#1 $\sigma(z) = \dfrac{1}{1+e^{-z}}$

$\sigma'(z) = \dfrac{-(-e^{-z})}{(1+e^{-z})^2} = \dfrac{e^{-z}}{(1+e^{-z})^2} = \dfrac{1}{1+e^{-z}} \cdot \dfrac{e^{-z}}{1+e^{-z}} = \dfrac{1}{1+e^{-z}}\left(1 - \dfrac{1}{1+e^{-z}}\right) = \sigma(z)\{1-\sigma(z)\}$

#2-1

$b = -0.1$, $w_1 = -1$, $w_2 = 1$ 로 선정하였다. (임의로)

		⟨Pred⟩	⟨label⟩
$(0,0)$:	$-1\times0 + 1\times0 + 0.1 = -0.1 < 0 \to$ class ①	1	1
$(0,1)$:	$-1\times0 + 1\times1 - 0.1 = 0.9 \geq 0 \to$ " 1	1	1
$(1,1)$:	$-1\times1 + 1\times1 - 0.1 = -0.1 < 0 \to$ " ①	0	1
$(1,0)$:	$-1\times1 + 1\times0 - 0.1 = -1.1 < 0 \to$ " 0	0	0

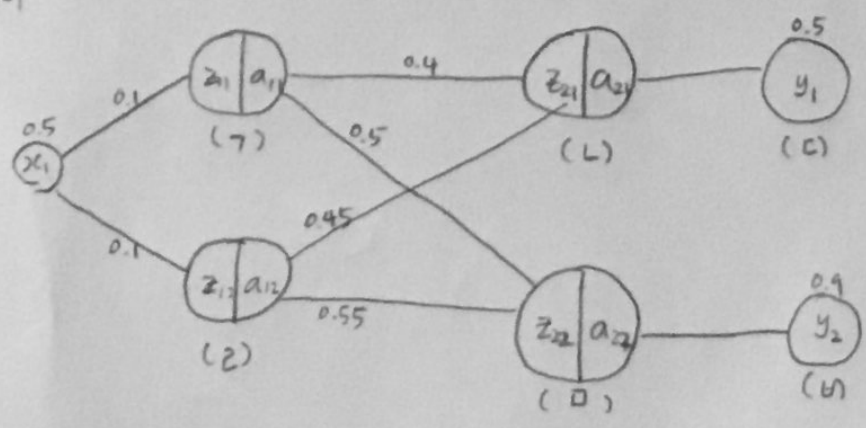$\Rightarrow$ $(0,0)$과 $(1,1)$은 분류가 제대로 되지 않았다.

#2-2

$\eta = 0.5$, $w_i \leftarrow w_i + \eta(y-o)x_i$. $(0,0)$을 이용해 학습시키자.

$$\begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix} \leftarrow \begin{bmatrix} -1 \\ 1 \\ -0.1 \end{bmatrix} + 0.5\times(1-0)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0.4 \end{bmatrix}$$

	⟨Pred⟩	⟨label⟩
$(0,0)$: $-1\times0 + 1\times0 + 0.4 = 0.4 \geq 0 \to 1$	1	1
$(0,1)$: $-1\times0 + 1\times1 + 0.4 = 1.4 \geq 0 \to 1$	1	1
$(1,1)$: $-1\times1 + 1\times1 + 0.4 = 0.4 \geq 0 \to 1$	1	1
$(1,0)$: $-1\times1 + 1\times0 + 0.4 = -0.6 < 0 \to 0$	0	0

1회 학습 후 모든 경우 제대로 분류된 것을 확인할 수 있다.

#3-1

(ㄱ) $z_{11} = 0.1\times0.5 = 0.05$, $a_{11} = \sigma(0.05) = 0.5124974\ldots \approx 0.5125$

(ㄴ) $z_{21} = 0.4a_{11} + 0.45a_{12} = 0.435625$, $a_{21} = \sigma(0.435625) = 0.6072161\ldots \approx 0.6072$

(ㄷ) $y_1 = a_{21} = 0.6072$

(ㄹ) $z_{12} = 0.1\times0.5 = 0.05$, $a_{12} = \sigma(0.05) = 0.5124974\ldots \approx 0.5125$

(ㅁ) $z_{22} = 0.5a_{11} + 0.55a_{12} = 0.538125$, $a_{22} = \sigma(0.538125) = 0.6313761\ldots \approx 0.6314$

(ㅂ) $y_2 = a_{22} = 0.6314$

#3-2

$$L = MSE = \frac{1}{n} \sum (y_i - \hat{y}_i)^2 = \frac{1}{2} \{ (0.5 - 0.6072)^2 + (0.9 - 0.6314)^2 \} = 0.0418189$$

#3-3

$$\frac{\partial L}{\partial a_{21}} = \frac{1}{2} \cdot 2 (y_1 - \hat{y}_1) \cdot (-1) = \hat{y}_1 - y_1 = 0.1072$$

$$\frac{\partial L}{\partial a_{22}} = \hat{y}_2 - y_2 = -0.2686$$

$$\frac{\partial a_{21}}{\partial z_{21}} = a_{21} \cdot (1 - a_{21}) = 0.2385, \qquad \frac{\partial a_{22}}{\partial z_{22}} = a_{22} (1 - a_{22}) = 0.2327$$

$$\frac{\partial z_{21}}{\partial a_{11}} = w_{11}^2 , \quad \frac{\partial z_{22}}{\partial a_{11}} = w_{21}^2$$

$$\frac{\partial a_{11}}{\partial z_{11}} = a_{11} \cdot (1 - a_{11}) = 0.2498$$

$$\frac{\partial z_{11}}{\partial w_{11}'} = x_1$$

$$\therefore \frac{\partial L}{\partial w_{11}^2} = \frac{\partial L}{\partial a_{21}} \cdot \frac{\partial a_{21}}{\partial z_{21}} \cdot \frac{\partial z_{21}}{\partial w_{11}^2} = 0.1072 \times 0.2385 \times a_{11} \approx 0.0131$$

$$\frac{\partial L}{\partial w_{11}'} = \frac{\partial L}{\partial a_{21}} \cdot \frac{\partial a_{21}}{\partial z_{21}} \cdot \frac{\partial z_{21}}{\partial a_{11}} \cdot \frac{\partial a_{11}}{\partial z_{11}} \cdot \frac{\partial z_{11}}{\partial w_{11}'} + \frac{\partial L}{\partial a_{22}} \cdot \frac{\partial a_{22}}{\partial z_{22}} \cdot \frac{\partial z_{22}}{\partial a_{11}} \cdot \frac{\partial a_{11}}{\partial z_{11}} \cdot \frac{\partial z_{11}}{\partial w_{11}'}$$

$$= 0.1072 \times 0.2385 \times 0.4 \times 0.2498 \times 0.5 + (-0.2686) \times 0.2327 \cdot 0.5 \cdot 0.2498 \cdot 0.5$$

$$\approx -0.0026$$

$$\therefore w_{11}' \leftarrow 0.1 - 0.5 \times \frac{\partial L}{\partial w_{11}'} = 0.1013$$

$$w_{11}^2 \leftarrow 0.4 - 0.5 \times \frac{\partial L}{\partial w_{11}^2} = 0.39345$$