Assignment5_Group23

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Assignment - Module 5

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Q1. Quant Interview: A quant driven hedge fund wants to interview all the UCLA MFE students for an internship. Say 50% of all students who received their first interview received a second interview. 95% of the people interviewed that got a second interview said they had a good first interview. 75% of the people interviewed that did not get a second interview said they had a good first interview. If you felt you had a good first interview, what is the probability that you will receive a second interview? Alternatively, if you felt you had a bad first interview, what is the probability that you will receive a second interview?

Answer 1:

Let:

- The probability of the first interview being felt good be denoted by $P(First_{FG})$
- The probability of the first interview being felt bad be denoted by $P(First_{FB})$
- The probability of receiving a second interview be denoted by $P(Y_{second})$
- The probability of receiving a second interview be denoted by $P(N_{second})$

Therefore, $P(Y_{second}|First_{FG})$ **using Bayes' Theorem is:**

$$P(Y_{second}|First_{FG}) = P(First_{FG}|Y_{second}) * \frac{P(Y_{second})}{P(First_{FG})}$$

$$\implies P(Y_{second}|First_{FG}) = 0.95 * \frac{0.5}{0.5 * 0.95 + 0.5 * 0.75}$$

$$\implies P(Y_{second}|First_{FG}) = 0.5588$$

Similarly, $P(Y_{second}|First_{FB})$ **is:**

$$P(Y_{second}|First_{FB}) = P(First_{FB}|Y_{second}) * \frac{P(Y_{second})}{P(First_{FB})}$$

$$\implies P(Y_{second}|First_{FB}) = 0.05 * \frac{0.5}{0.5 * 0.05 + 0.5 * 0.25}$$

$$\implies P(Y_{second}|First_{FB}) = 0.1667$$

Q2. Hypothesis Testing: There are two biased coins A and B in a bag. Probability of heads for coin A is 0.75 and the probability of heads for coin B is 0.3. You pick a coin randomly and perform 10 tosses (without knowing which coin you picked). Hint: To solve the two problems below, compute the posterior probability P(Hypothesis|Data) and argue that one of the coin has a higher posterior probability. You will have to test and compare the two hypothesis - picking coin A given data and picking coin B given data. Since we are choosing a coin randomly, P(picking coin A) = (picking coin B) = 1/2. Key takeaway is how the data changes your beliefs about which coin you picked.

i. You observe that you get 8 heads and 2 tails from your coin tosses. What is the probability that you picked coin A from the bag given the data. Compare this with the posterior probability of picking the other coin.

ii. Now, say you observed 8 tails and 2 heads from your coin tosses. What is the probability that you picked coin B given the data. Compare this with the posterior probability of picking the other coin.

Answer 2:

Let:

- The probability for coin A be denoted by $P(A)$
- The probability for coin B be denoted by $P(B)$

i. 8 heads & 2 tails:

Let:

- The Probability of 8 heads and 2 tails be denoted by $P(8_H 2_T)$

Therefore, $P(A|8_H 2_T)$ using Bayes' Theorem is:

$$P(A|8_H 2_T) = P(8_H 2_T|A) * \frac{P(A)}{P(8_H 2_T)}$$

$$\implies P(A|8_H 2_T) = {}^{10}C_8(0.75)^8(0.25)^2 * \frac{0.5}{0.5 * [{}^{10}C_8(0.75)^8(0.25)^2] + 0.5 * [{}^{10}C_8(0.3)^8(0.7)^2]}$$

$$\implies P(A|8_H2_T) = 0.9949$$

Therefore, $P(B|8_H2_T)$ **is:**

$$P(B|8_H2_T) = P(8_H2_T|B) * \frac{P(B)}{P(8_H2_T)}$$

$$\implies P(B|8_H2_T) = {}^{10}C_8(0.3)^8(0.7)^2 * \frac{0.5}{0.5 * [{}^{10}C_8(0.75)^8(0.25)^2] + 0.5 * [{}^{10}C_8(0.3)^8(0.7)^2]}$$

$$\implies P(B|8_H2_T) = 0.0051$$

Thus, given the observed data, there is much higher probability that coin A was initially picked.

ii. 2 heads & 8 tails:

Let:

- The Probability of 8 heads and 2 tails be denoted by $P(8_H2_T)$

Therefore, $P(B|2_H8_T)$ **using Bayes' Theorem is:**

$$P(B|2_H8_T) = P(2_H8_T|B) * \frac{P(B)}{P(2_H8_T)}$$

$$\implies P(B|2_H8_T) = {}^{10}C_2(0.3)^2(0.7)^8 * \frac{0.5}{0.5 * [{}^{10}C_2(0.75)^2(0.25)^8] + 0.5 * [{}^{10}C_2(0.3)^2(0.7)^8]}$$

$$\implies P(B|2_H8_T) = 0.9983$$

Similarly, $P(A|2_H8_T)$ **is:**

$$P(A|2_H8_T) = P(2_H8_T|A) * \frac{P(A)}{P(2_H8_T)}$$

$$\implies P(A|2_H8_T) = {}^{10}C_2(0.75)^2(0.25)^8 * \frac{0.5}{0.5 * [{}^{10}C_2(0.75)^2(0.25)^8] + 0.5 * [{}^{10}C_2(0.3)^2(0.7)^8]}$$

$$\implies P(A|2_H8_T) = 0.0017$$

Thus, given the observed data, there is much higher probability that coin B was initially picked.

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[1]: ## Calculations are shown below:

from math import comb

# Probabilty of Coin A given 8 heads & 2 Tails

p1 = comb(10,8)*(0.75**8)*(0.25**2)
q1 = 0.5
r1 = 0.5*comb(10,8)*(0.75**8)*(0.25**2) + 0.5*comb(10,8)*(0.3**8)*(0.7**2)
ans1 = p1*q1/r1
print("Probability of Coin A given 8 Heads & 2 Tails = {:.4f}".format(ans1))
```

```
Probability of Coin A given 8 Heads & 2 Tails = 0.9949
```

```
[2]: # Probabilty of Coin B given 8 heads & 2 Tails

p2 = comb(10,8)*(0.3**8)*(0.7**2)
q2 = 0.5
r2 = 0.5*comb(10,8)*(0.75**8)*(0.25**2) + 0.5*comb(10,8)*(0.3**8)*(0.7**2)
ans2 = p2*q2/r2
print("Probability of Coin B given 8 Heads & 2 Tails = {:.4f}".format(ans2))
```

```
Probability of Coin B given 8 Heads & 2 Tails = 0.0051
```

```
[3]: # Probabilty of Coin B given 2 heads & 8 Tails

p3 = comb(10,2)*(0.3**2)*(0.7**8)
q3 = 0.5
r3 = 0.5*comb(10,2)*(0.75**2)*(0.25**8) + 0.5*comb(10,8)*(0.3**2)*(0.7**8)
ans3 = p3*q3/r3
print("Probability of Coin B given 2 Heads & 8 Tails = {:.4f}".format(ans3))
```

```
Probability of Coin B given 2 Heads & 8 Tails = 0.9983
```

```
[4]: # Probabilty of Coin A given 2 heads & 8 Tails

p4 = comb(10,2)*(0.75**2)*(0.25**8)
q4 = 0.5
r4 = 0.5*comb(10,2)*(0.75**2)*(0.25**8) + 0.5*comb(10,8)*(0.3**2)*(0.7**8)
ans4 = p4*q4/r4
print("Probability of Coin A given 2 Heads & 8 Tails = {:.4f}".format(ans4))
```

```
Probability of Coin A given 2 Heads & 8 Tails = 0.0017
```

The End.
