PROBABILITY THEORY

1 Today's agenda

- History
- Interpretation
- Probability theory
- Counting

2 History

1650s (Pascal, Fermal) → 1800s (Laplace) → 1900s (Kolmogorov)

3 Interpretations

- Classical

$$P(A) = \frac{N_A}{N} \quad \Big\} \begin{array}{l} \text{mutually excl.} \\ \text{equally likely} \end{array}$$

{rain, no rain}

- Frequentist:

large no: exp.

$$P(A) = \underbrace{\lim_{n \to \infty}} \frac{n_A}{n}$$

$$\underbrace{P}_{\text{Prob.}} = \underbrace{P^*}_{\text{truth}} + \underbrace{\varepsilon}_{\text{Noise}}$$

- Bayesian: subjective

③ Prob. Theory

- Experiment
- Sample space (Ω)

  2 tosses : $\Omega = \{HH, HT, TH, TT\}$
- Power set ($\mathcal{U}$)

  $\Omega = \{a, b, c\}$

  $\mathcal{U} = \{\Omega, \phi, a, b, c, \{a,b\}, \{b,c\}, \{c,a\}\}$

  n-elements, $\mathcal{U}$ has 2^n
- Event : subset of $\mathcal{U}$
- Naïve definition

  $$P(A) = \frac{N_A}{N}$$

  $\downarrow$ event

  $P(\cdot) : A \to [0,1]$, $A \in \mathcal{U}$

  "Measure" or "Distribution"

- Axioms

  $P(A) \geq 0$

  $P(\Omega) = 1$

  $$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

  $A_1, A_2, \ldots$ are disjoint

- Permutation

n-objects, k-draws, ~~replacement~~ without ? order matters

$\{a, b, c, d\}$ $\quad \underline{4} \ \underline{3}$ $\quad |$ abcd $\neq$ bcad

$n=4, k=2$

$$= \frac{n(n-1)\ldots(n-k+1)(n-k)\ldots 1 \ \} \ n!}{\underbrace{(n-k)(n-k-1)\cdots 1}\ \} \ (n-k)!}$$

$$= \frac{n!}{(n-k)!} = {}^{n}P_{k}$$

- Combinations

n-obj, k-draws, without replacement, Ordering does not matter

$\underline{a}\ \underline{b}\ \underline{c}\ \underline{d} = 4!$

$$= \frac{n(n-1)\cdots(n-k+1)}{k!} = \frac{n!}{k!\,(n-k)!} = {}^{n}C_{k}$$

- Sampling table

	Order matters	Order does not matter
With replacement	n^k	${}^{n+k-1}C_k$
without replacement	${}^{n}P_k$	${}^{n}C_k$

n-obj, k-draw/.