BAN402 – Project 3

Fall 2023

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Part A

In this task we want to forecast the price index for existing dwellings (PIFED), using current price information in the open market. We have formulated the following model to achieve this:

Sets

$t \in T\text{: } set\ of\ time\ periods = \{0, 1,\ 2,\ \ldots 120\}$

Parameters

$PIFED_t\text{: the } actual\ value\ of\ \text{the } price\ index\ in\ quarter\ t$

Decision variables

$L_t\text{: } level\ at\ time$
$\mathrm{R_t}\text{: } trend\ at\ time\ t$
$\gamma\text{: } smoothing\ constant\ for\ \text{the}\ \ level\ L,\ where\ 0.01 \leq \gamma \leq 0.99$
$\beta\text{: } smoothing\ constant\ for\ \text{the}\ trend\ T,\ where\ 0.01 \leq \beta \leq 0.99$
$F_t\text{: the } forecast\ for\ index\ at\ time\ t$ (predicted before t)

Objective function:

$$\min MAPE = \frac{100}{120}\sum_{t=1}^{120}\left|\frac{F_t - PIFED_t}{PIFED_t}\right|$$

Constraints:

Forecast constraint:
$F_{t+1} = L_t + \mathrm{R_t}$

Level update constraint:
$L_{t+1} = \gamma * PIFED_{t+1} + (1-\gamma) * (L_t + R_t)$

Trend update constraint
$R_{t+1} = \beta * (L_{t+1} - L_t) + (1-\beta) * R_t$

The initial level and trend must adhere to specified values:
$L_0 = 18.8$
$R_0 = 0.5$

Non negativity constraints:

$L_t, F_t \geq 0$

When an optimal solution is found by a solver in a non-linear problem there is no guarantee that the found solution is a global optimum and not a local optimum. We therefore experimented with changing the lower bound of β and γ, as to see if we could further improve our objective value. By changing the lower bound of β to 0.011, we achieved a marginally better (smaller) MAPE, which confirms our suspicion that the original objective value represented a local optimum. This was however only an improvement on the 5 decimal level and will therefore not be considered further in the tasks below.

The model is shown in AMPL-code below:

PartA.mod

```
set T; # Set of time periods

param PIFED{T}; # Observed Price Index for Existing Dwellings after period t

var gamma >= 0.01, <= 0.99; # Smoothing constant for level
var beta >= 0.011, <= 0.99; # Smoothing constant for trend

var L{T} >= 0; # Level for period t
var R{T}; # Trend for period t
var F{T} >= 0; # Forecast for period t (made before t)

# Objective function to minimize the mean absolute percentage error (MAPE)
minimize MAPE:
    (100/120) * sum{t in T:t>0} abs((F[t] - PIFED[t]) / PIFED[t]);

subject to

# Constraints to update level and trend based on expressions (2) and (3)
UpdateLevel{t in T: t<120}:
    L[t+1] = gamma * PIFED[t+1] + (1 - gamma) * (L[t] + R[t]);

UpdateTrend{t in T: t<120}:
    R[t+1] = beta * (L[t+1] - L[t]) + (1 - beta) * R[t];

# Constraint to calculate the forecast for the next period based on expression (1)
Forecast{t in T:t<120}:
    F[t+1] = L[t] + R[t];

# Initial conditions for level and trend
InitialLevel:
    L[0] = 18.8;

InitialTrend:
    R[0] = 0.5;
```

Figure 1 - Model implemented in AMPL

a) The optimal value of MAPE = 2.20279 %

b) The optimal values of γ and β are 0.715168 and 0.0254128 respectively.

c) The maximum absolute percentage error was in period 64, with a value of 10.30 %

d) The minimum absolute percentage error was in period 62, with a value of ≈ 0

e) To find the forecast for the first quarter of 2023 we can use the level- and trend-values of t = 120. We can then manually calculate:

$$F_{121} = 1.17064 + 139.591 = 140.76$$

This gives us a percentage error of $\frac{140{,}76-140}{140} \approx 0.5\ \%$, which is very close to the prediction by SSB.

f)

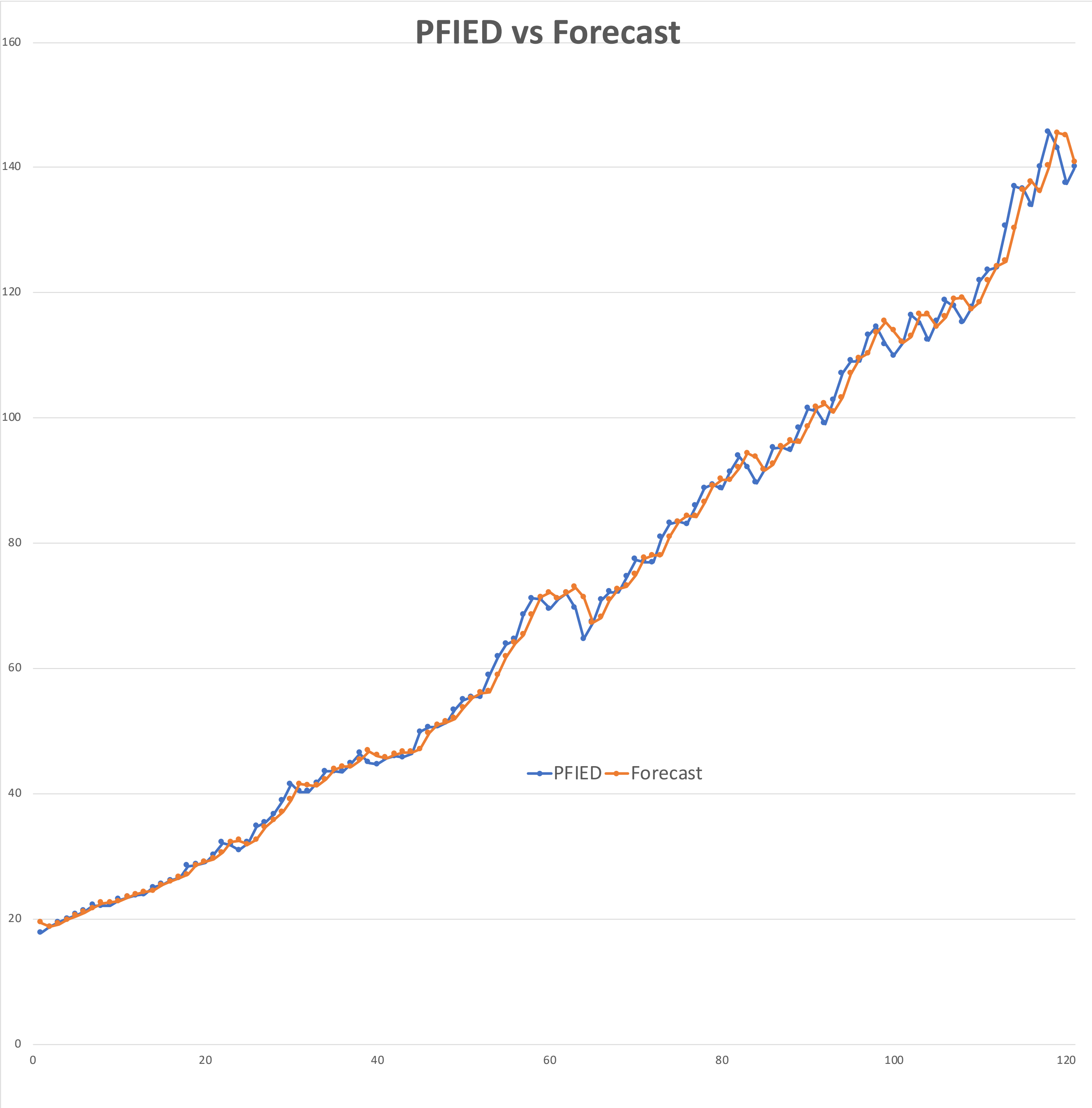


Figure 2 - PIFED and Forecast compared

The graph demonstrates that the forecast is usually quite aligned with the actual PIFED, which is also supported by the models rather small MAPE. We can see that there is slight a lag in our forecasts, which might be due to the smoothing constraints.

TASK B

1)

First, we must introduce a new parameter to set the average number of transplant centres in the districts:

$$Avg = \frac{\sum_{i \in J} h_i}{N}$$

We then modify constraint (7) to include both a lower and upper bound for the districts:

This constraint now ensures that if DSA k is chosen as a centre of a district, it cannot be assigned more than or less than 6 transplant centres from the average number defined in the parameter above.

$$Y_k * (Avg - 6) \leq \sum_{i \in J} h_i * W_{i,k} \leq Y_k * (Avg + 6), \forall\, k \in K$$

2)

Sets

First, we need to make a subset of J called T for DSA's on the top 10 list:

$$T: Set\ of\ the\ top\ 10\ DSA's, where\ T \subset J$$

Variables

We then create a new binary variable indicating if district k is a strong district:

$$S_k: 1\ if\ district\ k\ is\ a\ strong\ district, 0\ otherwise$$

New constraints

We introduce a new constraint ensuring that there cannot be more than 2 strong districts:

$$\sum_{k \in K} S_k \leq 2$$

Lastly, we must link the new variable to the number of top 10 DSA's:

This constraint ensures that S_k cannot be one if the number of top 10 DSA's in the district is below 3.

$$\sum_{i \in T} W_{i,k} \geq 3 * S_k, \forall\, k \in K$$

This constraint ensures that S_k must be one if the number of top 10 DSA's in the district is 3 or above.

$$\sum_{i \in T} W_{i,k} \leq 2 + M * S_k, \forall\, k \in K$$

Binary constraint:

$$S_k \in \{0, 1\}\ \forall\, k \in K$$

TASK C

Task 1

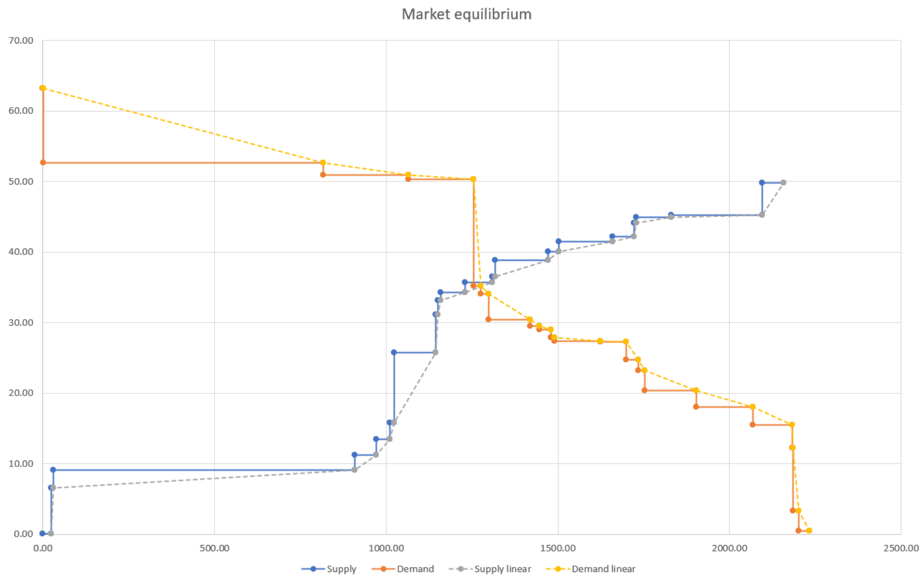
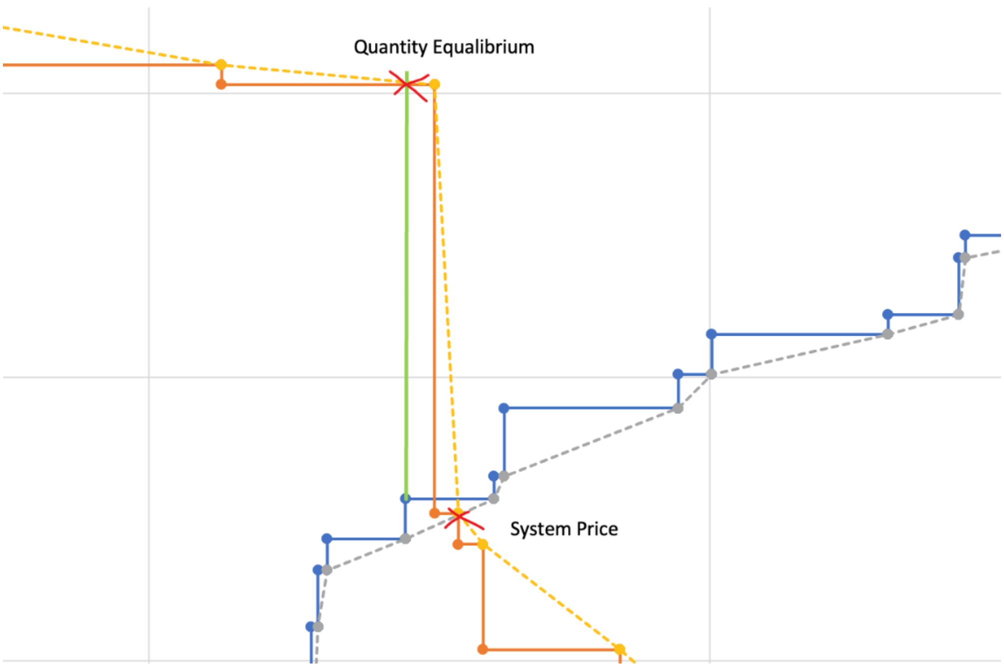


Figure 3 - market equilibrium in period 2

Our group is to look at the second period. When assessing the figure, the linearized curves cross just below the bid of 35.20. If we were to set a system price based on the figure, we would therefore propose a price just below 35.20; We find 35.15 to be a good approximation. There are four bids above this system price that should be accepted: 63.20, 52.70, 51 and 50.30. The last bid is not fulfilled in its entirety for the linearized system price, as it causes the last supply bid of 35.7 to no longer be accepted.



One advantage of using the step function curves is that they mirror the actual bids given in the market; they reflect real marked conditions more closely. On the other hand, it is harder to find a fair market equilibrium with the step curves, and one might lose some social surplus as a result. The linearized curves have the advantage of making it easier to find a clear system price by giving a unique intersection between the supply and demand curves. The linearized curves do however overestimate a customer's willingness to pay, and a supplier's willingness to supply. This might cause demand that would be fulfilled in the step function-curves, to no longer be fulfilled in the linearized equilibrium which is the case in our graph above. These rules are however well established, and the step function and linearized curve work well together in practice. In the step-function curve any price between the two last accepted bids would be a valid equilibrium. By using the linearized curves, you get a "fair" system price, giving a good compromise. In practice the step-curves decide which bids are to be accepted via the equilibrium system price set by the linearized curves.

Task 2

In this task we are to take the role of a new supplier in the market, trying to maximize profit for a power plant by setting the optimal bid. There are 20 suppliers and buyers in the market before our entry, and our plant can supply up to 1000 MW, which indicates that we will be able to affect the market equilibrium with our bid. We must therefore explore the compromise between a big volume or a higher price, as high volume drives down the system price, reducing revenue. Our goal is therefore to find the volume/price combination where the marginal gain of increasing the system price is zero.

2A)

We are here to make one bid per period, choosing one volume across all bids. By assessing the demand slopes of the market, we made the conclusion that the optimal strategy is to maximise bid volume and then experiment with the bid prices. The general tendency is that profit is reduced when sales volume is decreased, even if this leads to a higher system price. Our initial strategy was therefore to place an initial bid where our entire quantum of 1000 was accepted. If our bid is accepted in its entirety, our price should not impact the system price in most cases. However, we could still maximize the system price further if we are able to place our bid in a way that impacts the equilibrium point.

This is possible in one of two ways; either we have to be the bid setting the equilibrium ourselves, or we could place ourselves between the last and second to last bid to decrease the incline of the linearized curve setting the equilibrium. The next part of our strategy was therefore to place our bid as high as possible on the demand-curve, without sacrificing volume. By doing this we managed to be the last accepted bid without sacrificing any volume in hour 4 and 5, increasing profit as a result. When we no longer could increase our price without sacrificing volume we explored if it was possible to increase our profits by sacrificing some volume for an increased price. This would be the case if $PriceIncrease * New\ quantum > OldProfit * Old\ quantum$. To optimize further we increased our price by one unit at a time from our initial starting point, until we saw a large loss in volume. We improved our profit in period 1 by doing this.

Initial starting point

Time period	Bid volume	Accepted volume	Bid price	System price	Profit
1	1000	1000	7	13.28	3 280
2	1000	1000	9	18.31	8 310
3	1000	1000	10	27.54	17 540
4	1000	1000	10	20.99	10 990
5	1000	1000	10	32.03	22 030
				Total profit	62 150

Final values

Time period	Bid volume	Accepted volume	Bid price	System price	Profit
1	1000	972	14	14.08	3 965.76
2	1000	1000	9	18.31	8 310
3	1000	1000	27	27.54	17 540
4	1000	1000	21	21.23	11 230
5	1000	1000	32	32.24	22 240
				Total profit	63 285.76

Figure 4 - our bids

2B)

In this revised case, we can bid different volumes for different time periods. This opens the opportunity to aim our bid towards specific demand bids in the market. The most attractive points to target are the points on the demand curve where the price elasticity is low or where there is a bid with large amounts of volume. By targeting these points, we can increase our price whilst sacrificing as little volume as possible. The rule from the previous task also applies here. If we can revise our bid to a point where $PriceIncrease * New\ quantum > OldProfit * Old\ quantum$ the volume change will be profitable. By exploring prices and volumes around the large bids we managed to increase our profit for the first and second hour by targeting the 16.5 and 27.3 bids respectively.

Revised bids

Time period	Bid volume	Accepted volume	Bid price	System price	Profit
1	800	800	16	16.5	5 200.00
2	680	675	25	25.17	10 240
3	1000	1000	27	27.54	17 540
4	1000	1000	21	21.23	11 230
5	1000	1000	32	32.24	22 240
				Total profit	66 449.75

Task 3

In this task we will use the data "blockbids_2.dat". We are still able to supply the same volume at the same cost as before. We are now to place one block bid over four consecutive periods. From the data, we can see that supplied volume and prices are substantially higher in period 1 than in period 5; we will therefore set a block bid for period 1-4.

As discussed previously, a higher sales volume offsets the lower system price in most cases. Our initial strategy is therefore to start with a bid volume of 1000 MW, as we did in 2A).

When placing our initial bid of 1000 MW, we decided to undercut the lowest previously accepted block bid by setting the price to 16.4. As you can see from the average system price, we get a higher average price per unit than any of the accepted block bids. It is therefore not in our interest to try to push any of the competing block bids out of the market, as we would have to reduce our average price per unit to do so. In principle, this means that the price of our bid would not matter if it were to be accepted. This means that we gain no benefit from increasing our price, as this could see us pushed out of the market if another bid were to undercut us. We therefore set our price to equal our production cost so that we will never take a loss on an accepted block bid. To optimize our profit further we again explored the relationship between price and volume in our bid. If $Avg\ Price\ Increase * New\ total\ volume > Old\ avg\ profit * \ Old\ total\ volume$ we would benefit from reducing our bid size. By reducing our bids in intervals of 10 we found that the optimal profit was found with a bid volume of 960. The results are presented on the next page.

Bid volume		1000.00
Bid price	€	16.40
Time period	**System price**	
1	€	30.17
2	€	16.58
3	€	27.45
4	€	19.94
Avg. System price	€	23.54
Total profit	€	54 140.00

Figure 6 - initial block bid

Bid volume		960.00
Bid price	€	10.00
Time period	**System price**	
1	€	31.71
2	€	17.30
3	€	27.48
4	€	20.39
Avg. System price	€	24.22
Total profit	€	54 604.80

Figure 5 - optimised block bis

Accepted blockbids

Price	Volume
€ 16.50	18
€ 22.00	85
€ 16.40	1000

Figure 8 - initial accepted block bids

Accepted blockbids

Price	Volume
€ 16.50	18
€ 22.00	85
€ 10.00	960

Figure 7 - revised accepted block bids

TASK D

Sets

$k \in K$: $Set\ of\ counties$

$j \in J$: $Set\ of\ municipalities$

$\overline{J_k} = set\ of\ municipalities\ to\ county\ k,\ \text{where}\ \overline{J_k} \subset J, \forall\ k \in K$

$i \in I$: $Set\ of\ companies$

$b \in B$: $Set\ of\ bids$

Parameters

$P_b = Price\ of\ bid\ b, \forall\ b \in B$

$D_i = Discount\ offered\ by\ company\ i, \forall\ i \in I$

$T_i = Treshold\ parameter\ for\ company\ i, \forall\ i \in I$

$a_{i,b} = Binary\ parameter\ inidcating\ if\ bid\ b\ is\ placed\ by\ company\ i, \forall\ i \in I, b \in B$

$m_{j,b} = Binary\ parameter\ indicating\ if\ municipality\ j\ is\ contained\ in\ bid\ b, \forall j \in J, b \in B$

$MaxN = Maximum\ number\ of\ companies\ that\ can\ have\ bids\ accepted$

$M = A\ sufficiently\ large\ value\ based\ on\ available\ data.$

Variables

$x_b = Binary\ variable\ indicating\ if\ bid\ b\ is\ accepted, \forall\ b \in B$

$y_i = Binary\ variable\ indicating\ if\ company\ i\ exceeds\ the\ treshold\ for\ discount, \forall\ i \in I$

$z_{i.k} = Binary\ variable\ indicating\ if\ company\ i\ has\ an\ accepted\ bid\ in\ county\ k, \forall i \in I, k \in K$

$w_i = Binary\ variable\ indicating\ if\ company\ i\ has\ an\ accepted\ bid, \forall\ i \in I$

Objective function

The objective function aims to minimize cost: $Price\ of\ all\ bids - discount\ awarded.$

$$Minimize\ cost = \sum_{b \in B} P_b * x_b - \sum_{i \in I} D_i * y_i$$

Constraints

All municipalities must be covered by at least one bid:

$$\sum_{b \in B} m_{j,b} * x_b \geq 1, \forall\ j \in J$$

All counties must be served by at least two different companies:

$$\sum_{i \in I} z_{i,k} \geq 2, \forall\ k \in K$$

The total number of companies with an accepted bid must be below MaxN:

$$\sum_{i \in I} w_i \leq MaxN$$

The discount can only be applied if the total price for all of company i's accepted bids exceeds T_i:

$$\sum_{b \in B} a_{i,b} * P_b * x_b \geq T_i * Y_i \text{ ,} \forall\, i \in I$$

Linking constraints

This constraint ensures that if company i has any bid accepted, w_i must equal 1.

$$\sum_{b \in B} x_b * a_{i,b} \leq M * w_i \text{ ,} i \in I$$

This constraint ensures that w_i cannot be one if company i has no bids accepted.

$$\sum_{b \in B} x_b * a_{i,b} \geq w_i \text{ ,} i \in I$$

This constraint ensures that $z_{i,k}$ must be 1 if company i has a bid accepted which includes a municipality from county k.

$$\sum_{b \in B} \sum_{j \in J_k} a_{i.b} * m_{j,b} * x_b \leq M * z_{i.k}, \forall\, i \in I, k \in K$$

This constraint ensures that $z_{i,k}$ cannot be 1 if company i has no bid including a municipality from county k accepted.

$$\sum_{b \in B} \sum_{j \in J_k} a_{i.b} * m_{j,b} * x_b \geq z_{i.k}, \forall\, i \in I, k \in K$$

Binary constraints:

$x_b \in \{0, 1\}\ \ \forall\, b \in B$

$y_i \in \{0, 1\}\ \forall\, i \in I$

$z_{i,k} \in \{0, 1\}\ \forall i \in I, k \in K$

$w_i \in \{0, 1\}\ \forall\, i \in I$