Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia Engineering 29 (2012) 2838 – 2842
Procedia Engineering
www.elsevier.com/locate/procedia

2012 International Workshop on Information and Electronics Engineering (IWIEE)

An Improved PDE Based Super-Resolution Reconstruction Algorithm

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Abstract

In this paper, we present an improved partial differential equation (PDE) model for multi-frame image super-resolution reconstruction. Our proposed PDE model is achieved by using an adaptively weighting function, which combines the Total Variation (TV) regularization with fourth-order partial differential equations (PDE) regularization. Experiments show the effectiveness and practicability of the proposed method and demonstrate its superiority to some existing SR methods, both in its visual effects and in quantitative terms.

Keywords: Partial differential equation, Super-resolution reconstruction, Total Variation.

1. Introduction

Image super-resolution (SR) reconstruction is currently an active research topic in image processing. This technique is to reconstruct a high-resolution (HR) image from a set of low-resolution (LR) images that are noisy, blurred, shifted and down-sampled. In many civilian and military applications, such as remote sensing [1], video surveillance [2], medical diagnostics [3], super-resolution reconstruction has already been proved useful.

There have been lots of techniques for super-resolution presented in the literatures [4-9]. These can be are roughly categorized into two classes, namely, frequency domain methods and spatial domain methods. In the frequency domain, most of the earlier SR reconstruction algorithms were based on the property of

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doi:10.1016/j.proeng.2012.01.400

shifting of Fourier transforms. More recently, DCT-based and wavelet transform-based SR methods have also been proposed. However, although frequency domain methods are computationally attractive, they have some limitations because they are not able to incorporate any spatial domain prior knowledge in their formulation. Therefore, many spatial domain methods have been successfully developed to overcome the aforementioned drawbacks. Recently, the use of the total variation (TV) function [10] has become popular in super resolution. The most useful property of TV function is to preserve edges in the reconstruction. However, TV function has the disadvantage that smoothes signal into piecewise constants, i.e., the so-called staircase effect. To overcome this spurious effect, four-order partial differential equations (PDEs) [11] have been of special interest over the last few years. Although fourth-order diffusion methods reduce blocky effects, they are more severely prone to the over-smoothness in the image restoration. Inspired by this, in this paper we proposed a novel improved SR reconstruction algorithm for image sequences by adaptively combining the TV filter with a fourth-order PDE filter. Experimental results indicate that the new SR algorithm can achieve good tradeoff between noise removal and edge preservation.

The remainder of the paper is organized as follows. Section 2 briefly describes the image degradation model of the SR reconstruction. In Section 3, we propose the new SR image reconstruction algorithm. Experimental results are shown in Section 4. Our conclusion is drawn in Section 5.

2. Image Degradation Model

In SR image reconstruction, it is necessary to set up a suitable imaging degradation model, which is to relate the desired reference high-resolution (HR) image to all the observed low-resolution (LR) images. Suppose that the HR image and LR images are denoted in the vector form, Y_k is the k th LR frame, which is represented as $Y_k = [y_{k,1}, y_{k,2}, \cdots, y_{k,N_1 \times N_2}\}^T$, where $k = 1,2,\cdots,p$ with p being the number of LR and images and $N_1 \times N_2$ is the size of each observed LR image. X is the HR image of size $qN_1 \times qN_2$, which is represented as $X = \{x_1, x_{2,} \cdots, x_{qN_1 \times qN_2}\}^T$, and the parameter q is the down-sampling factor for the horizontal and vertical directions, respectively. The relationship between an ideal HR image and an LR image can be described as [6]:

$$Y_k = D_k B_k M_k X + N_k \qquad \text{for } 1 \le k \le p \tag{1}$$

where M_k stands for the geometric warp matrix of size $q^2 N_1 N_2 \times q^2 N_1 N_2$; B_k is the blurring matrix of size $q^2 N_1 N_2 \times q^2 N_1 N_2$, which represents the point spread function (PSF) of the camera sensor; D_k is an $N_1 N_2 \times q^2 N_1 N_2$ down-sampling matrix for presenting the loss of resolution in the observed images; n_k is the normally distributed additive noise of size $N_1 N_2 \times 1$.

3. The Proposed Super-resolution Reconstruction Method

3.1. The PDE Models

TV regularization is a successful approach to recover images with sharp edges. This method was first proposed by Rudin et al. [10] in image processing field. The standard TV norm looks like:

$$\gamma_{TV}(X) = \int_\Omega |\nabla X| dxdy = \int_\Omega \sqrt{\left|\nabla X^2\right| + \beta}\, dxdy \tag{2}$$

Here, Ω is the 2-dimensional image space, β is a small positive parameter which ensures differentiability. Using the gradient descent method, the corresponding Euler equation of (2) is solved by:

$$\frac{\partial \gamma_{TV}(X)}{\partial t} = \nabla\left(\frac{\nabla X}{|\nabla X|}\right) \tag{3}$$

In order to resolve formation of staircase artifacts on the ramp edges, a fourth order PDE for noise removal was proposed by You and Kaveh [11]. The energy function is defined as:

$$\gamma_{FPED}(X) = \iint_{\Omega} f\left(\left|\nabla u^2\right|\right) dxdy \tag{4}$$

where $f(\bullet) \geq 0$ is an increasing function associated with the diffusion coefficient. The corresponding Euler equation of (3) can be solved through the following gradient descent procedure:

$$\frac{\partial \gamma_{FPDE}(X)}{\partial t} = -\nabla^2\left[f'\left(\left|\nabla^2 X\right|\right)\frac{\nabla^2 X}{\left|\nabla^2 X\right|}\right] = -\nabla^2\left[g\left(\left|\nabla^2 X\right|\right)\nabla X^2\right] \tag{5}$$

Here, $g(\bullet)$ is a nonlinear diffusivity function which would inhibit diffusion across edges.

3.2. The Proposed Adaptive Combination Model

TV regularization is a successful approach to recover images with sharp edges. Higher-order PDEs are known to recover smoother surfaces. Both methods have their strengths and weaknesses depending on the characteristics of the image of interest. According to this, we want to generate a new solution by combining the two methods with an adaptive weighting function τ . To emphasize the restoration properties for TV, the weighting function τ is 1 along edge; and to emphasize the restoration properties for FPDE, the weighting function τ is $0 \leq \tau < 1$ in smooth regions and small jumps (staircase effect) caused by TV. Thus, the combined regularization term can be written as:

$$\gamma(X) = \tau \times \gamma_{TV}(X) + (1-\tau)\gamma_{FPDE}(X) \qquad \textit{for } \tau \in (0,1) \tag{6}$$

Based on the properties of the two models, a new weighting function is defined as:

$$\tau = \begin{cases} 1 & |\nabla e| \geq C \\ \sin\left(\dfrac{\pi|\nabla e|}{2C}\right) & |\nabla e| < C \end{cases} \tag{7}$$

where $|\nabla e|$ is obtained by scaling the values of $|\nabla X|$ to the interval [0, 1]. The parameter C is a positive constant in the interval [0, 1].

In order to obtain more desirable SR results, the ill-posed problem should be stabilized to become well-posed. As reference [12], we proposed to use regularization techniques that can estimate the desired HR image by the following cost function minimization:

$$\hat{X} = Arg\min_X\left[\sum_{k=1}^{N}\left\|D_k B_k M_k X - Y_k\right\|_2^2 + \lambda\gamma(X)\right] = Arg\min_X\left[\begin{array}{l}\sum_{k=1}^{N}\left\|D_k B_k M_k X - Y_k\right\|_2^2 \\ +\lambda\left[\tau \times \gamma_{TV}(X) + (1-\tau)\gamma_{FPDE}(X)\right]\end{array}\right] \tag{8}$$

where the first term is the data fidelity term, which measures the reconstruction error to ensure the pixels in the reconstructed HR image are close to real values, the second term as the regularization term is to control the smoothness of the reconstructed HR image and λ is the regularization parameter which provides a tradeoff between the data fidelity term and the regularization term.

By using steepest descent algorithm, we can find the solution to the following minimization problem:

$$X^{n+1} = X^n - \beta\nabla E(X) = X^n - \beta\left\{\begin{array}{l}\sum_{k=1}^{N} M_k^T B_k^T D_k^T (D_k B_k M_k X^n - Y_k) \\ -\lambda\left\{\tau\nabla\left(\frac{\nabla X}{|\nabla X|}\right) + (1-\tau)\left\{-\nabla^2\left[g\left(\left|\nabla^2 X\right|\right)\nabla X^2\right]\right\}\right\}\end{array}\right\} \tag{9}$$

where n is the iteration number, and β is a scalar defining the step size in the direction of the gradient.

4. Experiments

In this section, the application results of the proposed SR algorithm are presented. We also compared the performance of the proposed algorithm with that of the bi-cubic interpolation and the TV based SR algorithm. Parameters of all the algorithms in our comparison are set such that the most visually appealing results and the highest PSNR of the reconstructed HR images can be produced. The peak signal-to-noise ratio (PSNR) is used to evaluate the performance of the algorithm.

(a) (b) (c) (d)

Fig. 1. Experimental results of the "lena" sequence. (a) One of the LR images. (b) The result of bi-cubic interpolation. (c) The Reconstructed result of TV SR algorithm (PSNR=29.6860, $\beta = 0.1$, $\lambda = 4$). (d) The reconstructed result of the proposed algorithm (PSNR=30.1773, $\beta = 0.2$, $\lambda = 6$, $C = 1/6$, $k = 2$).

In the experiment, the well-known "Lena" image of size 256×256 is shifted by 8 different motion vectors, blurred, and down-sampled by 2 to obtain 8 observations. Then, zero-mean Gaussian-noise with 0.002 variance was added to each LR images. Fig. 1(a) shows one of the observed low-resolution degraded images. Fig.1 (b) is one of LR images interpolated by the bi-cubic algorithm. Super-resolved images with TV algorithm and our proposed algorithm are shown in Fig. 1(c)-(d), respectively. From the comparison of the results of Fig.1, we can find that the overall visual quality of the SR reconstruction images of Fig.1 (c) and (d) are much better than that of the interpolated result of Fig.1 (b), and Fig.1 (d) has the better visually enhanced result and higher PSNR value than the TV algorithm in the smooth region.

5. Conclusion

In this paper, we proposed a novel PDE-based SR algorithm to reconstruct a HR image from multiple LR images. The objective function of the algorithm consists of a data fidelity term and a combined regularization term, which is the adaptively weighted average of the TV regularization and fourth-order PDE regularization. The proposed algorithm has been tested in different cases. Experimental results show that it can effectively remove visual artifacts and keep well information of edges in the reconstructed HR images.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (No. 60963012), by the Key Project of Chinese Ministry of Education (No. 211087), by the Natural Science Foundation of Jiangxi Province (No. 2010GZS0052), and by the Science and Technology Research Project of the Education Department of Jiangxi Province (No. GJJ11415).

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