

Probability Take Home Questions 1

1. There are 10 people. What is the number of ways in which you can split them into a team of 6 and a team of 4? Additionally, what are the number of ways in which you can split them into two teams of 5 each?
2. Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
Hint: The goal is to prove this mathematically (not using Venn diagrams). Try writing $A \cup B$ as union of three disjoint sets and then apply axiom 3 that we discussed in class.
3. Two people take turns trying to sink a basketball into a net. Person 1 succeeds with probability $1/3$ and person 2 succeeds with probability $1/4$. What is the probability that person 2 succeeds before person 1. Additionally, compute the probability that person 1 succeeds before person 2.
4. Suppose we toss a fair coin until we get exactly two heads. Describe the sample space S . What is the probability that exactly k tosses are required?
5. **Birth Month Problem** - This is a variation of the birthday problem. How many people are needed in a room to make it possible that atleast two people have the same birth month with a probability of 50%. There are 12 months in a year and probability of being born in any month is the same. *Hint:* Try expressing the probability of atleast 2 people having the same birth month as a complementary event.
6. **(optional) Birth Month Paradox Advanced Version** - In the above question, we assumed that the probability of being born in any month is equal. Historically, month of August has seen the largest number of births compared to any other month. If we were to incorporate this new information, say probability of being born in August is p and probability of being born in any other month is q . What would be your approach to solving the above problem with this new information? *Hint :* The trick here would still be to write the probabilities in terms of its complementary event. Try enumerating possible combinations and compute the probability using the non-uniform distribution.