Assignment4_Group23

August 1, 2021

Assignment - Module 4

01 August 2021

Group 23: Aman Jindal | Yuhang Jiang | Daniel Gabriel Tan | Qining Liu

Q1.There are 10 people. What are the number of ways in which you can split them into a team of 6 and a team of 4? Additionally, what are the number of ways in which you can split them into two teams of 5 each?

Answer 1:

- Number of ways to split the 10 people in teams of 6 and 4 are:

$$^{10}C_6 = {}^{10}C_4$$

- Number of ways to split the 10 people in teams of 5 and 5 are:

$$\frac{^{10}C_5}{2}$$

This is because the two groups have identical count of people. Hence, the division by two.

Q2. Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Hint: The goal is to prove this mathematically (not using Venn diagrams). Try writing $A \cup B$ as union of three disjoint sets and then apply axiom 3 that we discussed in class.

Answer 2:

- Partitioning a Set: For any two sets A and B, B and $A \cap B^c$ are disjoint. Thus,

$$A \cup B = B \cup (A \cap B^c) \qquad (1)$$

- For every finite sequence of n disjoint events $A_1, ..., A_n$, the probability of the union of the events is equal to the summation of their individual properties. Thus,

$$Pr(A \cup B) = Pr(B) \cup Pr(A \cap B^c) \qquad (2)$$

- For any two events A and B, the events $A \cap B^c$ and $A \cap B$ are disjoint. Thus,

$$A = (A \cap B) \cup (A \cap B^c) \qquad (3)$$

$$\implies Pr(A) = Pr(A \cap B) + Pr(A \cap B^c) \qquad (4)$$

$$\implies Pr(A \cap B^c) = Pr(A) - Pr(A \cap B) \qquad (5)$$

- Hence, from (1) and (5), we get,

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B) \qquad (6)$$

Q.E.D.

Q3. Two people take turns trying to sink a basketball into a net. Person 1 succeeds with probability 1/3 and person 2 succeeds with probability 1/4. What is the probability that person 2 succeeds before person 1. Additionally, compute the probability that person 1 succeeds before person 2.

Answer 3:

- Let the probability of Person 1 succeeding, should the Person 1 go first, be denoted by `x`.
- Let the probability of Person 2 succeeding, should the Person 2 go first, be denoted by `y`.

Thus,

$$x = \frac{1}{3} + \frac{2}{3}(1 - y) \qquad (1)$$

$$y = \frac{1}{4} + \frac{3}{4}(1 - x) \qquad (2)$$

- Thus for each person the probability of winning is the sum of the following two components viz.
  - Component A: Probability of the person winning on the first turn. For instance, $\frac{1}{3}$ in case of Person 1.
  - Component B: It is the product of the following two probabilities:
    * Probability of the person losing on the first turn. For instance, $\frac{2}{3}$ in case of Person 1.
    * Probability of the person winning if he had started second which is equal to 1 - the probability of the other person winning if the other person took the turn first. For instance, $1 - y$ in case of Person 1.
- Thus, on solving (1) and (2) above, we get $x = \frac{2}{3}$ and $y = \frac{1}{2}$

Thus, Probability of Person 2 succeeding first is $\frac{1}{2}$. Probability of Person 1 succeeding first is $\frac{2}{3}$.

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