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Prediction of the Maximum Amplitude and Timing of Sunspot Cycle 24

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Abstract Precursor techniques, in particular those using geomagnetic indices, often are used in the prediction of the maximum amplitude for a sunspot cycle. Here, the year 2008 is taken as being the sunspot minimum year for cycle 24. Based on the average *aa* index value for the year of the sunspot minimum and the preceding four years, we estimate the expected annual maximum amplitude for cycle 24 to be about 92.8 ± 19.6 (1-sigma accuracy), indicating a somewhat weaker cycle 24 as compared to cycles 21 – 23. Presuming a smoothed monthly mean sunspot number minimum in August 2008, a smoothed monthly mean sunspot number maximum is expected about October 2012 ± 4 months (1-sigma accuracy).

Keywords Sun: sunspot number · Sun: prediction technique, geomagnetic activity index *aa*

1. Introduction

Solar outbursts cause inclement space weather that sometimes wrecks havoc on technological systems on which our society is progressively more dependent. These outbursts involve the sudden release of energy that is stored in stressed coronal magnetic fields. They occur on a wide variety of scales depending upon the available free magnetic energy, which, however, is drawn from sunspot magnetic fields. Thus the predictions of solar and geomagnetic activity are important. Various techniques, namely, even/odd behavior, precursor, spectral, climatology and neural networks have been used in the past for the prediction of the solar activity. Many researchers (Ohl, 1966; Kane 1978, 2007; Thompson, 1993; Jain, 1997; Hathaway and Wilson, 2006) have used the 'precursor' technique to predict the solar activity. Also the report entitled "*Solar Cycle 23 Project: Summary and Panel Findings*" (Joselyn

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et al., 1997) has mentioned precursor techniques as being the most successful. Jain (1997) used a 'precursor' technique to predict the amplitude of the solar cycle using the geomagnetic activity *aa* index and predicted the maximum annual mean sunspot number for cycle 23 to be 166.2, which, however, was found to be higher than the observed values of 120. Had he used an error estimate, perhaps his forecast would have been within the error limits.

The high level of geomagnetic activity occurs not only at sunspot maximum but also in the following two to four years, thereby supporting the idea of the 'extended solar cycle' where a solar cycle really begins some years before solar minimum and where two solar cycles co-exist on the Sun for a number of years. Wilson and Hathaway (2008a, 2008b) observed that the variation of the *aa* index usually peaks after a sunspot maximum, which appears to be directly related to increased solar wind speed, which probably is the result of high-speed streams from coronal holes. The prediction of maximum amplitude of a sunspot cycle using various aspects of the *aa* index has been pursued for many years (since the 1960s) by many authors. For example, Ohl (1966, 1971) and Wilson (1990) showed that the *aa* index values in the few years prior to a sunspot cycle minimum can be used to gauge the size of the next unfolding sunspot cycle. The level of geomagnetic activity near the time of solar activity minimum has been found to be a reliable indication of the amplitude of the following solar activity maximum (Hathaway and Wilson, 2006). In this view, we are motivated to predict the amplitude of the maximum annual mean sunspot number of solar cycle 24 using the technique employed by Jain (1997); however, with error estimates. In Section 2 we briefly describe the data of sunspot numbers and the *aa* geomagnetic index used in the current investigation. In Section 3, we present the prediction techniques and results obtained. We compare our findings to predictions by many other investigators in Section 4.

2. Data

To predict the amplitude of a solar cycle, a few precursor techniques employ geomagnetic activity indices *viz.* Ap and *aa*. The *aa* indices are derived using a homogeneous series of data from two nearly antipodal observatories, where magnetic data have been obtained since 1868. The *aa* index represents the activity level at an invariant magnetic latitude of about 50°. The two observatories were Greenwich (1868 – 1925) in the northern hemisphere and Melbourne (1868 – 1919) in the southern hemisphere. Greenwich was replaced by Abinger in 1926 and by Hartland in 1957. Melbourne was substituted by Toolangui in 1920 and by Canberra in 1980. The data are normalized by cross-correlation of the instruments distributed over the globe and over time, and therefore may be considered homogeneous over the period under current study.

The annual geomagnetic *aa* indices are obtained from the website: ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/RELATED_INDICES/AA_INDEX/AA_YEAR for the period 1868 – 2007. Monthly *aa* values for 2008 (January – November) are acquired from the following website: ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/RELATED_INDICES/AA_INDEX/AA_MONTH. Following the method of Svalgaard, Cliver, and Le Sager (2004) and Wilson and Hathaway (2006), the values of *aa* prior to 1957 were increased by 3 nT in the present study to compensate for change in the geographical latitudes of the magnetometers used in determining the *aa* index.

The relative sunspot number (International Sunspot Number), Ri, is an index of the activity of the entire visible disc of the Sun. It is determined each day at a given observing station without reference to preceding days using the form $\mathrm{Ri} = K(10g + s)$, where g is the number of sunspot groups and s is the total number of distinct spots. The scale factor K (usually less than unity) depends on the observer and is intended to effect the conversion to the scale originating in the work of Wolf. The relative sunspot number Ri (international) is derived from the statistical treatment of data originating from more than twenty-five observing stations. For our current investigation, the data for the yearly sunspot numbers for the period 1868 – 2007 are taken from the following website: ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/YEARLY and for 2008 (monthly) is obtained from the following website: ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/MONTHLY.PLT

3. Analysis and Results

3.1. Defining the Sunspot Minimum Year of Cycle 24

In our investigation the year of the sunspot minimum of solar cycle 24 is of greatest importance and thus at least its approximate determination is explicitly necessary. According to "The Weekly" report by NOAA/Space Weather Prediction Center (available at http://www.swpc.noaa.gov/weekly/pdf2008/prf1688.pdf) the first sunspot of solar cycle 24 was observed on 4 January 2008 and it was numbered NOAA AR 10981. Later, NOAA AR 10990 and 10993 were observed in April and May 2008 respectively, and these have also been classified as sunspots of cycle 24. However, simultaneously, sunspots of solar cycle 23 have also been appearing near the equator until and even after mid 2008. Thus, the year 2008 is considered as the transition period from one cycle to the next. The trend of observed annual mean sunspot number and annual mean *aa* index for the year 1992 – 2008 is shown in Figure 1. Monthly *aa* values for 2008 are identified at the website only for January – November. A December *aa* value can be estimated to be about 14.4 +/− 4.3, based

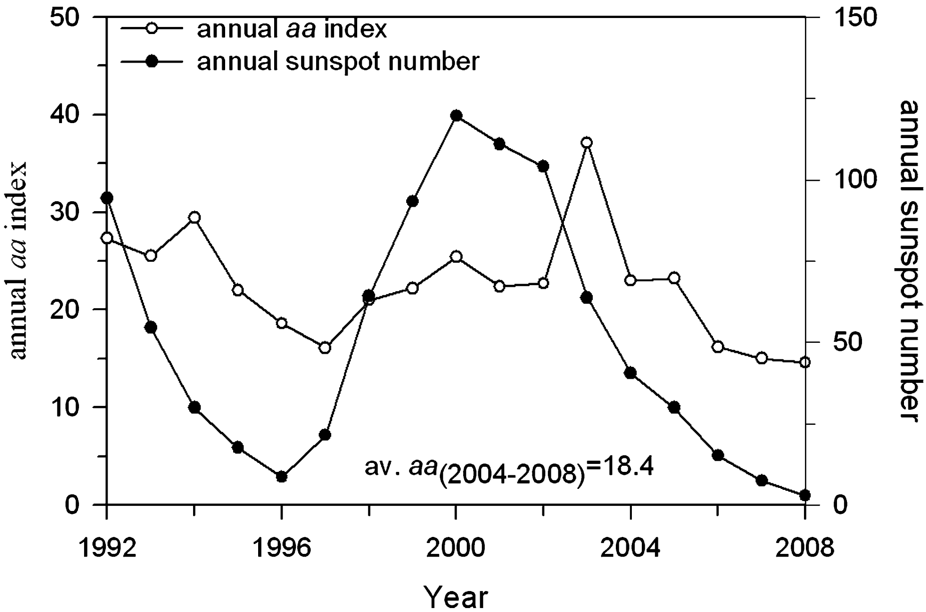

Figure 1 The observed annual mean *aa* index and annual sunspot number for the period of 1992 – 2008. Note that the annual mean *aa* index for the period of 2004 – 2008 is 18.4.

on the average adjusted December *aa* values for each of the previous 12 years of a sunspot minimum. Hence, the 2008 *aa* value can be estimated to be about 14.6 and the average *aa* value for the sunspot minimum year together with the preceding four years (2004 – 2008) is estimated to be about 18.4. We notice from the figure that the annual mean *aa* index ranges from 16.1 (in 1997) to 37.1 (in 2003), which is indicative of minimum and maximum geomagnetic activity, respectively, during this period. The annual mean sunspot number for the year 2008 is found to be 2.9 (lower than 7.5 in 2007), which is well within the range of a typical sunspot minimum value (Dabas *et al.*, 2008). Therefore in the current study we consider 2008 as the year of the sunspot minimum and August 2008 as the month of the year of the sunspot minimum of solar cycle 24.

3.2. Prediction of the Maximum Annual Mean Sunspot Number

Following to method described by Jain (1997), we examine the level of geomagnetic activity to predict the amplitude of solar cycle 24. The annual mean sunspot number for the period 1868 – 2008 and the annual mean of geomagnetic activity *aa* index for the period 1868 – 2008 are considered in the present investigation. We have determined $(aa_n^*)_{\mathrm{dsc}}$, an average of the geomagnetic *aa* index for the year of the sunspot minimum and the preceding four years of the descending phase of the *n*th cycle (*i.e.* in total five years) and compared it with the observed maximum annual mean sunspot number $(R_{n+1})^{\max}$ of the next, $(n+1)$th, cycle. The variation of the observed amplitude $(R_{n+1})^{\max}$ for the $(n+1)$th cycle is plotted as a function of $(aa_n^*)_{\mathrm{dsc}}$ as shown in Figure 2. We obtained the best linear fit to the data with the correlation coefficient (r) of 0.89. Figure 2 demonstrates the validation of Ohl's

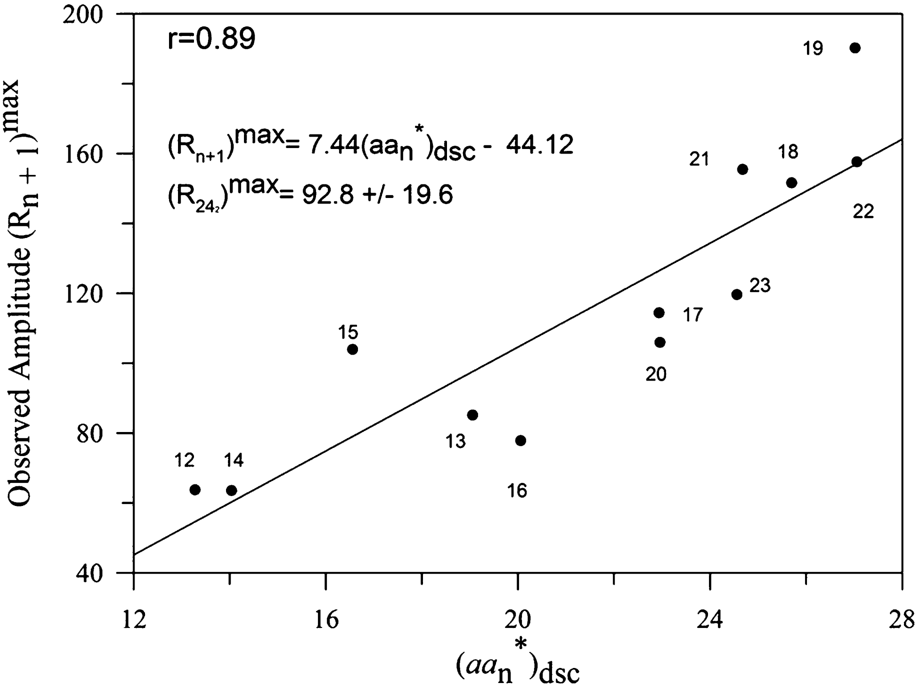

Figure 2 Plot of $(R_{n+1})^{\max}$ of the $(n+1)$th cycle as a function of $(aa_n^*)_{\mathrm{dsc}}$. The solid line is the best fit with a correlation coefficient of $r = 0.89$.

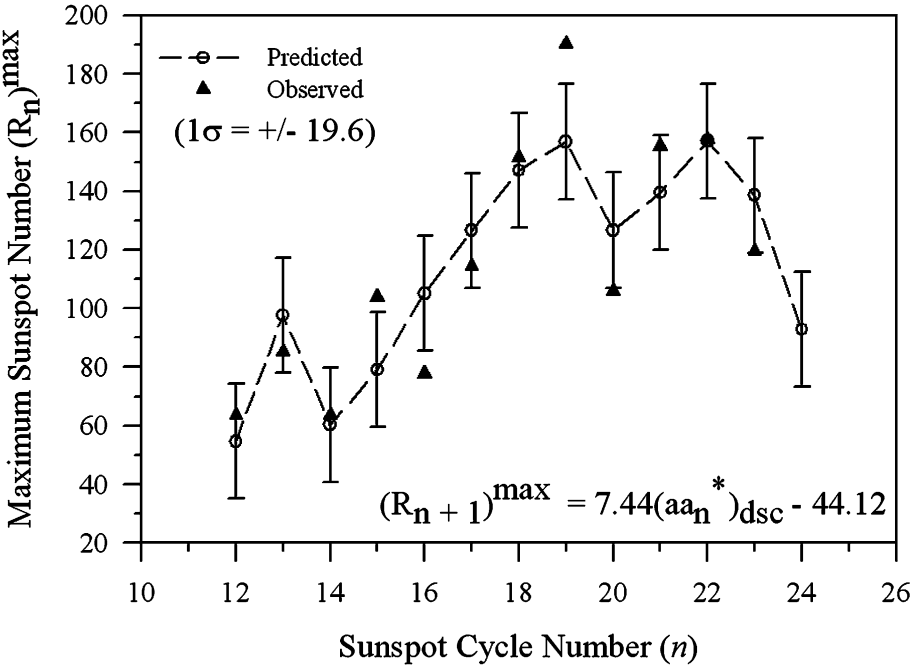

Figure 3 Variation of the predicted (open circle) and observed (triangle) maximum annual mean sunspot number $(R_n)^{\max}$ as a function of sunspot cycle number (n). The predicted $(R_n)^{\max}$ are connected with a dashed line.

precursor method for deducing the size of the maximum amplitude for a sunspot cycle. Ohl (1966) observed that the geomagnetic activity level during the declining phase of a solar cycle was related to the maximum level of solar activity of the next cycle.

The linear equation derived from the fit of the data ranging for cycle 11 to 23 is of the following form:

$$(R_{n+1})^{\max} = 7.44(aa_n^*)_{\mathrm{dsc}} - 44.12. \quad (1)$$

Using Equation (1), we predict the maximum annual mean sunspot number $(R_{n+1})^{\max}$ for cycles 12 to 24 using $(aa_n^*)_{\mathrm{dsc}}$ of the previous cycle. The predicted (open circle) and observed (triangle) maximum annual mean sunspot number, $(R_n)^{\max}$ for cycle 12 to 23 as well as the value predicted for cycle 24 are shown in Figure 3. The amplitude of the predicted annual mean sunspot number of cycle 24 is found to be 92.8 ± 19.6 (1-sigma accuracy). The maximum amplitude of cycle 24 is estimated to be about 92.8 ± 35.5 (the 90% prediction interval). This suggests that there is only a 5% chance that $(R_{24})^{\max}$ is expected to exceed 128.3 or to be below 57.3, unless cycle 24 proves to be a statistical outlier. Our predicted amplitude of 92.8 ± 19.6 for cycle 24 is in agreement with the predictions made by a few other investigators (Kane, 1999; Wang *et al.*, 2002; de Meyer, 2003; Sello, 2003; Duhau, 2003; Schatten, 2005; Svalgaard, Cliver, and Kamide, 2005; Xu *et al.*, 2008).

3.3. Prediction of the Ascending Period of Cycle 24

The ascent duration of a solar cycle is observed to be inversely correlated with the maximum amplitude of a solar cycle. Waldmeier (1935) showed that there is an inverse correlation be-

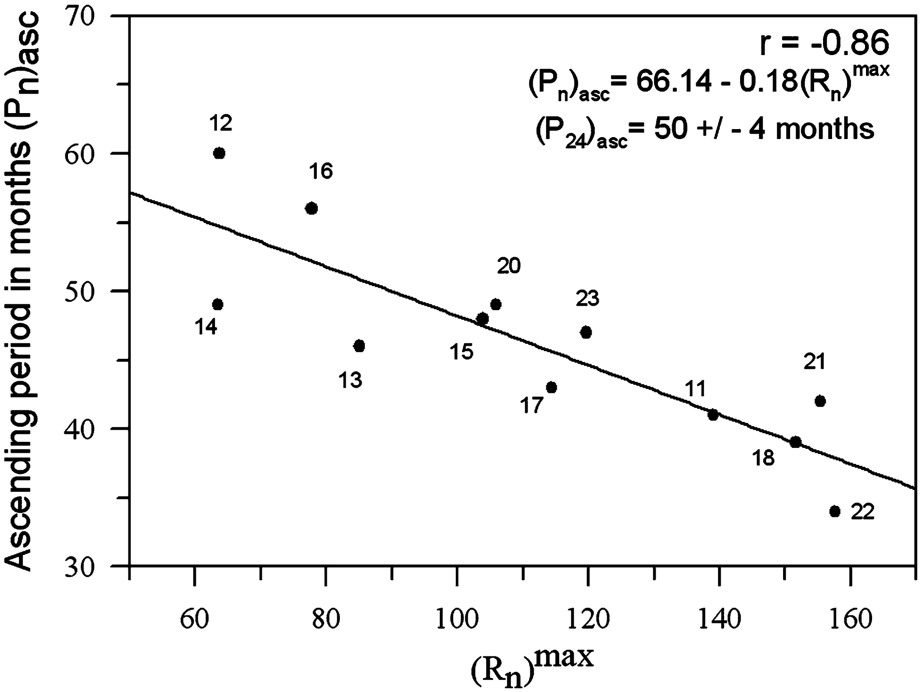

Figure 4 The ascending period (in months), $(P_n)_{\mathrm{asc}}$, is plotted as a function of $(R_n)^{\max}$ for cycles 11 to 23 (cycle 19 is omitted). The solid line is the best fit with $r = -0.86$.

tween the length of the ascending duration of a solar cycle and the peak sunspot number of that cycle. This phenomenon is often called the "Waldmeier effect". To predict the ascending period for solar cycle 24, we studied the relationship between the ascending period in months, $(P_n)_{\mathrm{asc}}$ (often simply defined as the elapsed time in months from sunspot minimum amplitude to sunspot maximum amplitude) and $(R_n)^{\max}$. In this investigation, we have considered the ascending period in months for cycles 11 to 23; however, excluding cycle 19 to improve the correlation. The statistical relation between the maximum annual mean sunspot number, $(R_n)^{\max}$ and the corresponding $(P_n)_{\mathrm{asc}}$ for cycle 11 to 23 is shown in Figure 4. This figure unambiguously shows that the $(P_n)_{\mathrm{asc}}$ of a solar cycle decreases with the increase in maximum annual mean sunspot number, which is the manifestation of the "Waldmeier effect". Figure 4 is the best linear fit with a negative correlation coefficient of ~ 0.86 and a standard error of estimate of 4 months. Figure 4 gives a linear relationship which can be expressed in the form of the following relation:

$$(P_n)_{\mathrm{asc}} = 66.14 - 0.18(R_n)^{\max}. \tag{2}$$

From Equation (2) the $(P_n)_{\mathrm{asc}}$ for cycle $11-23$ is calculated. Considering $(R_{24})^{\max} = 92.8 \pm 19.6$ (*cf.* Section 3.2) in relation (2) suggests $(P_n)_{\mathrm{asc}} = 50 \pm 4$ months with 1-sigma accuracy or 50 ± 8 months being the 90% prediction interval. Provided that the minimum amplitude indeed occurred in August 2008, the maximum amplitude would be expected to be about October 2012 $\pm$ 8 months, inferring only a 5% chance that the maximum amplitude for cycle 24 will occur after June 2013 or before February 2012.

4. Discussion

In the current investigation, we have used the long term data of sunspot numbers and *aa* indices from year 1868 to 2008 to predict the amplitude of sunspot cycle 24 employing the 'precursor technique' of Jain (1997). However, the prediction by Jain (1997) for cycle 23 was not accompanied by error estimates. Further, in our view, the linear relation derived by him (Equation (1) in his paper) to predict the sunspot amplitude for cycle 23 requires modification. For example, if we use our relation (1) to predict the amplitude of cycle 23, then we obtain $(R_{23})^{\max} = 138.6$ with error estimates $\pm\,19.6$ (*cf.* Figure 3), which is closer to the observed 120. This suggests that the precursor technique employed by Jain (1997) is successful if the current linear relation with error estimates is considered.

In the present investigation, we have predicted the maximum amplitude of cycle 24 considering the error estimates. We predict the maximum amplitude for cycle 24 to be 92.8 ± 19.6 (1-sigma accuracy), which is expected to peak in October 2012 $\pm\,4$ months (1-sigma accuracy). Our results suggest that cycle 24 will be about 40%, 41% and 22% weaker than cycle 21, 22 and 23, respectively. Further, our prediction is found to be in agreement with the predictions made by some investigators (Kane, 1999; Wang *et al.*, 2002; de Meyer, 2003; Sello, 2003; Duhau, 2003; Schatten, 2005; Svalgaard, Cliver, and Kamide, 2005; Xu *et al.*, 2008) but in disagreement with many others who have predicted either an acute minimum or an extraordinarily high amplitude of cycle 24. Recently, Wilson and Hathaway (2009) have published a technical report (NASA/TP-2009-215687, February 2009), which appears on the website http://solarscience.msfc.nasa.gov/papers.shtml. They have identified 11 statistically important single-variate fits and 22 statistically important bi-variate fits for estimating the size of the sunspot maximum amplitude, applying the fits to cycle 24. The weighted mean prediction of 11 statistically important single-variate fits is 116 ± 34 and that of 22 statistically important bi-variate fits is 112 ± 32. Many investigators have used different "precursor techniques", and their predictions for the maximum amplitude for solar cycle 24 appear to be varying between 75 (Svalgaard, Cliver, and Kamide, 2005) and 190 (Li, Gao, and Su, 2005). Further, using the solar polar magnetic field strength, Svalgaard, Cliver, and Kamide (2005) have predicted that the approaching cycle ($\sim$2011 maximum) will have a peak smoothed monthly sunspot number of 75 ± 8, making it potentially the smallest cycle in the last 100 years.

We propose the following two hypotheses to explain the expected low amplitude of cycle 24. First, a given sunspot cycle is an extended sunspot cycle composed of two sunspot cycles, one main and the second sympathetic. The latter begins two-three years after the main cycle, and it produces a rather stronger geomagnetic activity. While comparing the geomagnetic activity during the descending phase of cycles 21, 22 and 23 we found that the *aa* index $(aa_n^*)_{\mathrm{dsc}}$ was 27.06, 24.56, and 18.4 respectively indicating that the magnitude of the sympathetic cycle has been decreasing since cycle 21. The weak geomagnetic activity during the descending phase of solar cycle 23 is an indicator of a low amplitude of solar cycle 24. It appears that the sympathetic cycle plays an important role in governing the amplitude, length and activity of the solar cycle. Our second hypothesis is based on long term periodicities of sunspots over and above a 11-year primary sunspot cycle. The $\sim$200-year periodicity (Nordemann, Rigozo, and de Faria, 2005; Ma and Vaquero, 2009 and many previous studies) is well known and we propose that the low amplitude in cycle 24 may be an epoch of this periodicity, which previously occurred in 1816, approximately 196 years ago. However, there might be other possible mechanisms to switch over the Sun to lower amplitude of the solar activity such as the Wolf – Gleissberg cycle, which has a periodicity of about 80 – 100 years. However, we would like to mention that in this paper we consider August 2008 as the month of the year of the sunspot minimum of solar cycle 24, and it should be noted with caution that the Sun is still showing a decreasing trend of sunspot activity, and the Sun remained almost spotless between September 2008 and May 2009 (average $R = 1.8$). In this view, the annual mean *aa* index during the descending phase of cycle 23, considering up to May 2009, has further come down to ~ 17, which, however, predicts the maximum amplitude of cycle 24 to be ~ 82.

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