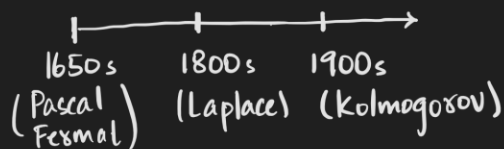


PROBABILITY THEORY

① Today's agenda

- History
- Interpretation
- Probability theory
- Counting

② History



③ Interpretations

- classical
$$P(A) = \frac{N_A}{N}$$

}

mutually excl.

equally likely

$\{\text{rain, no rain}\}$

- Frequentist:

large no. exp.

$$P(A) = \lim_{n \rightarrow \infty} \frac{n_A}{n}$$

$$\underbrace{P}_{\text{Prob.}} = \underbrace{P^*}_{\text{truth}} + \underbrace{\varepsilon}_{\text{Noise}}$$

- Bayesian: Subjective

③ Prob. Theory

- Experiment

- Sample space (Ω)

2 tosses : $\Omega = \{HH, HT, TH, TT\}$

- Power set (\mathcal{U})

$\Omega = \{a, b, c\}$

$\mathcal{U} = \{\Omega, \emptyset, \{a, b, c\}, \{a, b\}, \{b, c\}, \{c, a\}\}$

n -elements, \mathcal{U} has 2^n

- Event : subset of \mathcal{U}

- Naïve definition

$$P(A) = \frac{N_A}{N}$$

↓
event

$P(\cdot) : \mathcal{A} \rightarrow [0, 1], A \in \mathcal{U}$

"Measure" or "Distribution"

- Axioms

$$P(A) \geq 0$$

$$P(\Omega) = 1$$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

A_1, A_2, \dots are disjoint

- Permutation

n -objects, k -draws, ^{without} ^{order} replacement matters

$\{a, b, c, d\}$ 4 3 | $abcd \neq bcad$

$n=4, k=2$

$$= \frac{n(n-1) \dots (n-k+1) \underbrace{(n-k) \dots 1}_{(n-k)!}}{(n-k)!} \cdot n!$$

$$= \frac{n!}{(n-k)!} = {}^n P_k$$

- Combinations

n -obj, k -draws, without replacement, ordering does not matter

$$\underline{a} \ \underline{b} \ \underline{c} \ \underline{d} = 4!$$

$$= \frac{n(n-1) \dots (n-k+1)}{k!} = \frac{n!}{k!(n-k)!} = {}^n C_k$$

- Sampling table

	Order matters	Order does not matter
with replacement	n^k	$n+k-1 \ C_k$
without replacement	${}^n P_k$	${}^n C_k$

n -obj, k -draw/.