NHH

BAN402 – Project 2

Fall 2023

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Part A

Task 1.

In this problem we want to minimize costs of designating hubs, while at the same time making sure that every country is i is served by at *least one* hub. To solve this problem, we recommend using a modified set covering problem from lecture 8. Our adopted model for solving the problem is presented below.

Sets

$i, j \in I$: *set of countries*

Parameters

t_{ij}: *travel time from country i to country j*
c_i: *cost of establishing a hub in country i*
$\bar{t}$: *maximum allowable travel time for flight from country i to country j*

The values for the parameters would be assigned in this manner:

c_i: *directly from a provided list of establisment costs for each country i*
t_{ij}: *travel times from the* $18 * 18$-matrix provided with flight times between every pair of countries i
$\bar{t}$: *assigned as a single provided value, representing the maximum allowable flight time*

Decision variables
x_i: 1 *if making a hub in country i*, 0 *otherwise*
y_{ij}: 1 *if country j is served by hub i*, 0 *zero otherwise*

Objective function

$$\min z = \sum_{i \in I} c_i x_i$$

Constraints

Every country must j be served by at least one hub i:

$$\sum_{i \in I} x_i y_{ij} \geq 1 \ \forall j \in I$$

Flight time from country i to country with hub j cannot exceed $\bar{t}$.

$$t_{ij} y_{ij} \leq \bar{t} \quad \forall\, i, j \in I$$

Logical constraint, country j can only by served by a hub i if it is already established:

$$y_{ij} \leq x_i \ \ \forall i, j \in I$$

Infeasibility

This problem could be infeasible if there are countries i that cannot be reached from any potential hub within the time $\bar{t}$.

Task 2

In the revised problem we would advise using a modified version of the facility location problem from lecture 7.

Sets

$i, j \in I: set\ of\ countries$

Parameters

$r_{ij} = induced\ cost\ if\ local\ manager\ of\ country\ j\ reports\ to\ hub\ \ manager\ in\ country\ i$

$c_i = cost\ of\ establishing\ a\ hub\ in\ country\ i$

$m_i = maximum\ number\ of\ local\ managers\ that\ can\ report\ to\ the\ hub\ in\ country\ i$

Decision variables

$x_i: 1\ if\ making\ a\ hub\ in\ country\ i, 0\ otherwise$

$y_{ij}: 1\ if\ the\ local\ manager\ of\ country\ j\ reports\ to\ the\ hub\ manager\ in\ country\ i, 0\ otherwise$

Objective function

$$\min z = \sum_{i \in I} c_i x_i + \sum_{i \in I} \sum_{j \in I} r_{ij} y_{ij}$$

Constraints

Every local manager must report to exactly one hub manager:

$$\sum_{i \in I} y_{ij} = 1\ \forall j \in I$$

Local managers can only report to an already existing hub.

$$y_{ij} \leq x_i$$

The maximum number of local managers per established hubs must be less than m_i for all i:

$$\sum_{j \in I} y_{ij} \leq m_i x_i \forall i \in I$$

Infeasibility

The problem can be infeasible if there is a local manager in country j that cannot report to a local manager at hub i within the managerial limit m_i.

Part B

In this model we will adapt and modify the model from the article “Developing Optimal Student Plans of Study” by R. Alan Bowman (2021), to answer new scenarios.

Task 1

To solve this new scenario, we will add a new subset $C_{Hard} \subset C$.

We will then introduce a new constraint in the model to make sure that Nils preferences are accounted for; i.e. that at most one hard subject can be assigned per term t, for all terms:

$$\sum_{i:c_i \in C_{Hard}, t_i = t} x_i \leq 1, \ \ \forall t \in T$$

Task 2

In this scenario, Lisa wants to make sure that she does not take two or more hard courses in the same term for more than two terms.

<u>Decision variables:</u>

To account for this, we have added a new binary decision variable:

$b_t = 1 \; if \; Lisa \; takes \; 2 \; or \; more \; hard \; courses \; in \; term \; t, \; 0 \; otherwise, \; for \; each \; term \; t \in T$

<u>New constraints:</u>

This binary constraint forces b_t to be 1 if 2 courses are taken, otherwise it remains 0. Here, M is a parameter with a large number.

$$\sum_{i:c_i \in C_{Hard}, t_i = t} x_i - 2 \geq -M * (1 - b_t), \forall t \in \ T$$

We also need to limit the number of terms Lisa can take two or more hard courses:

$$\sum_{t \in T} b_t \leq 2$$

Task 3

Abel would like to take two hard courses for each term t. To account for this, we introduce two new decision variables:

$d_t = binary\ variable,\ 1\ if\ Abel\ takes\ exactly\ two\ \mathrm{hard}\ courses\ in\ term\ t\ and\ 0\ otherwise.$

Number of hard courses taken for each term:

$$\mathrm{h}_t = \sum_{i:c_i \in C_{Hard}, t_i = t} x_i$$

We then introduce two linking constraints for these new variables. We used the “big M”-method to ensure that if d_t is 1, then h_t must be 2:

$$\mathrm{h}_t \geq 2d_t$$
$$\mathrm{h}_t \leq 2 + (1 - d_t)M$$

We then revised the objective function, adding the 10 awardpoints:

$$Max = \sum_i [(RP(c_i) + PR(m_i) - 0.00001ss_i)x_i - 0.01y_i - 0.01z_i] + 0.001 \sum_{i:c_i \in L} t_i x_i + 10 \sum_{t \in T} d_t$$

Part C

For this task we will formulate a linear mixed integer programming model for the oil company Petro&BAN. The model will be plan and decide on transportation, storage, and refining of different products to meet demand and maximize profit over a defined time. The model is enclosed in the .zip-file. We have three .run-files: one for displaying the answers in task 1, one displaying the answers in task 2, and one “TaskCRapport.run” displaying the flow characteristics of our model.

Task 1

Sets

$j \in J: set\ of\ crude\ oils = \{\mathrm{CrA}, \mathrm{CrB}\}$

$b \in B: set\ of\ components = \{\mathrm{lowqfuel}, \mathrm{distilA}, \mathrm{distilB}, \mathrm{naphtha1}, \mathrm{naphtha2}\}$

$p \in P: set\ of\ products = \{premium, regular, distilF, super\}$

$sp \in SP: set\ of\ salable\ products = \{premium, regular, distilF, super, lowqfuel\}$

$i \in I: set\ of\ CDU = \{1,\ 2\}$

$d \in D: set\ of\ depots = \{D1, D2\}$

$k \in K: set\ of\ markets = \{K1, K2 \ldots K12\}$

$t \in T: set\ of\ days\ = \{0,\ 1,\ 2 \dots 12\}$

$lowqfuel \subset B: product\ that\ can\ be\ directly\ sent\ to\ depots\ from\ the\ blending\ department$

Parameters

$Cmain_i = cost\ of\ maintenance\ on\ CDU\ i$

$E_j = amount\ of\ crude\ oil\ j\ arriving\ each\ day$

$A_{j,b,i} =\ amount\ of\ component\ b\ obtained\ from\ rening\ one\ unit\ of\ crude\ oil\ j\ in\ CDU\ i$

$R_{b,p} = amount\ of\ component\ b\ needed\ for\ product\ p$

$S_{sp} = sales\ price\ of\ salable\ product\ sp$

$Cref_{j,\,i}\ = cost\ of\ refining\ one\ unit\ of\ crude\ oil\ j\ at\ CDU\ i$

$minR = \min refined\ oil\ that\ day$

$maxR = \max refined\ oil\ that\ day$

$Cprod_p = cost\ of\ producing\ one\ unit\ of\ product\ p$

$Ctra1 = cost\ of\ transporting\ one\ unit\ from\ the\ refining\ to\ the\ blending\ department$

$Ctra2_d = cost\ of\ transporting\ one\ unit\ from\ refining\ department\ to\ depot\ d$

$Ctra3_d = cost\ of\ transporting\ one\ unit\ of\ blended\ \ product\ p\ \ to\ depot\ d$

$Ctra4_{d,k}\ = cost\ of\ shipping\ one\ unit\ of\ product\ from\ depot\ d\ to\ market\ k$

$Cinvi = cost\ of\ storing\ one\ unit\ of\ crude\ oil\ at\ the\ refining\ department$

$Cinvb = cost\ of\ storing\ one\ unit\ of\ component\ at\ the\ blending\ department\ per\ day$

$Cinvp_d = cost\ of\ storing\ one\ unit\ of\ product\ at\ depot\ d$

$\delta_{p,k,t} = maximum\ demand\ for\ product\ p\ from\ market\ k\ in\ day\ t$

$Izero_{sp,d} = inventory\ of\ \text{saleable}\ product\ sp\ in\ depot\ d$

$Ifinal_{sp,d} = inventory\ of\ saleable\ product\ sp\ in\ depot\ d$

Decision variables

$x_{i,t} = 1\ if\ CDU\ i\ is\ running\ on\ day\ t, 0\ otherwise$

$y_{i,t} = 1\ if\ CDU\ has\ maintenance\ on\ day\ t, 0\ otherwise$

$IR_{j,t} = inventory\ of\ crude\ oil\ j\ at\ refining\ department\ at\ the\ end\ of\ day\ t$

$IB_{b,t} = units\ in\ inventory\ of\ component\ b\ on\ end\ of\ day\ t\ in\ blending\ department$

$IP_{sp,d,t} = units\ in\ inventory\ of$ seleable saleable $product\ sp\ \ in\ depot\ d\ at$ end of $day\ t$

$z_{j,i,t} = units\ of\ crude\ oil\ j\ destilled\ at\ CDU\ i\ on\ day\ t$

$c_{b,\ t} = units\ of\ component\ b\ produced\ on\ day\ t$

$u_{b,t} = units\ of\ component\ b\ sent\ to\ blending\ department\ at\ day\ t$

$BD_{Lowqfuel,i,d,t} = amount\ of\ component\ lowqfuel\ sent\ from\ CDU\ i\ to\ depot\ d\ at\ day\ t$

$w_{p,t} = amount\ of\ product\ p\ produced\ at\ blending\ department\ on\ day\ t$

$PD_{p,d,t} = amount\ of\ product\ p\ sent\ to\ depot\ d\ from\ the\ blending\ department\ at\ day\ t$

$v_{sp,d,k,t} = amount\ of\ saleable\ product\ p\ sent\ from\ depot\ d\ to\ market\ k\ on\ day\ t$

Objective function

$$\begin{aligned}\max Profit = \ & \sum_{sp\in SP}\sum_{k\in K}\sum_{d\in D}\sum_{t\in T:0<t<12} S_{sp} * v_{sp,k,d,t} - \sum_{j\in J}\sum_{i\in I}\sum_{t\in T} Cref_{j,i} * z_{j,i,t} \\ & - \sum_{i\in I}\sum_{t\in T} Cmain_i * y_{i,t} - \sum_{j\in J}\sum_{t\in T} Cinvi * IR_{j,t} \\ & - \sum_{i\in I}\sum_{d\in D}\sum_{t\in T} Ctra2_d * BD_{lowqfuel\in B,i,d,t} \\ & - \sum_{b\in B}\sum_{t\in T} Ctra1 * u_{b,t} - \sum_{p\in P}\sum_{t\in T} Cprod_p * w_{p,t} - \sum_{b\in B}\sum_{t\in T} Cinvb * IB_{b,t} \\ & - \sum_{p\in P}\sum_{d\in D}\sum_{t\in T} Ctra3_d * PD_{p,d,t} - \sum_{sp\in SP}\sum_{d\in D}\sum_{t\in T} Cinvp_d * IP_{sp,d,t} \\ & - \sum_{sp\in SP}\sum_{k\in K}\sum_{d\in D}\sum_{t\in T} Ctra4_{d,k} * u_{sp,k,d,t}\end{aligned}$$

Constraints

Starting and final values for inventories and units transported between departments:

$IR_{j,t=0} = 0\ \ \forall j \in J$

$IB_{b,t=0} = 0\ \ \forall b \in B$

$IP_{sp,d,t=0} = Izero_{sp,d}\ \ \forall sp \in SP, d \in D$

$$\sum_{i\in I} z_{j,i,t=0} = 0 \forall j \in J$$

$w_{p,t=0} = 0\ \ \forall p \in P$

Final values in inventory at day 12:

$$IP_{sp,d,t=12} \geq Ifinal_{sp,d} \forall sp \in SP, d \in D$$

Current inventory of stored crude oil must be equal to yesterday's inventory plus todays incoming supply:

$$IR_{j,t} = IR_{j,t-1} + E_j - \sum_{i \in I} z_{j,i,t} \; \forall j \in J, t \in T: t > 0$$

CDU's cannot be both running and in maintenance:

$$y_{i,t} + x_{i,t} \leq 1 \;\; \forall i \in I, t \in T: t > 0$$

At least one CDU must be running for all days $t > 0$:

$$\sum_{i \in I} x_{i,t} \geq 1 \;\; \forall t \in T : t > 0$$

Each CDU must be maintained at least once in the last 6 days of the planning period:

$$\sum_{t=7}^{12} y_{i,t} \geq 1 \;\; \forall i \in I$$

A CDU cannot produce more than its max limit or less than its minimum limit on a given day:

$$\sum_{j \in J} z_{j,i,t} \leq maxR_i * x_{i,t} \quad \forall i \in I, t \in T : t > 0$$

$$\sum_{j \in J} z_{j,i,t} \geq minR_i * x_{i,t} \;\; \forall i \in I, t \in T : t > 0$$

Components created cannot exceed the total sum of components created from the destilled crude on a given day:

$$c_{b,t} = \sum_{j \in J, i \in I} z_{j,i,t} * A_{j,b,i} \; \forall b \in B, t \in T$$

The amount of components sent to the blending department cannot exceed the total amount produced on that day:

$$u_{b,t} = c_{b,t} \forall b \in B \; with \, b \neq' lowqfuel', t \in T$$

The amount of components sent to all depots cannot exceed the amount produced on that day. This is only eligible for the "lowqfuel"-component, which is immediately resealable to the marked:

$$\sum_{\boldsymbol{d} \in \boldsymbol{D}} \boldsymbol{BD}_{\boldsymbol{'lowqfuel',i,d,t}} = \sum_{j \in J} z_{j,i,t} * A_{j,lowqfuel,i} \;\; \forall i \in I, t \in T$$

The amount of components in inventory must be equal to yesterdays inventory and yesterdays production, except for "lowqfuel", which is sent directly to depots:

$$IB_{b,t} = IB_{b,t-1} + u_{b,t-1} - \sum_{p\in P} w_{p,t} * R_{b,p} \;\; \forall b \in B \; with \, b \neq' lowqfuel', t \in T : t > 0$$

There must be enough components in inventory on a given day to produce the products produced on that day:

$$\sum_{p\in P} w_{p,t} * R_{b,p} \leq IB_{b,t-1} + u_{b,t-1} \;\; \forall \, b \in B, t \in T : t > 0$$

Products sent to depot must equal the total amount of product produced for that day:

$$\sum_{d\in D} PD_{p,d,t} = w_{p,t} \forall p \in P, t \in T$$

The amount of saleable products in inventory at the end of day t must equal the sum of yesterdays inventory, saleable products produced yesterday, minus the sum of products sent to markets today:

$$IP_{p,d,t} = IP_{p,d,t-1} + PD_{p,d,t-1} - \sum_{k\in K} v_{p,d,k,t} \, \forall d \in D, p \in P, t \in T : t > 0$$

The same applies to lowqfuel:

$$IP_{'lowqfuel',d,t} = IP_{'lowqfuel',d,t-1} + \sum_{iinI} BD_{'lowqfuel',i,d,t-1} - \sum_{kinK} v_{'lowqfuel',d,k,t} \, \forall d \in D,$$

$$t \in T : t > 0$$

The sum of saleable products sent to marked k cannot exceed the storage capacity in depot d:

$$\sum_{k\in K} v_{p,d,k,t} \leq IP_{p,d,t-1} + PD_{p,d,t-1} \;\; \forall d \in D, \; p \in P, \; t \in T : t > 0$$

The same applies to lowqfuel:

$$\sum_{k\in K} v_{'lowqfuel',d,k,t} \leq IP_{'lowqfuel',d,t-1} + PD_{'lowqfuel',d,t-1} \;\; \forall \, d \in D, \; t \in T : t > 0$$

Products sent to market on day t-1 must be less or equal to the demand for day t:

$$\sum_{d\in D} v_{sp,d,k,t-1} \leq \delta_{sp,k,t} \;\; \forall k \in K, sp \in SP, t \in T \; with \, t > 0$$

Non negativity-condition for all non-binary variables:

$$IR_{j,t}, IB_{b,t}, IP_{sp,d,t}, z_{j,i,t}, c_{b,t}, u_{b,t}, BD_{Lowqfuel,i,d,t}, w_{p,t}, PD_{p,d,t}, v_{sp,d,k,t} \geq 0$$

$$\forall \, b \in B, j \in J, i \in I, d \in D, k \in K, p \in P, sp \in S, t \in T$$

a)

The optimal profit is **$ 3 820 080.**

As we can see from figure 1, CDU 1 is closed for maintenance on day 7, and CDU 2 is closed for maintenance on day 12.

Days	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
CDU 1	0	0	0	0	0	0	**1**	0	0	0	0	0
CDU 2	0	0	0	0	0	0	0	0	0	0	0	**1**

Figure 1 - maintenance schedule for CDU's

Days	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
CDU 1	1	1	1	1	1	1	0	1	1	1	1	1
CDU 2	1	1	1	1	1	1	1	1	1	1	0	0

Figure 2 - CDU running schedule

From figure 2 we can see the CDU running schedule: CDU 1 is not operating on day 7, and CDU 2 is not operating on day 11 and 12. Note that the CDU's are not allowed to operate on day 0.

b)

To check if there is any unsatisfied demand, we ask AMPL to print out the difference: $\delta_{sp,k,t} - \sum_{d \in D} v_{sp,\, d,\, k,\, t-1}$ for each marked and product. The results are presented in figure 3. The reason for the unsatisfied demand is that the profit of satisfying lowqfuel-demand is much lower than for all other products. Our model is therefore prioritizing to satisfy the demand of the other products, thus maximizing profit.

market	destilF	lowqfuel	premium	regular	super
K1	0	25	0	0	0
K2	0	0	0	0	0
K3	0	60	0	0	0
K4	0	124,8	0	0	0
K5	0	75	0	0	0
K6	0	276,2	0	0	0
K7	0	0	0	0	0
K8	0	15	0	0	0

Figure 3 - unsatisfied demand

c)

By looking at the variable $IR_{j,t}$ at t = 12 we see that we have 1150 units of CrA and 1435 of CrB. This is primarily because of the time constraint of our model; the model still receives 575 units of both crude oils on day 10-12, but the model will not use this supply for production since the products won't reach the market within our time horizon. Furthermore, since there is no defined demand for crude oils for $t \leq 12$ in our model, it will simply just induce costs to send the crude oils to the CDU's on day 11, without being able to recuperate the costs by selling it in the market. This leads to a "pile up" of crude oils in the refining department at the end of our planning period. If the company Oil&Ban wants to avoid this model behavior, they need to expand the planning horizon to compensate for the time offset, so that there is defined a demand δ for market, and allowing the production and transporting of products after receiving crude oil on the last day of the planning horizon.

"Lowqfuel" is the only product able to reach the market on day 12 after being produced on day 10. Still, the demand this day is satisfied with the planned production, so there will be a buildup of crude form day 10 and onwards.

d)

There are two components that have inventories higher than 100 at the end of day 12. distilA has an inventory of ≈ 740.6 and naptha2 has an inventory of 280.9. This is because destilB and naptha1 are used more intensely by more products, and are in shorter supply, while there is always a surplus of especially destilA after day 1. Both 'distilA' and 'naptha2' require pairing with another components to create products. As the planning period progresses, their inventories accumulate due to the more resource demanding nature of the other components.

Task 2

In this scenario we will make modify the model to take into account the revised cost structure, modelling fixed costs dependant the mode of operations on the CDU's.

New parameters:

$FixedCost_i$: $the\ fixed\ cost\ of\ running\ CDU\ i$

RC: $reduced\ cost\ for\ running\ both\ CDU's$

New decision variables:

$r_t = 1\ if\ both\ CDU's\ are\ running, 0\ otherwise$

Modified objective function:

$$\begin{aligned}
\max Profit = & \sum_{sp \in SP} \sum_{k \in K} \sum_{d \in D} \sum_{t \in T: 0<t<12} S_{sp} * v_{sp,k,d,t} - \sum_{j \in J} \sum_{i \in I} \sum_{t \in T} Cref_{j,i} * z_{j,i,t} \\
& - \sum_{i \in I} \sum_{t \in T} Cmain_i * y_{i,t} - \sum_{j \in J} \sum_{t \in T} Cinvi * IR_{j,t} \\
& - \sum_{i \in I} \sum_{d \in D} \sum_{t \in T} Ctra2_d * BD_{lowqfuel \in B,i,d,t} \\
& - \sum_{b \in B} \sum_{t \in T} Ctra1 * u_{b,t} - \sum_{p \in P} \sum_{t \in T} Cprod_p * w_{p,t} - \sum_{b \in B} \sum_{t \in T} Cinvb * IB_{b,t} \\
& - \sum_{p \in P} \sum_{d \in D} \sum_{t \in T} Ctra3_d * PD_{p,d,t} - \sum_{sp \in SP} \sum_{d \in D} \sum_{t \in T} Cinvp_d * IP_{sp,d,t} \\
& - \sum_{sp \in SP} \sum_{k \in K} \sum_{d \in D} \sum_{t \in T} Ctra4_{d,k} * u_{sp,k,d,t} - \sum_{i \in I} \sum_{t \in T} (FixedCost_i * x_{i,t} - RC * r_t)
\end{aligned}$$

New constraints:

Initial value of r_t is set to 0

$$r_{t=0} = 0$$

Days	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
CDU 1	1	1	1	1	1	1	**0**	1	1	1	1	1
CDU 2	1	1	1	1	1	**0**	1	1	1	1	**0**	**0**

Figure 3 - modified CDU operation plan

Linking constraint for r_t:

$$r_t = \sum_{i \in I} x_{i,t} - 1$$

Days	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
CDU 1	0	0	0	0	0	0	**1**	0	0	0	0	0
CDU 2	0	0	0	0	0	0	0	0	0	0	0	**1**

Figure 4 - modified CDU maintenance schedule

Results

The optimal profit is now **$ 2 774 380.** The CDU operation plan is presented in figure 4. CDU 2 is now closed on day 6 in order to save running costs. This causes some unmet demand in distilF as well as higher unmet demand in lowqfuel. This indicates that the model saves more by reducing the fixed cost of running CDU2 on day 6 than it earns by meeting the new unsatisfied demand.

Part D

Task 1

Sets

$n \in N$: $Set\ of\ cities$

$D \subset N$: $Set\ of\ cities\ that\ can\ be\ depots\ within\ N$

$t \in T$: $Set\ of\ trucks$

Parameters

$C_{i,j}$: $Traveltime\ between\ two\ cites\ i\ and\ j$

l_d: $Cost\ of\ loading\ in\ depot\ d$

Variables

$x_{i,j,t}$: $1\ if\ truck\ t\ drives\ from\ city\ i\ to\ city\ j, 0\ otherwise$

y_d: $1\ if\ city\ d\ is\ a\ distrobutioncenter, 0\ otherwise$

$z_{d,t}$: $1\ if\ truck\ t\ starts\ from\ depot\ d, 0\ otherwise$

T_{max}: $Maximum\ traveltime\ for\ a\ truck$

Objective function

$Minimize\ Max\ Truck\ Distance: T\_max$

Constraints

Every truck route must be below the maximum truck distance:

$$\sum_{i \in N} \sum_{j \in N, j \neq i} c_{i,j} * x_{i,j,t} + \sum_{d \in D} l_d * z_{d,t} \leq T_{Max}, \qquad \forall\ t \in T$$

Every city must be entered from somewhere:

$$\sum_{i \in N, i \neq j} \sum_{t \in T} x_{i,j,t} = 1, \qquad \forall\ j \in N$$

If a truck enters a city, it must also leave the city:

$$\sum_{i \in N, i \neq j} x_{i,j,t} = \sum_{k \in N, k \neq j} x_{j,k,t}, \qquad \forall\ j \in N, t \in T$$

A truck can't immediately return to the city it came from:

$$x_{i,j,t} + x_{j,i,t} \leq 1, \qquad \forall\ i \in N, j \in N, t \in T, i \neq j$$

Every truck must start form their assigned depot:

$$z_{d,t} \leq \sum_{j \in J, j \neq d} x_{d,j,t}, \qquad \forall\ t \in T, d \in D$$

Every truck must visit exactly 4 cities including its depot:

$$\sum_{i \in N} \sum_{j \in N, j \neq i} x_{i,j,t} = 4, \qquad \forall\, t \in T$$

There must be exactly 4 depots:

$$\sum_{d \in D} y_d = 4$$

There can only be one truck per depot:

$$\sum_{t \in T} z_{d,t} \leq 1, \forall\, d \in D$$

Every truck must belong to a depot:

$$\sum_{d \in D} z_{d,t} = 1, \forall\, t \in T$$

Linking the z and y variables:

$z_{d,t} \leq y_d, \forall\, d \in D, t \in T$

Truck	Length
T1	46,34
T2	39,74
T3	50,88
T4	47,26
Total	184,22

Figure 7 - length of trips for each truck t

The model above will minimize the longest time needed by a truck to run its route. Below are the route times for the different trucks.

Truck	Depot
T1	S5
T2	C2
T3	N1
T4	S4

Figure 5 - designated depots d for trucks t

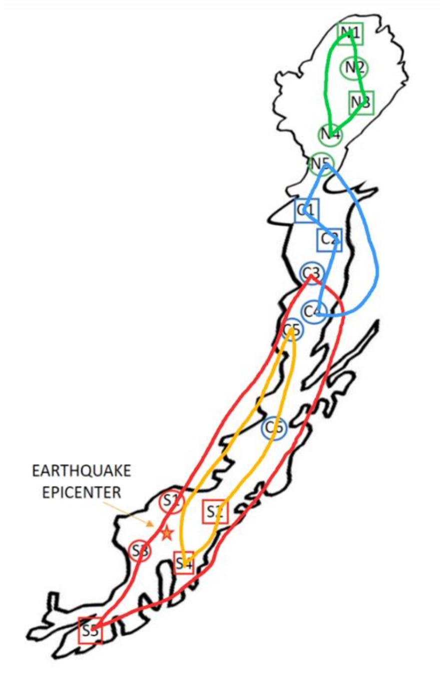

Figure 6 - route map

We do however think that it is possible to optimize this model further. In its current shape it only minimizes the longest route, and then chooses the other 3 at random. To further minimize the length of the other 3 routes we can hardcode in the maximum route length before minimizing the total length of all trucks. We remove our T_{Max} variable and replace it in our constraint with the previous minimized max length of 50,88. We then change our objective function to minimize total distance for all trucks as shown below.

Minimize total truck distance:

$$\sum_{i \in N} \sum_{j \in N, j \neq i} \sum_{t \in T} c_{i,j} * x_{i,j,t}$$

Truck	Length
T1	26,32
T2	46,38
T3	35,82
T4	50,88
Total	159,4

Figure 9 - optimized routes

Truck	Depot
T1	S5
T2	S2
T3	C2
T4	N1

Figure 10 - optimized depot allocation

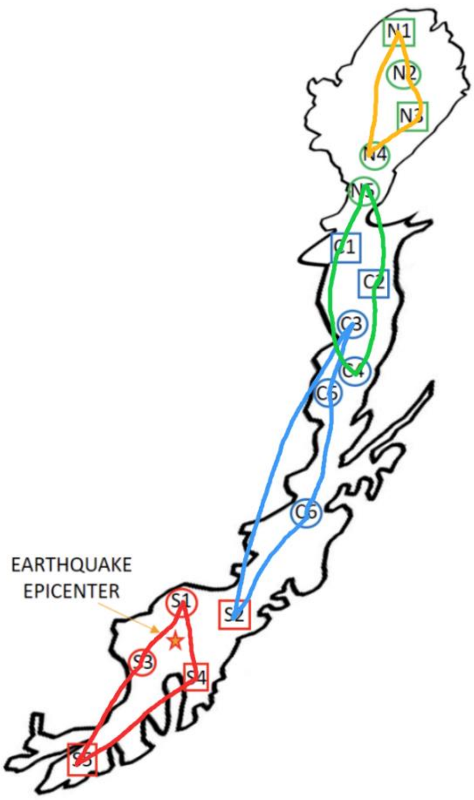

Figure 8 - optimized routes

As presented, we further optimize our model, reaching an objective value of 24,82 by doing another step of optimization.

Task 2

In this revised model we want to calculate an overall emergency score, based on the trucks order of visits to cities who have been affected differently.

Revised sets

$H \subset N$: $Set\ of\ high\ impact\ H \subset cities\ within\ N$

$M \subset N$: $Set\ of\ medium\ impact\ cities\ within\ N$

Revised parameters

$H_{1..3}$: $Impact\ score\ for\ high\ impact\ cities\ visited\ in\ position\ 1-3$

$H_{1..3}$: $Impact\ score\ for\ medium\ impact\ cities\ visited\ in\ position\ 1-3$

New variables

$s_{t,i,j}$: $1\ if\ truck\ t\ drives\ from\ city\ i\ to\ city\ j\ second$

$r_{t,i,j}$: $1\ if\ truck\ t\ drives\ from\ city\ i\ to\ city\ j\ third$

Revised objective function

Maximize Emergency Score:

$$\sum_{i \in H, i \in D} \sum_{t \in T} z_{i,t} * HI_1 + \sum_{i \in M, i \in D} \sum_{t \in T} z_{i,t} * MI_1 +$$

$$\sum_{i \in N} \sum_{j \in H, j \neq i} \sum_{t \in T} s_{i,j,t} * HI_2 + \sum_{i \in N} \sum_{j \in M, j \neq i} \sum_{t \in T} s_{i,j,t} * MI_2 +$$

$$\sum_{i \in N} \sum_{j \in H, j \neq i} \sum_{t \in T} r_{i,j,t} * HI_3 + \sum_{i \in N} \sum_{j \in M, j \neq i} \sum_{t \in T} r_{i,j,t} * MI_3$$

New constraints

Each truck must go from a depot d to a second city j.

$$z_{d,t} \leq \sum_{j \in J, j \neq d} s_{d,j,t}, \forall\ t \in T, d \in D$$

Each truck must go from a second city j to a third city k.

$$s_{i,j,t} \leq \sum_{k \in J, k \neq j} r_{j,k,t}, \forall\ t \in T, i \in N, j \in N, j \neq i$$

Second and third city cannot be the same city.

This constraint might be considered redundant because the constraint setting the maximum number of cities, together with the constraint telling the model that the third city must be after the second effectively implements this constraint. However, we find that the introduction of the below constraint helps the preprocessing of the model enough that the saved time justifies keeping it.

$$s_{i,j,t} + r_{i,j,t} \leq 1, \forall\ t \in T, i \in N, j \in N, j \neq i$$

Each truck can only have one second city.

$$\sum_{i \in N} \sum_{j \in N, j \neq i} s_{i,j,t} = 1, \forall\ t \in T$$

Each truck can only have one third city.

$$\sum_{i \in N} \sum_{j \in N, j \neq i} r_{i,j,t} = 1, \forall\ t \in T$$

Each second city s must have a x counterpart:

$$s_{i,j,t} \leq x_{i,j,t}, \forall t \in T, i \in N, j \in N, j \neq i$$

Each third city must have a x counterpart:

$$r_{i,j,t} \leq x_{i,j,t}, \forall t \in T, i \in N, j \in N, j \neq i$$

Figure 13 - truck routes in revised scenario

Truck	Depot
T1	S2
T2	C1
T3	C2
T4	S4

Figure 12 - depot allocation in revised scenario

Truck	Score
T1	300
T2	370
T3	370
T4	420
Total	1460

Figure 14 - emergency score for each truck

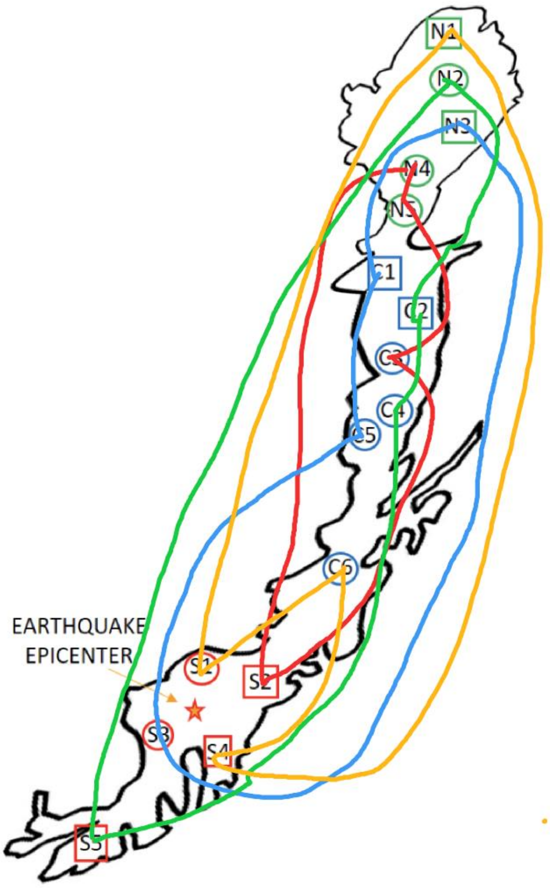

Figure 11 - truck routes in revised scenario

The overall emergergency score in this revised scenario is 1460. The route patterns are now prioritizing visits to high impact cities first, and then moving on to medium impact cities.

Task 3

To implement this condition in the model we first introduce a new set A, and a param CurrentA to our model:

$a \in A: Set\ of\ percentage\ values = \{100\%, 95\%, 90\%, 85\%, 80\%, 75\%\}$

$CurrentA: Current\ value\ of\ a \in A\ the\ model\ uses$

We then modify the objective function from D2 to a constraint in the model from D1. All the variables, parameters and constraints from D2 also follow over to the new model D3. The new constraint is shown below.

$$\sum_{i\in H, i\in D}\sum_{t\in T} z_{i,t} * HI_1 + \sum_{i\in M, i\in D}\sum_{t\in T} z_{i,t} * MI_1 +$$
$$\sum_{i\in N}\sum_{j\in H, j\neq i}\sum_{t\in T} s_{i,j,t} * HI_2 + \sum_{i\in N}\sum_{j\in M, j\neq i}\sum_{t\in T} s_{i,j,t} * MI_2 +$$
$$\sum_{i\in N}\sum_{j\in H, j\neq i}\sum_{t\in T} r_{i,j,t} * HI_3 + \sum_{i\in N}\sum_{j\in M, j\neq i}\sum_{t\in T} r_{i,j,t} * MI_3 \leq CurrentA * 1460$$

We then modify our run-file to script through each iteration of $a \in A$ by using a forloop:

```
for {a in A}{
        let CurrentA := a;
        solve;
```

Alpha	Emergency Score	Longest route
100 %	1460	75,18
95 %	1390	61,18
90 %	1340	55,14
85 %	1290	53,04
80 %	1190	50,88
75 %	1190	50,88

As we can see from the results above, the longest route decreases rapidly with the decreasing value of Alpha. In a real-world scenario, you might want to consider reducing the alpha to 90% as this seems to be a good tradeoff between emergency score and the longest route. This would however depend on the urgency of reaching the medium impact zones.

References

Bowman, R. A. (2021). Developing Optimal Student Plans of Study. *INFORMS Journal on Applied Analytics*, 409-421.