BAN402 – Project 1

Fall 2023

Candidate: 66 & 14

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Part A

Task 1.

In this problem we want to determine an optimal emission reduction plan for a factory, which seeks to minimize costs. The factory needs to choose between the usage of three emission reduction methods, and the intensity of usage of these, to reach the emission reduction goals set by the government.

Indexes and sets:

G: $set\ of\ emission\ gasses = \{\text{CO2, CFC, N2O}\}$
M: $set\ of\ methods = \{\text{LEF, EP, SCR}\}$

Parameters:

$Cost_m = cost\ per\ unit\ of\ intensity\ of\ method\ m,\ \ m \in M$
$Reduction_{g,m} = reduction\ of\ gas\ g\ with\ method\ m, m \in M, g \in G$
$Target_g = target\ reduction\ for\ gas\ g, g \in G$

Decision variables:

$x_m \geq 0$ $\quad$ $units\ of\ intensity\ of\ method\ m \in M$

Objective function:

Our objective function aims to minimize the induced cost for using the emission reduction methods.

$$\min \text{Cost} = \sum_{m \in M} x_m * Cost_m$$

Constraints:

The total reduction in emission gas g must equal or exceed the target set by the government for gas, for all emission gases.

$$\sum_{m \in M} Reduction_{g,m} * x_m \geq Target_g \ \forall g \in G$$

Solution

The optimal solution is shown in figure 1.

This gives us an objective value of ≈ \$ 12 888.9. This solution is unique.

Method	Units used
EP	11.1111
LEF	0
SCR	1.33333

Figure 1 – units of intensity used

Task 2.

As we can see from the sensitivity report in figure 2, the emission constraint is satisfied in the optimal solution. We see this from the slack variables in the AMPL-display, with $N_2 0$ reduction = 0, and CO_2-reduction = $-2.84217^{14} \approx 0$. This implies that the emission constraints are binding. Furthermore, we can see that the company has exceeded its goal for reduction in CFC emissions, with a slack value of 21,78. The CFC demand is therefore not binding in the optimal solution.

Gas	Shadow price	R.H.	Upper bound	Lower bound	Slack
CFC	0,00	100,00	121,78	0,00	21,78
C02	77,78	140,00	144,00	90,00	0,00
N20	11,11	180,00	280,00	175,00	0,00

Figure 2 – sensitivity analysis

Task 3.

To assess how the optimal cost is sensitive to the emission targets set by the government, we have look at the shadow prices. They signal how much the optimal solution will change for one unit increase/decrease in the RHS value of the emission constraint, unless we increase/decrease the RHS value outside the upper or lower bound.

If the government emission target for N_2O is reduced to 155 thousand kg (a relaxation of the model), we will move below the lower bound of 175 tons, and the shadow prices will not be valid. It is therefore impossible to assess what the reduction in cost will be without running our model again with the revised data. If we do run the model again with the new emission goal, the reduction in cost is:

$\$\,12\,888.9 - \$12\,833.3 = \$\,55.6$

The revised model is called "Project1_A3_reduction", and is included in the .zip-file.

On the other hand, if the N_2O emission reduction target is increased to 220 (restriction), we can use the shadow prices to determine the new, increased cost, since we are within our upper bound. The new price will increase with $(220 - 180) * \$\,11.11 = \$\,444.4$.

The new price will therefore be $ 12 888.9 + $ 444.4 = $ 13 333.3

Task 4.

To assess how changes in the cost coefficients will affect the optimal solution we use the upper and lower bounds of the cost coefficients. The lower bounds are 750, 1288.89 and 458.33 for EP, LEF and SCR respectively. The upper bounds are 1200 and 733,33 for EP and SCR. The upper bound for LEF is

$\approx \infty$. This is because there is no use of LEF method in the optimal solution, and further increase in prices will not change the optimal solution, considering all other factors remain unchanged. For the other methods, the optimal solution will not change if they stay within the bounds described.

Given that the current cost for EP is 1100, it is nearing its upper bound, and a 9% increase could alter the optimal solution. Similarly, SCR's current cost of 500 is close to its lower bound, and an 8% decrease might change the solution. Therefore, both EP and SCR are somewhat sensitive to cost variations.

We can from the sensitivity report see that reduced cost of LEF is $ 2011.11; for every unit we would use of the LEF method we would increase the costs of this amount. Therefore, a reduction in price for the LEF method of $3300 - 2950 = 350$ would not be sufficient to change the optimal solution.

Part B

Task 1.

In this problem we will determine an optimal weekly blending plan for a company which seeks to maximize revenue. The blending plan determines the cardboard blend for each product, and which supplier the cardboard should be acquired from. We have constructed the following model with the belonging data in AMPL to solve the problem. Product and cardboard are measured in number of tonnes in the model.

Sets

$C: set\ of\ cardboards = \{C1, C2, C3, C4\}$

$P: set\ of\ products = \{P1, P2, P3, P4\}$

$S: set\ of\ suppliers\ = \{S1, S2\}$

$T: set\ of\ product\ categories = \{A, B\}$

$Q: set\ of\ qualitytraits = \{Strength, Transparency\}$

Parameters

$MaxCardboard_{c,s} = Max\ available\ cardboard\ c\ from\ supplier\ s, c \in C, s \in S$

$CostCardboard_{c,s}\ = Price\ of\ cardboard\ c\ from\ supplier\ s, c \in C, s \in S$

$MinSupply_s = Minimum\ purchase\ value\ from\ supplier\ s, s \in S$

$Maxprod_t = Maximum\ production\ of\ product\ category\ t, t \in T$

$ProdCost_t = Production\ cost\ of\ product\ category\ t, t \in T$

$Type_{p,t} = Binary\ value\ indicating\ if\ product\ p\ belongs\ to\ productcategory\ t, t \in T, p \in P$

$MinProd_p = Minimum\ production\ of\ product\ p, p \in P$

$SalePrice_p = Saleprice\ of\ product\ p, p \in P$

$QualCardboard_{c,q} = Quality\ value\ of\ quality\ trait\ q\ in\ cardboard\ c, c \in C$

$QualProdMin_{p,q} = Minimum\ qualityvalue\ of\ quality\ trait\ q\ in\ product\ p, q \in Q, p \in P$

$QualProdMax_{p,q} = Maximum\ qualityvalue\ of\ quality\ trait\ q\ in\ product\ p, q \in Q, p \in P$

Decision variables

$x_{c,s} \geq 0 \quad Amount\ of\ cardboard\ c\ bought\ from\ supplier\ s, c \in C, s \in S$

$y_p \geq 0 \quad Amount\ produced\ of\ product\ p, p \in P$

$z_{c,p} \geq 0 \quad Amount\ of\ cardboard\ c\ used\ in\ product\ p, c \in C, p \in P$

Objective function

The first term in the objective function calculates the total revenue from the products, the second term calculates the total cost of purchasing cardboard from suppliers and the third term calculates the total production cost for all products.

$$Max: \sum_{p \in P} y_p * SalePrice_p - \sum_{c \in C} \sum_{s \in S} x_{c,s} * CostCardboard_{c,s} - \sum_{p \in P} \sum_{t \in T} y_p * ProdCost_t * Type_{p,t}$$

<u>Constraints</u>

Maximum capacity from supplier

$x_{c,s} \leq MaxCardboard_{c,s}, \forall\ c \in C, s \in S$

Bought cardboard c from supplier s can't exceed the maximum capacity for the supplier.

Minimum production

$y_p \geq MinProd_P, \forall\ p \in P$

Amount of product produced must be greater or equal to the demand.

Production capacity

$$\sum_{p \in P} y_p * Type_{p,t} \leq MaxProd_t, \forall\ t \in T$$

Total production of product category t can't exceed the production capacity.

Minimum supply

$$\sum_{c \in C} x_{c,s} * CostCardboard_{c,s} \geq MinSupply_s, \forall\ s \in S$$

The company must reach the minimum purchase value for each supplier.

Minimum quality

$$\sum_{c \in C} z_{c,p} * QualCardboard_{c,q} \geq y_p * QualProdMin_{p,q}, \forall\ p \in P, q \in Q$$

The average strength rating for the products produced cannot be less than the minimum requirement. The transparency minimum has been set to 0.

Maximum quality

$$\sum_{c \in C} z_{c,p} * QualCardboard_{c,q} \geq y_p * QualProdMax_{p,q}, \forall\, p \in P, q \in Q$$

The average transparency for the products produced cannot be higher than the maximum requirement. The strength maximum has been set to 100.

Cardboard flow

$$\sum_{p \in P} z_{c,p} = \sum_{s \in S} x_{c,s}\,, \forall\, c \in C$$

The amount of cardboard used in production must be equal to the cardboard bought.

Blending ratio

$$y_p = \sum_{c \in C} z_{c,p}\ \forall\, p \in P$$

The amount of produced product must equal the amount of cardboard used in the product. We have for simplicities sake set this ratio to 1:1, you could modify this constraint to implement another blending ratio.

Optimal solution

Figures 3, 4 and 5 show the optimal solution. Figure 3 shows the amount of cardboard bought from the suppliers, figure 4 show the amount produced of each product and figure 5 shows the blending mix of cardboard needed to produce the products.

The objective function returns a maximum profit of $ 2 915 520. In the optimal solution the company only supplies the minimum amount of product P1 and P3, it then fills the rest of the capacity in category A by producing P2. The model then makes as much of P4 as it´s able to with the available cardboard. The available production capacity for category B and the amount of available C1 is the binding constraints in the model.

Supplier/Cardboard	C1	C2	C3	C4
S1	0	25 955,70	0	911,39
S2	20 000	0	23 132,90	20 000

Cardboard/Product	P1	P2	P3	P4
C1	0	0	5 000	15 000
C2	0	5 955,70	5 000	15 000
C3	0	23 132,90	0	0
C4	9 000	11 911,40	0	0

Figure 5 – Blending mix

Product	Quantity
P1	9 000
P2	41 000
P3	10 000
P4	30 000

Figure 4 – Produced product

Preferred supplier

To conclude on which supplier is the overall most profitable supplier we must look at the slack for the Maximum capacity from supplier constraint. Using AMPL we get the following output shown in table 5.

Supplier/Cardboard	C1	C2	C3	C4
S1	0	14 044,30	25 000	24 088,60
S2	0	0	6 876,09	0

Figure 3 – Slack values for available cardboard

Looking at the slack values in table 5, supplier S1 has a considerable amount of unused supply across all cardboards (they have no supply of C1). In contrast, supplier S2 only has one significant slack value, which are still lower then S1's slack value for the same cardboard. Therefore, given the lower slack values, we can conclude that supplier S2 is the most profitable supplier as it has a higher degree of utilized supply.

Task 2.

a)

To find the maximum price the company should accept to pay per tonne of C1 from the new supplier, we must first calculate the minimum increase in profits:

$2\,915\,520 * 1{,}025 - 2\,915\,520 = 72\,888$

For 1000 tonnes of cardboard this means that each tonne must increase the profit by $\frac{72\,888}{1000} \approx 72{,}9$

We then implement our new supplier in the model and set the values for supply and cost of cardboards to zero, so we can analyse the shadow price. Using AMPL we get the following output shown in figure 7.

C1	Dual	Up	Current	Down
S1	134,18	1 286	0	0
S2	76,68	21 285,70	20 000	11 000
S3	134,18	1 285,71	0	0

Figure 4 – shadowprice for cardboard C1

The data in figure 7 shows us that a free tonne of cardboard C1 from supplier S3 will increase our profits by 134,18 for 1286 tonnes. From there we can conclude that the maximum price the company should accept to pay per tonne is: $134{,}18 - 72{,}9 = 61{,}3$

b)

To find the maximum price the company should accept to pay per tonne of the new cardboard C5, we can apply the same logic as above. In addition to the supplier we added in a), we need to

C5	Dual	Up	Current	Down
S1	95	5 000	0	0
S2	95	5 000	0	0
S3	95	5 000	0	0

Figure 5 – Shadow price for cardboard C5

implement our new cardboard C5 in our model by modifying our datafile. After implementing C5 in our data we get the following output shown in figure 8.

The data in figure 8 shows us that a free tonne of cardboard C5 from supplier S3 will increase our profits by 95 for 5000 tonnes. From there we can conclude that the maximum price the company should accept to pay per tonne is: $95 - 72{,}9 = 22{,}11$

Part C

Task 1.

Sets:

$R: set\ of\ regions = \{R1,\ R2\}$
$F: set\ of\ production\ facilities = \{F1,\ F2\}$
$M: set\ of\ markets\ = \{K1,\ K2 \ldots K15\}$

Parameters:

S_r: $\max available\ supply\ of\ salmon\ per\ region\ r\ , r \in R$

$D_m: monthly\ demand\ in\ market\ m, m \in M$

$C_{r,f}^{TRA1}$: $transporting\ cost\ per\ tonnes\ from\ region\ r\ to\ production\ facility\ f, r \in R, f \in F$

$C_{f,m}^{TRA2}$: $transporting\ cost\ per\ tonnes\ from\ production\ facility\ f\ to\ market\ m, f \in F, m \in M$

$V_f = maximum\ volume\ capacity\ at\ facility\ f, f \in F$

Decision variables:

$x_{r,f}\ \geq 0: tonnes\ of\ salmon\ sent\ from\ region\ r\ to\ facility\ f, r \in R, f \in F$
$y_{f,m} \geq 0: tonnes\ of\ salmon\ sent\ from\ facility\ f\ to\ market\ m\ f \in F, m \in M$

Objective function:

$$\min Cost = \sum_{r \in R} \sum_{\mathrm{f} \in \mathrm{F}} \mathrm{x}_{\mathrm{r,f}} * C_{r,f}^{TRA1} + \sum_{f \in F} \sum_{m \in M} y_{f,m} * C_{f,m}^{TRA2} \ ,$$

Constraints

<u>Supply constraint</u>

The total tonnes shipped from region r to facility f must not exceed available supply from region r, for all regions r.

$$\sum_{f \in F} x_{r,f} \leq S_r \ \forall\ r \in R$$

Demand constraint

The total amount of salmon sent from all facilities to market m must meet the demand in market m, for all markets.

$$\sum_{f \in F} y_{f,m} = D_m \forall\, m \in M$$

Capacity constraint:

The total amount of salmon shipped from facility f to market m can't exceed facility f´s capacity.

$$\sum_{m \in M} y_{f,m} \leq V_f \;\forall f \in F$$

Flow constraint

The total amount of salmon shipped from all regions to facility f must be equal to total amount of salmon shipped from facility f to market m, for all facilities.

$$\sum_{r \in R} x_{r,f} = \sum_{m \in M} y_{f,m} \forall f \in F$$

Optimal shipping plan

The optimal shipping plan for region to facility is presented in figure 9, and the optimal shipping plan for facility to market is presented in figure. Table 10 shows the transport plan from facility to marked. Table 9 shows the transportation plan from region r to facility f. The total transport cost for the shipping plan is $ **10 480.** The model is called "Project1_C1" in the zip folder.

	F1	F1
R1	150	0
R2	0	137

Figure 6 – region to facility

Facility/Market	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	Sum
F1	0	21	15	15	0	0	0	0	15	28	0	18	0	0	23	135
F2	17	0	0	0	22	15	20	12	0	0	19	0	25	7	0	137

Figure 7 – shipping plan for facility to market

Task 2.

Scenario 1 – increase in demand

To run this scenario, we added a new parameter:

$I_m = change\ in\ marked\ demand\ in\ marked\ m, m \in M$

We then changed the demand constraint:

$$\sum_{f \in F} y_{f,m} = D_m * \mathrm{I}_m \; \forall \; m \in M$$

In the data file, we have specified a 25% increase for each market m. This model is called "Project1_C2_increase".

This revised model is infeasible; since we have added a 25 % increase in aggregated market demand, the production facilities will not be able to process all the salmon from region r to facility t. The aggregated demand goes from a total of $135 + 137$ = 272 tonnes of salmon, to $272 * 1.25 = 340$ tonnes, which is above the facilities combined production capacity of 300.

<u>Scenario 2 – increase in demand and introduction of penalty costs</u>

To deal with scenario 2, where it is allowed to not meet all demands in all markets at a penalty cost, we revised the model. To account for the penalty costs we have added a new parameter:

$P_m = penalty\ cost\ for\ not\ meeting\ demand\ in\ marked\ m, m \in M$

Then we added a new variable:

$z_m = unsatisfied\ demand\ in\ marked\ m, m \in M$

Revised objective function:

$$\min Cost = \sum_{r \in R} \sum_{\mathrm{f} \in \mathrm{F}} \mathrm{x}_{\mathrm{r,f}} * C_{r,f}^{TRA1} + \sum_{f \in F} \sum_{m \in M} y_{f,m} * C_{f,m}^{TRA2} + \sum_{m \in M} P_m * z_m$$

We added the following constraint:

$$z_m = D_m * I_m - \sum_{f \in F} y_{f,m} \; \forall \; m \in M$$

The new, optimal shipping plan for facility to marked is presented in figure 12, and the optimal shipping plan for region to facility is presented in figure 11. Table 12 shows the transport plan from facility f to market k. Table 11 shows the transportation plan from region r to facility f. The total transport cost for the optimal shipping plan is \$ $\mathbf{9122,5,}$ which is substantially lower than in the base case. This is because the penalty costs for some markets are substantially lower than the shipping costs. It is therefore more rational to take the penalty than shipping when the goal is to minimize costs. This is maybe not the intention of the seafood company; they want to maximize profits, and not only minimize costs.

By not shipping as much in the revised model, the facilities are also not fully utilized in the new shipping plan, leaving potential profits on the table. In the revised model, facility F1 is handling 117,5 tons form R1, and F2 is handling a total of 107,5 tons. This is below the maximum handling capacity of both facilities.

The transport costs here are a total of $7 022,50 and the penalty costs are a total of $ 2100.

The market with the biggest unsatisfied demand is market K13, with an unsatisfied demand of 31,25 tons of salmon. This is again because the shipping costs per ton, $75 or $21, are both higher than the penalty cost, $ 20.

	F1	F2
R1	117,5	
R2		107,5

Figure 8- shipping plan for region to facility

Facility/Market	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15
F1	0,00	26.25	18.75	18.75	0	0	0	0	18.75	35,00	0	0	0	0	0
F2	21.25	0	0	0	27.5	18.75	25	15	0	0	0	0	0	0	0

Figure 9 – shipping plan for facility to market

Part D

Task 1.

a)

Yes, the model could be infeasible. If a large number of users want to reach an end state $s_{k,f}^{end}$ in such a manner that these end states cannot be met without overloading the grid; $\sum_{k \in K} x_{k,t} \leq c_t \forall t \in T$, the model would be infeasible.

Also, if a user sets such a high value for $s_{k,f}^{end}$ that the charging rate needed, $x_{k,t}$ to reach the end state $s_{k,f}^{end}$ would violate the constraint $x_{k,t} \leq m_k \ \forall k \in K,\ t \in T$, the model would be infeasible.

Furthermore, if an user sets an end state *lower* than the start state, the model would also be infeasible, since $s_{k,i}^{start} + \sum_{t \in \{i, \dots, f\}} x_{k,t} = s_{k,f}^{end} \ \forall (k,\ i,\ f) \in U$ would require $x_{k,t}$ to be negative and break the non-negativity constraint $x_{k,t} \geq 0 \ \forall k \in K,\ t \in T$. In practice, all these scenarios can easily be prevented, if there is an app or another system checking for these logical errors when trying to prompt illogical data that would break the model.

b)

A model can be considered unbounded if there exists no constraints that limit the growth of the decision variable, in turn allowing the objective function to approach infinity ($\rightarrow \infty$).

This is not the case for this model. We can see this by looking at the following constraints:

(5) $x_{k,t} \geq 0 \ \forall k \in K,\ t \in T$	This constraint sets a lower bound on the decision variable.
(3) $x_{k,t} \leq m_k \ \forall k \in K,\ t \in T$)	This constraint sets an upper bound on the decision variable.
(4) $\sum_{k \in K} x_{k,t} \leq c_t \forall t \in T$	this constraint also sets an upper bound on the decision variable.

These constraints limit the maximum charge per time period, per vehicle k, $x_{k,t}$ the decision variable will always be limited, and the objective function will not be able to approach infinity, $\rightarrow \infty$.

c)

No, the model would still not be unbounded by the presence of negative prices for some time periods.

This would on the other hand allow the cost z to be negative if the price $p_t < 0$ for a certain time interval. This implies that users are getting paid for consuming electricity during those time periods. The objective function then minimizes costs by maximizing the aggregated charging for all vehicles k connected during these periods. While the negative prices can influence the charging behaviour, they will not resolve the potential infeasibility issues previously discussed. If the input data or constraints are inconsistent, for example, the combined desired state of charge for all vehicles exceeds the available grid capacity, the model can be infeasible.

Task 2.

To account for the smoothing condition, we have added two new constraints:

$$\sum_{k \in K} x_{(k,t)} - \sum_{k \in K} x_{(k,t-1)} \leq \delta \; \forall \; t \in \{2,\ 3 \ldots 24\}$$

$$\sum_{k \in K} x_{(k,t-1)} - \sum_{k \in K} x_{(k,t)} \leq \delta \; \forall \; t \in \ \{2,\ 3 \ldots 24\}$$

These constraints ensure that the difference in total power consumption between any two consecutive hours does not exceed δ whether it's an increase or decrease. By using this pair of constraints instead of absolute values, we maintain the linearity of the model.

By adding these constraints, we are imposing a restriction on the model. This can lead to more suboptimal charging scheduling. The model might not fully utilize periods with low electricity prices, or not reduce consumtion during high price periods, as the model will now try to maintain a smooth power consumption profile. Since the model now cannot scale up or down the power usage solely based on demand and pricing, it is reasonable to believe the aggregated charging costs for all vehilces will increase. On the other hand, it is reasonable to believe the reason the smoothing condition is implementet is to reduce strain and potential costs on the grid itself. The overall utility for society might therefore be a net positive.

Task 3.

a)

How a price increase of 10 NOK at 12 and price reduction of 10 NOK at 20 would impact the optimal objective value v^* depends on how the total consumption is spread throughout the day. In figure 13 from Bjørndal et al. (2023), we can see the optimal charging pattern for smart charger users. As we can see the smart charger does not consume much power at 12, presumably because the power is expensive at this time. A price increase for 12 would therefore not alter the behaviour of the model by much, as consumption in this period is already de-prioritized. The price decrease at 20 would however impact the model by a bit as it is in the starting interval of where the model increases its consumption.

Assuming the price decrease of 10 NOK makes the time period less expensive than another prioritized time period, the model would increase its consumption in this time-period as it would cause the objective value v^* to decrease from the previous optimal value of 37 500 NOK.

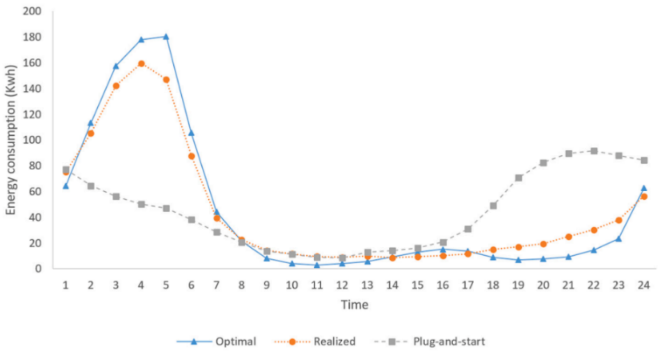

Figure 10 – Bjørndal et al., 2023

b)

As we have discussed earlier figure 13 shows us that the model de-prioritizes the time interval 9-11 presumably because the price is high at this time. Given that the prices are known to the model before the charging period is started, a longer time period will give the model a better chance to optimize and minimize the induced charging costs. Therefore, starting the charging at the same time, while ending at 11, will never give higher costs than the first case. It will however be possible for it to reduce costs if the prices are lower in the extended charging period. Based on figure 13, however, it is likely that the electricity costs will higher in the time interval after 8. If this is the case, the w^* will be unchanged from the previous optimal value of NOK 37,500.

References

Bjørndal, E., Bjørdan, M., Bøe, E., Dalton, J., & Guajardo, M. (2023). *Smart home charging of electric vehicles using a digital platform.* Bergen: Elsevier.