Assignment5_Group23

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Assignment - Module 5

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Q1.Quant Interview: A quant driven hedge fund wants to interview all the UCLA MFE students for an internship. Say 50% of all students who received their first interview received a second interview. 95% of the people interviewed that got a second interview said they had a good first interview. 75% of the people interviewed that did not get a second interview said they had a good first interview. If you felt you had a good first interview, what is the probability that you will receive a second interview? Alternatively, if you felt you had a bad first interview, what is the probability that you will receive a second interview?

Answer 1:   **Let:** - The probability of the first interview being felt good be denoted by $P(First_{FG})$ - The probability of the first interview being felt bad be denoted by $P(First_{FB})$ - The probability of receiving a second interview be denoted by $P(Y_{second})$ - The probability of receiving a second interview be denoted by $P(N_{second})$

Therefore, $P(Y_{second}|First_{FG})$ using Bayes' Theorem is:

$$P(Y_{second}|First_{FG}) = P(First_{FG}|Y_{second}) * \frac{P(Y_{second})}{P(First_{FG})}$$

$$\implies P(Y_{second}|First_{FG}) = 0.95 * \frac{0.5}{0.5 * 0.95 + 0.5 * 0.75}$$

$$\implies P(Y_{second}|First_{FG}) = 0.55588$$

Similarly, $P(Y_{second}|First_{FB})$ is:

$$P(Y_{second}|First_{FB}) = P(First_{FB}|Y_{second}) * \frac{P(Y_{second})}{P(First_{FB})}$$

$$\implies P(Y_{second}|First_{FB}) = 0.05 * \frac{0.5}{0.5 * 0.05 + 0.5 * 0.25}$$

$$\implies P(Y_{second}|First_{FB}) = 0.1667$$

Q2. Hypothesis Testing: There are two biased coins A and B in a bag. Probability of heads for coin A is 0.75 and the probability of heads for coin B is 0.3. You pick a coin randomly and perform 10 tosses (without knowing which coin you picked). Hint: To solve the two problems below, compute the posterior probability P(Hypothesis|Data) and argue that one of the coin has a higher posterior probability. You will have to test and compare the two hypothesis - picking coin A given data and picking coin B given data. Since we are choosing a coin randomly, P(picking coin A) = (picking coin B) = 1/2. Key takeaway is how the data changes your beliefs about which coin you picked.
i. You observe that you get 8 heads and 2 tails from your coin tosses. What is the probability that you picked coin A from the bag given the data. Compare this with the posterior probability of picking the other coin. **ii.** Now, say you observed 8 tails and 2 heads from your coin tosses. What is the probability that you picked coin B given the data. Compare this with the posterior probability of picking the other coin.

Answer 2: **Let:** - The probability for coin A be denoted by $P(A)$ - The probability for coin B be denoted by $P(B)$

i. 8 heads & 2 tails: **Let:** - The Probability of 8 heads and 2 tails be denoted by $P(8_H 2_T)$

Therefore, $P(A|8_H 2_T)$ using Bayes' Theorem is:

$$P(A|8_H 2_T) = P(8_H 2_T|A) * \frac{P(A)}{P(8_H 2_T)}$$

$$\implies P(A|8_H 2_T) = {}^{10}C_8 (0.75)^8 (0.25)^2 * \frac{0.5}{{}^{10}C_8 (0.75)^8 (0.25)^2 + {}^{10}C_8 (0.3)^8 (0.7)^2}$$

$$\implies P(A|8_H 2_T) = 0.4974$$

Therefore, $P(B|8_H 2_T)$ is:

$$P(B|8_H 2_T) = P(8_H 2_T|B) * \frac{P(B)}{P(8_H 2_T)}$$

$$\implies P(B|8_H 2_T) = {}^{10}C_8 (0.3)^8 (0.7)^2 * \frac{0.5}{{}^{10}C_8 (0.75)^8 (0.25)^2 + {}^{10}C_8 (0.3)^8 (0.7)^2}$$

$$\implies P(A|8_H 2_T) = 0.0026$$

Thus, given our observed data, there is much higher probability that coin A was initially picked.

ii. 2 heads & 8 tails: **Let:** - The Probability of 8 heads and 2 tails be denoted by $P(8_H2_T)$

Therefore, $P(B|2_H8_T)$ using Bayes' Theorem is:

$$P(B|2_H8_T) = P(2_H8_T|B) * \frac{P(B)}{P(2_H8_T)}$$

$$\implies P(B|2_H8_T) = {}^{10}C_2(0.3)^2(0.7)^8 * \frac{0.5}{{}^{10}C_2(0.75)^2(0.25)^8 +{}^{10} C_2(0.3)^2(0.7)^8}$$

$$\implies P(B|2_H8_T) = 0.4492$$

Similarly, $P(A|2_H8_T)$ is:

$$P(A|2_H8_T) = P(2_H8_T|A) * \frac{P(A)}{P(2_H8_T)}$$

$$\implies P(A|2_H8_T) = {}^{10}C_2(0.75)^2(0.25)^8 * \frac{0.5}{{}^{10}C_2(0.75)^2(0.25)^8 +{}^{10} C_2(0.3)^2(0.7)^8}$$

$$\implies P(A|2_H8_T) = 0.0008$$

Thus, given our observed data, there is much higher probability that coin A was initially picked.

```
[15]:  ## Calculations are shown below:

       from math import comb

       # Probabilty of Coin A given 8 heads & 2 Tails

       p = comb(10,8)*(0.75**8)*(0.25**2)
       q = 0.5
       r = comb(10,8)*(0.75**8)*(0.25**2) + comb(10,8)*(0.3**8)*(0.7**2)
       ans = p*q/r
       print("Probability of Coin A given 8 Heads & 2 Tails = {:.4f}".format(ans))
```

Probability of Coin A given 8 Heads & 2 Tails = 0.4974

```
[10]:  # Probabilty of Coin B given 8 heads & 2 Tails

       p = comb(10,8)*(0.3**8)*(0.7**2)
       q = 0.5
       r = comb(10,8)*(0.75**8)*(0.25**2) + comb(10,8)*(0.3**8)*(0.7**2)
       ans = p*q/r
       print("Probability of Coin B given 8 Heads & 2 Tails = {:.4f}".format(ans))
```

Probability of Coin B given 8 Heads & 2 Tails = 0.0026

```
[16]:  # Probabilty of Coin B given 2 heads & 8 Tails

       p = comb(10,2)*(0.3**2)*(0.7**8)
       q = 0.5
       r = comb(10,2)*(0.75**2)*(0.25**8) + comb(10,8)*(0.3**2)*(0.7**8)
       ans = p*q/r
       print("Probability of Coin B given 2 Heads & 8 Tails = {:.4f}".format(ans))
```

Probability of Coin B given 2 Heads & 8 Tails = 0.4992

```
[17]:  # Probabilty of Coin A given 2 heads & 8 Tails

       p = comb(10,2)*(0.75**2)*(0.25**8)
       q = 0.5
       r = comb(10,2)*(0.75**2)*(0.25**8) + comb(10,8)*(0.3**2)*(0.7**8)
       ans = p*q/r
       print("Probability of Coin A given 2 Heads & 8 Tails = {:.4f}".format(ans))
```

Probability of Coin A given 2 Heads & 8 Tails = 0.0008

The End.