Fig 3 FORECAST OF RETURNS AND VALUE AT RISK**

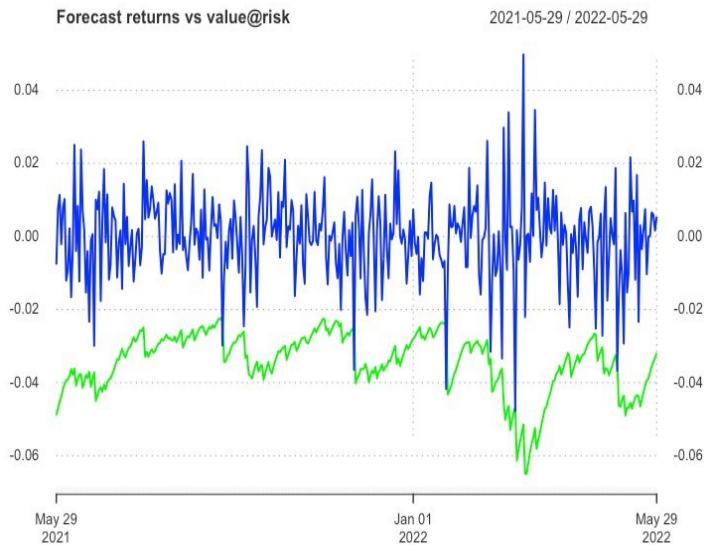


Fig. 3,

Finally, we calculated Value-at-risk in the out-of-sample period and plotted it with the returns of our portfolio: Portfolio returns vs Value-at-Risk, 2021/05/29 –2022/05/29. The losses of the portfolio were higher than the assumed VaR in 1,36% of days.

## CHAPTER 4 CONCLUSIONS

The effectiveness of symmetric and asymmetric GARCH models based on the normal distribution, Student-t distribution, skewed Student-t distribution, and parameterized models is evaluated in this article. JSU's approach to estimating and forecasting, including the distribution of residual value at risk of a portfolio of a selected instrument. The relative performance of GARCH-type models was tested using the daily returns of the portfolio consisting of the stock index. The period of examination was from 1 January 2009 to 31 May 2022; this is a suitable amount of time, taking into account both peaceful and difficult years. The need for proper financial regulation has been underscored by the rising interest of international investors in participating in developing financial

markets and the heightened volatility in these markets during times of crisis. Value at-risk quantification and forecasting.

Descriptive statistics for the portfolio Kurtosis and skewness were present, as shown by the index. The ARMA(1,2) model was determined to be suitable for modelling the series of logarithmic returns using AIC criterion. The residuals showed a strong ARCH presence, as well as clustering effects. Using the ARCH-LM test, the residuals from several estimated GARCH models revealed the absence of the ARCH effect. The models have the right statistical properties for both removing ARCH effects and autocorrelation. None of the eight models failed the Kupiec test with a 95% confidence level, according to the backtesting findings. The combination ARMA(1,2) with APARCH, GARCH, TS-GARCH, T GARCH, GJR-GARCH, EGARCH, and IGARCH, on the other hand, passed the Kupiec test with a 99% confidence level, passing seven out of the eight models that were evaluated. Three models were found to pass the joint Christoffersen test with a 95% confidence level ARMA(1,2)-TS GARCH(1,1) with a Student-t distribution of residuals; ARMA(1,2)-T GARCH(1,1) with a Student-t distribution of residuals; and ARMA(1,2)-EGARCH(1,1) with a parameterized JSU distribution of residuals. ARMA(1,2)-TS GARCH(1,1) with a Student-t distribution of residuals. Distribution of residuals. These three models seem to be suitable for capturing volatility clustering since they are all many exceptions that caused the failure. The other models were found to have rejected both the independence of failures and the null hypothesis of accurate exceedances and are therefore no longer acceptable. The finding is that the ARMA(1,2)-NARCH(1,1) model is insufficient for capturing volatility clustering (the Christoffersen test was rejected at a 95% confidence level).

Additionally, the Kupiec test at 95% and 99% confidence levels found that the right amount of exceptions were missed. This is the main justification for thinking that the ARMA(1,2)NARCH(1,1) model is not reliable for calculating VaR in connection to the portfolio stock market. The Pearson's Q test was not passed by any of the examined models, whether it was conducted at 90%, 95%, or 99% accuracy. With 95% and 99% confidence levels, Cerovic and Karadzic (2015) demonstrated that the Kupiec test was passed by the peaks-over-threshold model. This model, however, fails to meet the Christoffersen joint test for both 95% and 99%. We draw the conclusion from these findings that, although being accurate in terms of correct exceedances, the peaks-over-threshold approach for calculating VaR is insufficient to capture volatility

clustering on the Montenegrin stock market and should be improved. In this study, we discovered that ARMA(1,2)-TS GARCH(1,1) with Student-t residual distribution, ARMA(1,2)-T GARCH(1,1) model with Student-t residual distribution, and ARMA(1,2)- EGARCH(1,1) with parameterized JSU distribution of residuals all passed the joint Christoffersen test with a 95% confidence level. Despite failing the Kupiec test for a number of exceptions at a 95% confidence level, we may infer that these three models are suitable for capturing volatility clustering. For the evenly weighted portfolio we had chosen, we computed Value-at-Risk for both the in-sample and out-of-sample periods. It seems like the projections are rather accurate. It seems that our choice of the model AR(1)GARCH(2,1) was the right one. The losses of the portfolio that exceeded VaR, which were 1,17% in the in-sample period and 1,36% in the out-of-sample period, indicate that our findings, which support the accuracy of our estimate, were correct.

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## APPENDIX

**Table 3 DESCRIPTIVE STATISTICS OF STANDARDIZED RETURNS.**

	rEWP
nobs	1612.000000
NAs	0.000000
Minimum	-12.694867
Maximum	5.149658
1. Quartile	-0.452672
3. Quartile	0.504888
Mean	0.000000
Median	0.008413
Sum	0.000000
SE Mean	0.024907
LCL Mean	-0.048853



UCL Mean	0.048853
Variance	1.000000
Stdev	1.000000
Skewness	-1.428037
Kurtosis	19.269197

Source own calculation

**Table 4 GARCH MODEL FIT**

GARCH Model Fit				
Conditional Variance Dynamics				
GARCH Model: eGARCH(2,1)				
Mean Model : ARFIMA(1,0,0)				
Distribution: norm				
Optimal Parameters				
	Estimate	Std. Error	t value	Pr(> t )
mu	-0.000114	0.000294	-0.38755	0.698346
ar1	0.008413	0.028308	-0.29719	0.766318
omega	-0.250629	0.006938	-36.12565	0.000000
alpha1	-0.192607	0.035809	-5.37868	0.000000

# Information Criteria

Akaike	-5.5409
Bayes	-5.5142
Shibata	-5.5409
Hannan-Quinn.	-5.5310

alpha2	0.175986	0.036492	4.82257	0.000001
beta1	0.968927	0.001154	839.2786	0.000000
gamma1	0.297285	0.047467	6.26301	0.000000
gamma2	-0.174298	0.037754	-4.61667	0.000004

## Robust Standard Errors

	Estimate	Std. Error	t value	Pr(> t )
mu	-0.000114	0.000563.	-0.20201	0.839910
ar1	-0.008413	0.029144	-0.28867	0.772833
omega	-0.250629	0.020557.	-12.19164	0.000000
alpha1	-0.192607	0.097985	-1.96568	0.049335
alpha2	0.175986	0.095276	1.84711	0.064731
beta1	0.968927	0.002468.	392.61109	0.000000
gamma1	0.297285	0.122387	2.42906	0.015138
gamma2	-0.174298	0.105677	-1.64935	0.099077

LogLikelihood: 4473.952

SOURCE :own calculation

**TABLE 5 WEIGHTED LJUNG-BOX TEST ON  
STANDARDIZED RESIDUALS**

	Statistic.	p-value
Lag[1]	0.3615	0.5477
Lag[2*(p+q)+(p+q)-1][2]	1.4766	0.4503
Lag[4*(p+q)+(p+q)-1][5]	3.9788	0.2288
d.o.f=1		

H0: No serial correlation

source :own calculation

**TABLE 7: Weighted Ljung-Box Test on Standardized Squared Residuals**

	statistic	p-value
Lag[1]	0.02485	0.8747
Lag[2*(p+q)+(p+q)-1][8]	4.35726	0.4477
Lag[4*(p+q)+(p+q)-1][14]	8.68040	0.3117
d.o.f=3		

### Weighted ARCH LM Tests

	Statistic	Shap	Scale	P-Value
ARCH Lag[4]	1.116	0.500	2.000	0.2908
ARCH Lag[6]	1.250	1.461	1.711	0.6774
ARCH Lag[8]	6.677	2.368	1.583	0.1171

### Nyblom stability test

Joint Statistic: 1.5435

Individual Statistics:

mu	0.14621
ar1	0.04237
omega	0.12144
alpha1	0.08514
alpha2	0.10610
beta1	0.12247
gamma1	0.07752
gamma2	0.10544

### TABLE 8 SUMMARY OF SOME STATISTICS

#### Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35. 0.47 0.75

### Sign Bias Test

t-value	prob	sig
---------	------	-----

Sign Bias	0.8976	0.3695
Negative Sign Bias	0.3039	0.7613
Positive Sign Bias	0.6624	0.5078
Joint Effect	0.8518	0.8371

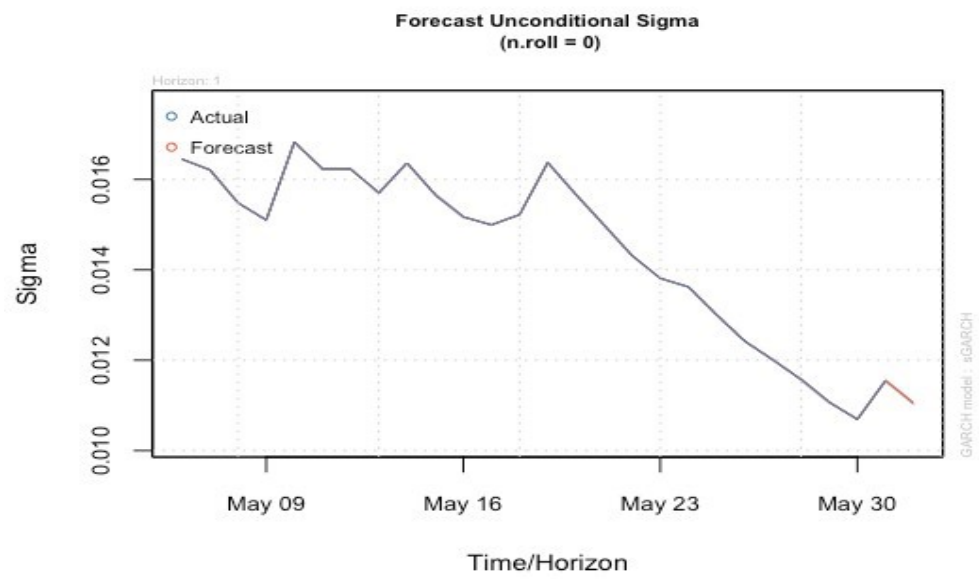
**Adjusted Pearson Goodness-of-Fit Test:**

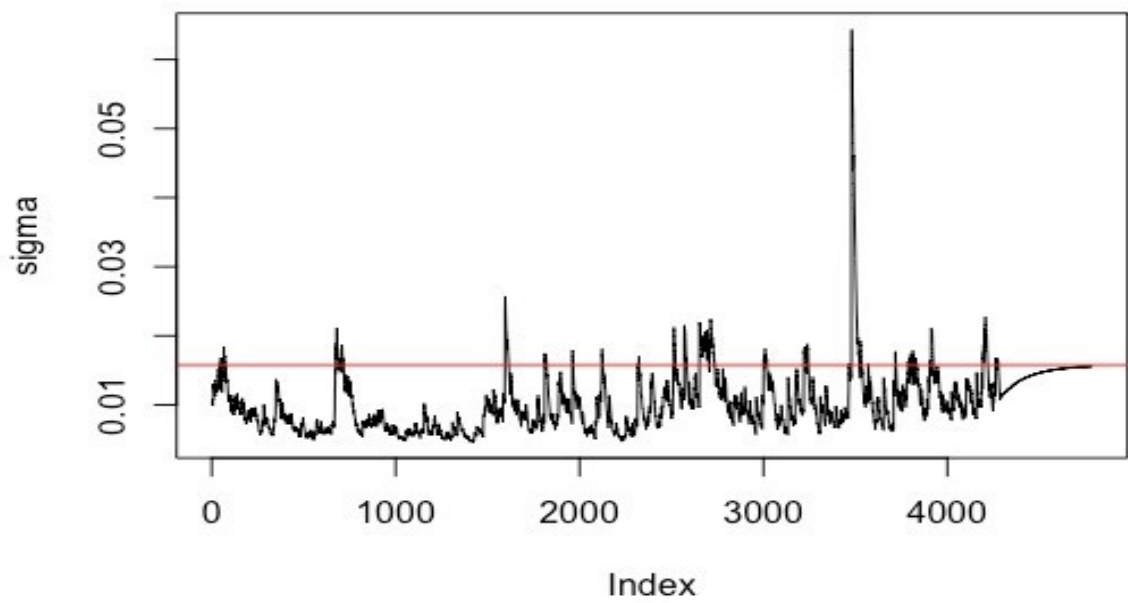
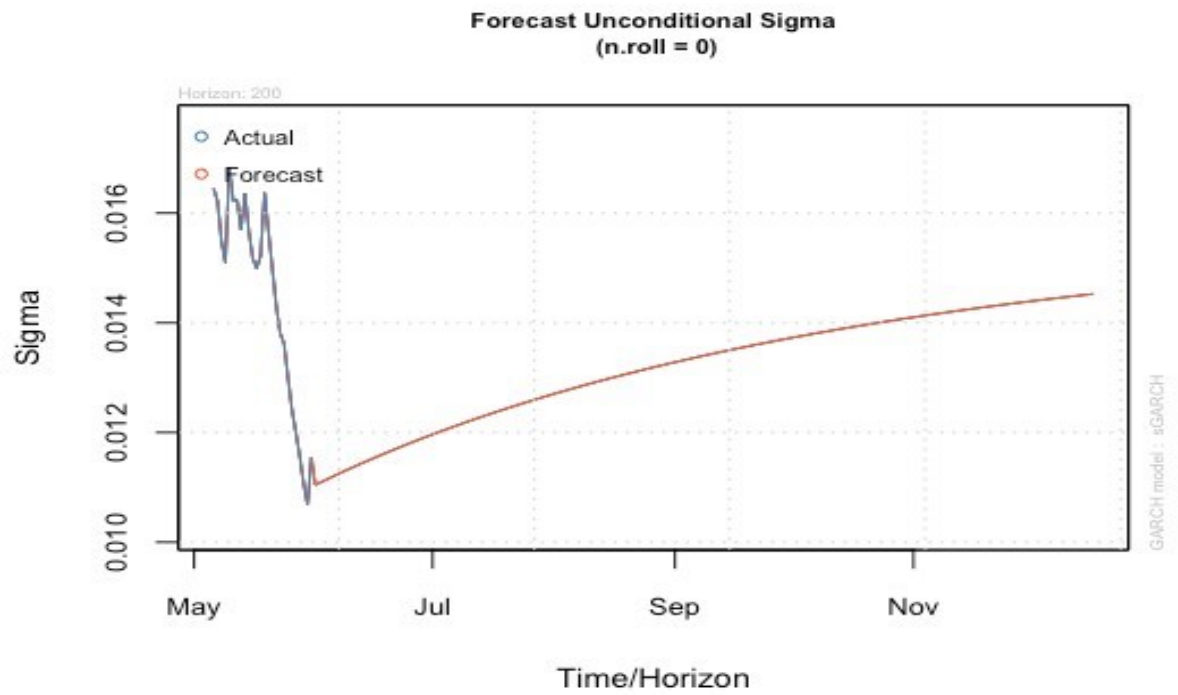
group		statistic	p-value(g-1)
1	20	110.4	6.707e-15
2	30	137.3	5.119e-16
3	40	140.7	2.032e-13
4	50	164.0	2.553e-14

Elapsed time : 1.336498

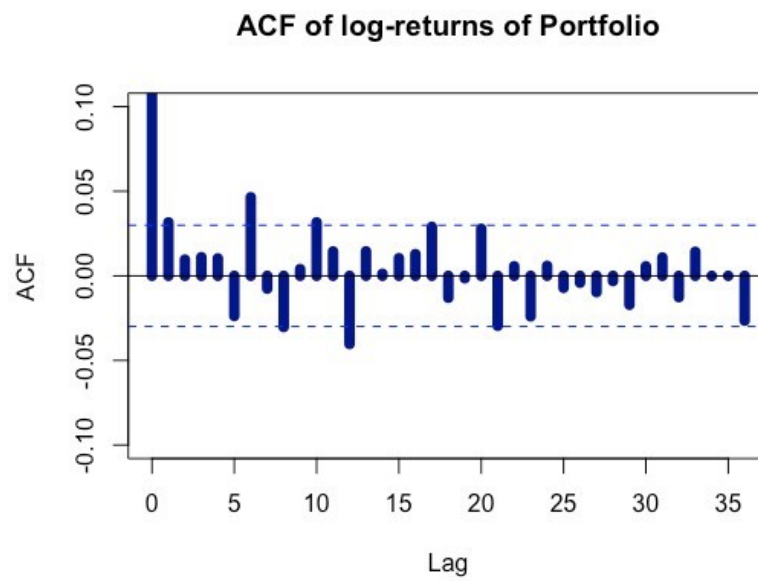
SOURCE :own calculation

**Fig 3, FORECASTING UNCONDITIONAL SIGMA**

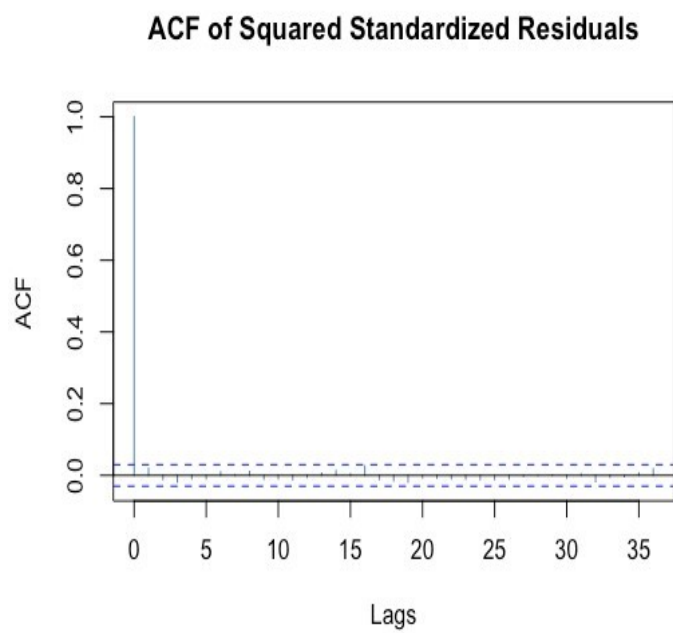




**Fig<sub>4</sub> ACF OF LOG- RETURN OF PORTFOLIO**

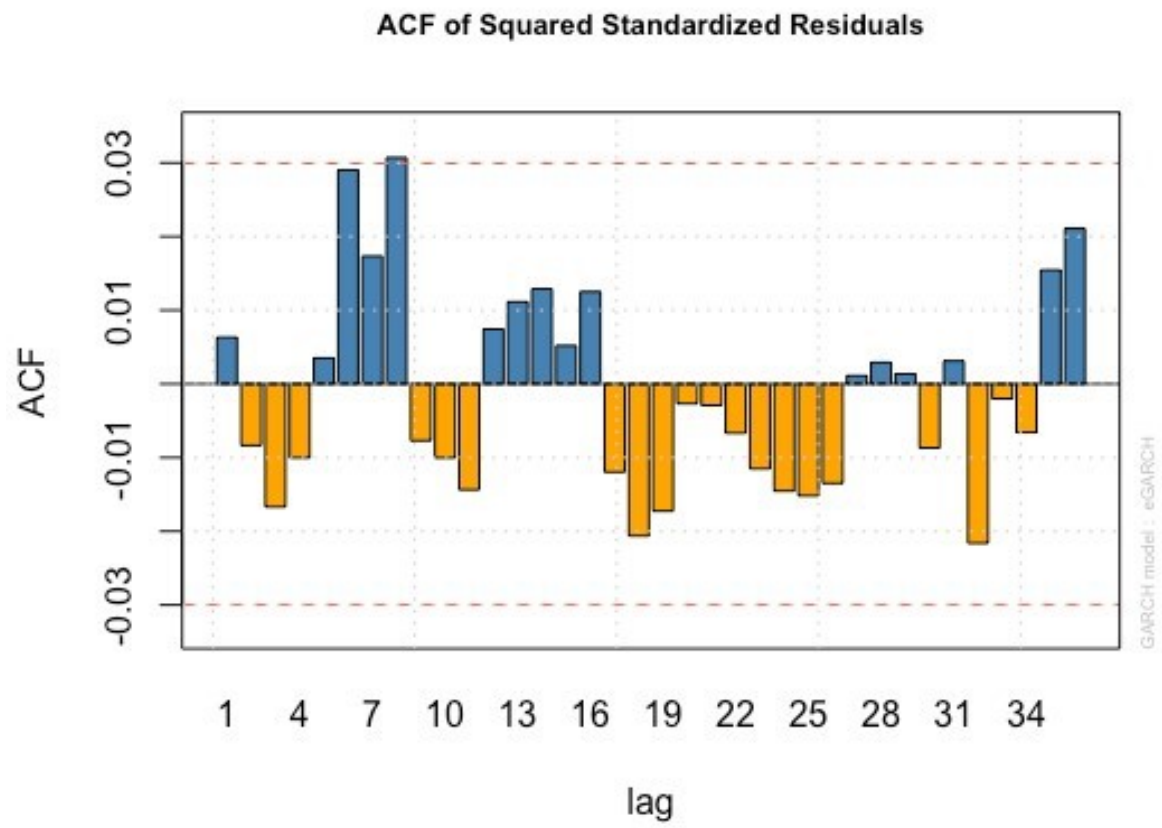


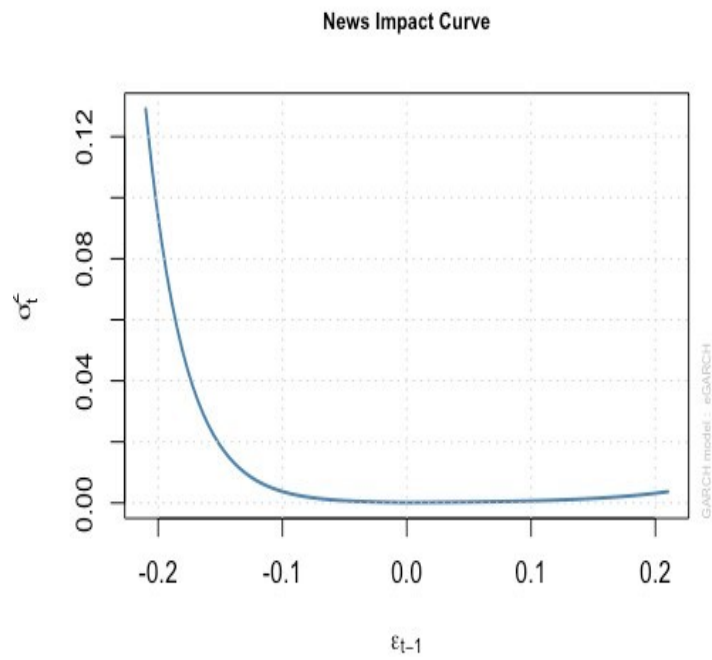
**Fig<sub>5</sub> ACF OF SQUARED STANDARDIZED RESIDUALS  
GERCH MODEL**



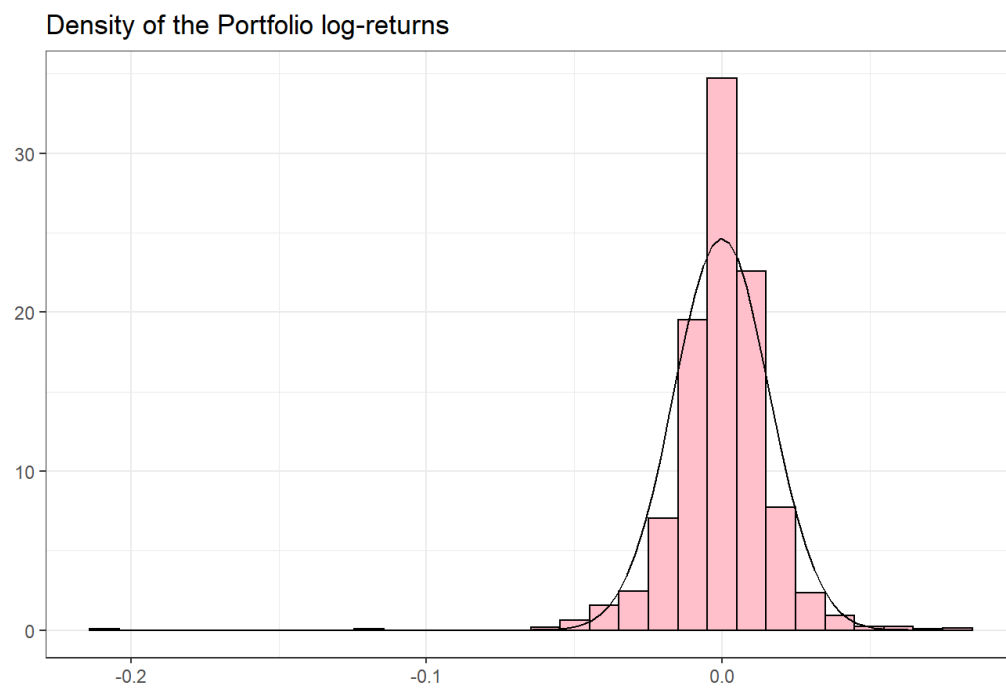


**Fig6 PLOT OF eGARCH MODELING**





**Fig7 DENSITY OF THE PORTIFOLIO LOG-RETURNS**



source: own calculations