Probability Take Home Questions 2

1. **Quant Interview**: A quant driven hedge fund wants to interview all the UCLA MFE students for an internship. Say 50% of all students who received their first interview received a second interview. 95% of your friends that got a second interview said they had a good first interview. 75% of your friends that did not get a second interview said they had a good first interview. If you felt you had a good first interview, what is the probability that you will receive a second interview? Alternatively, if you felt you had a bad first interview, what is the probability that you will receive a second interview?

Solution - Defining events as {Good,Bad} and {Pass,Fail}.

$$P(\text{Pass}) = P(\text{Fail}) = 0.5$$
$$P(\text{Good}|\text{Pass}) = 0.95$$
$$P(\text{Good}|\text{Fail}) = 0.75$$
$$P(\text{Good}) = P(\text{Good}|\text{Pass}) \cdot P(\text{Pass}) + P(\text{Good}|\text{Fail}) \cdot P(\text{Fail}) = (0.95)(0.5) + (0.75)(0.5) = 0.85$$
$$P(\text{Pass}|\text{Good}) = P(\text{Good}|\text{Pass}) \cdot \frac{P(\text{Pass})}{P(\text{Good})} = 0.95 \cdot \frac{0.5}{0.85} = 0.5588$$

Now considering the alternate situation -

$$P(\text{Bad}|\text{Pass}) = 0.05$$
$$P(\text{Bad}|\text{Fail}) = 0.25$$
$$P(\text{Bad}) = P(\text{Bad}|\text{Pass}) \cdot P(\text{Pass}) + P(\text{Bad}|\text{Fail}) \cdot P(\text{Fail}) = 0.5 * (0.05 + 0.25) = 0.15$$
$$P(\text{Pass}|\text{Bad}) = P(\text{Bad}|\text{Pass}) \cdot \frac{P(\text{Pass})}{P(\text{Bad})} = 0.05 \cdot \frac{0.5}{0.15} = 0.167$$

2. There are two biased coins A and B in a bag. Probability of heads for coin A is 0.75 and the probability of heads for coin B is 0.3. You pick a coin randomly and perform 10 tosses (without knowing which coin you picked). *Hint*: To solve the two problems below, compute the posterior probability $P(\text{Hypothesis}|\text{Data})$ and argue that one of the coin has a higher posterior probability. You will have to test and compare the two hypothesis - picking coin A given data and picking coin B given data. Since we are choosing a coin randomly, $P(\text{picking coin A}) = P(\text{picking coin B}) = 1/2$. Key takeaway is how the data changes your beliefs about which coin you picked.

(a) You observe that you get 8 heads and 2 tails from your coin tosses. What is the probability that you picked coin A from the bag given the data. Compare this with the posterior probability of picking the other coin.

(b) Now, say you observed 8 tails and 2 heads from your coin tosses. What is the probability that you picked coin B given the data. Compare this with the posterior probability of picking the other coin.

Solution Part A 8 Heads and 2 Tails:

$$P(\text{coin A}) = P(\text{coin B}) = 1/2$$
$$P(\text{Data}|\text{coin A}) = {}^{10}C_2(0.75)^8(0.25)^2 = {}^{10}C_2 \times 0.00626$$
$$P(\text{Data}|\text{coin B}) = {}^{10}C_2(0.3)^8(0.7)^2 = {}^{10}C_2 \times 0.00003$$

$$P(\text{Data}) = P(\text{Data}|\text{coin A})P(\text{coin A}) + P(\text{Data}|\text{coin B})P(\text{coin B}) = {}^{10}C_2\frac{1}{2}\cdot 0.00626 + {}^{10}C_2\frac{1}{2}\cdot 0.00003 = {}^{10}C_2 \times 0.00314$$

$$P(\text{coin A}|\text{Data}) = P(\text{Data}|\text{Coin A}) \cdot \frac{P(\text{coin A})}{P(\text{Data})} = 0.00626 \cdot \frac{0.5}{0.00314} = 0.996$$

$$P(\text{coin B}|\text{Data}) = P(\text{Data}|\text{Coin B}) \cdot \frac{P(\text{coin B})}{P(\text{Data})} = 0.00003 \cdot \frac{0.5}{0.00314} = 0.004$$

Solution Part B 8 Heads and 2 Tails:

$$P(\text{coin A}) = P(\text{coin B}) = 1/2$$

$$P(\text{Data}|\text{coin A}) = (0.75)^2(0.25)^8 = {}^{10}C_2 \times 0.00001$$

$$P(\text{Data}|\text{coin B}) = (0.3)^2(0.7)^8 = {}^{10}C_2 \times 0.00519$$

$$P(\text{Data}) = P(\text{Data}|\text{coin A})P(\text{coin A}) + P(\text{Data}|\text{coin B})P(\text{coin B}) = {}^{10}C_2\frac{1}{2}\cdot 0.00001 + {}^{10}C_2\frac{1}{2}\cdot 0.00519 = {}^{10}C_2 \times 0.0026$$

$$P(\text{coin A}|\text{Data}) = P(\text{Data}|\text{Coin A}) \cdot \frac{P(\text{coin A})}{P(\text{Data})} = 0.00001 \cdot \frac{0.5}{0.0026} = 0.0016$$

$$P(\text{coin B}|\text{Data}) = P(\text{Data}|\text{Coin B}) \cdot \frac{P(\text{coin B})}{P(\text{Data})} = 0.00519 \cdot \frac{0.5}{0.0026} = 0.9984$$