ScrollCompressor_Study

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参考文献：**Bell I H. Theoretical and experimental analysis of liquid flooded compression in scroll compressors[J]. Dissertations & Theses - Gradworks, 2011.**

目的**:** 翻译和注释**Bell I H.**的博士学位论文和软件**pdSIM**，为后面开发电动涡旋压缩机一维热力学模型做准备

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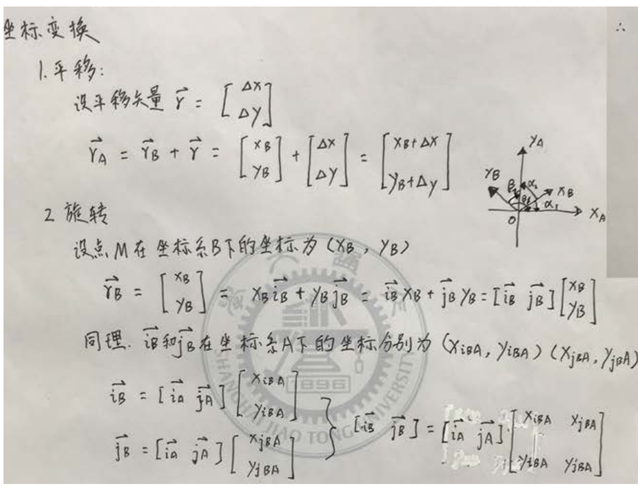
一、前期准备知识

1.1 坐标变换

1) 平移变换和旋转变换



坐标变换

1. 平移：

设平移矢量 $\vec{r} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$

$$\vec{r}_A = \vec{r}_B + \vec{r} = \begin{bmatrix} x_B \\ y_B \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x_B+\Delta x \\ y_B+\Delta y \end{bmatrix}$$

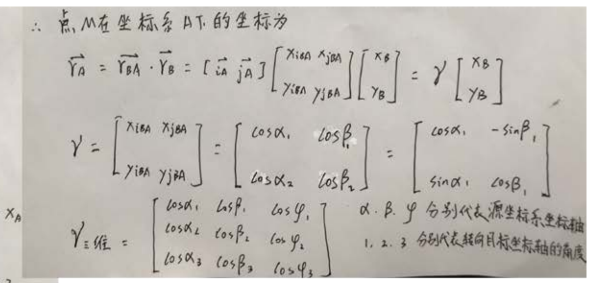
2. 旋转

设点M在坐标系B下的坐标为 (x_B, y_B)

$$\vec{r}_B = \begin{bmatrix} x_B \\ y_B \end{bmatrix} = x_B \vec{i}_B + y_B \vec{j}_B = \vec{i}_B x_B + \vec{j}_B y_B = [\vec{i}_B \ \vec{j}_B]\begin{bmatrix} x_B \\ y_B \end{bmatrix}$$

同理，$\vec{i}_B$ 和 $\vec{j}_B$ 在坐标系A下的坐标分别为 (x_{iBA}, y_{iBA}) (x_{jBA}, y_{jBA})

$$\left.\begin{aligned} \vec{i}_B &= [\vec{i}_A \ \vec{j}_A]\begin{bmatrix} x_{iBA} \\ y_{iBA} \end{bmatrix} \\ \vec{j}_B &= [\vec{i}_A \ \vec{j}_A]\begin{bmatrix} x_{jBA} \\ y_{jBA} \end{bmatrix} \end{aligned}\right\} [\vec{i}_B \ \vec{j}_B] = [\vec{i}_A \ \vec{j}_A]\begin{bmatrix} x_{iBA} & x_{jBA} \\ y_{iBA} & y_{jBA} \end{bmatrix}$$



∴ 点M在坐标系A下的坐标为

$$\vec{r}_A = \vec{r}_{BA} \cdot \vec{r}_B = [\vec{i}_A \ \vec{j}_A]\begin{bmatrix} x_{iBA} & x_{jBA} \\ y_{iBA} & y_{jBA} \end{bmatrix}\begin{bmatrix} x_B \\ y_B \end{bmatrix} = \gamma \begin{bmatrix} x_B \\ y_B \end{bmatrix}$$

$$\gamma = \begin{bmatrix} x_{iBA} & x_{jBA} \\ y_{iBA} & y_{jBA} \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & \cos\beta_1 \\ \cos\alpha_2 & \cos\beta_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & -\sin\beta_1 \\ \sin\alpha_1 & \cos\beta_1 \end{bmatrix}$$

$$\gamma_{三维} = \begin{bmatrix} \cos\alpha_1 & \cos\beta_1 & \cos\varphi_1 \\ \cos\alpha_2 & \cos\beta_2 & \cos\varphi_2 \\ \cos\alpha_3 & \cos\beta_3 & \cos\varphi_3 \end{bmatrix}$$

α. β. φ 分别代表源坐标系坐标轴

1, 2, 3 分别代表转向目标坐标轴的角度

1.2 面积积分和质心求解

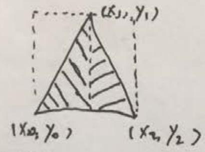
1) 三角形和曲线面元的推导

- 图中φ1和φ2的位置错了，应该是逆时针布置才对，遵循右手定律

面积推导

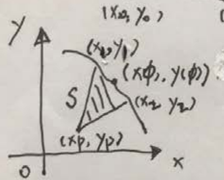
引理. 三角形面积 $A = \frac{1}{2}\left|[(x_0 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_0 - y_1)]\right|$

引理证明:



可用矢量叉乘的一半得到

对于



S的面积运用引理得

$$S = \frac{1}{2}[(x_1 - x_p)(y_1 - y_2) - (x_1 - x_2)(y_1 - y_p)]$$

$$ds = \frac{1}{2}[(x_1 - x_p)dy - (y_1 - y_p)dx]$$

$$ds = \frac{1}{2}\left[(x_1 - x_p)\frac{dy}{d\phi} - (y_1 - y_p)\frac{dx}{d\phi}\right]d\phi$$

$$= \frac{1}{2}\left[(x_1 - x_p)y_\phi - (y_1 - y_p)x_\phi\right]d\phi$$

$$\therefore A = \frac{1}{2}\int_{\phi_1}^{\phi_2}\left[(x - x_p)y_\phi - (y - y_p)x_\phi\right]d\phi.$$ (为保证面积为正, $\phi_1 \to \phi_2$ 取逆时针顺序)

2) 质心的推导

(2) 涡旋型线的重心

观察图 2-15 所示的圆渐开线涡旋型线的几何参数可知，涡旋型线的重心不可能正好在基圆中心线上，而是偏离基圆中心线。动涡盘基圆中心绕静涡盘基圆中心作圆周轨道运动时，使动涡盘产生离心力，这种离心力传递到主轴上，会引起主轴振动。为了很好地平衡动涡盘运动时的离心力，就必须把涡旋型线的重心移至基圆中心线上。首先应确定涡旋型线的重心位置。

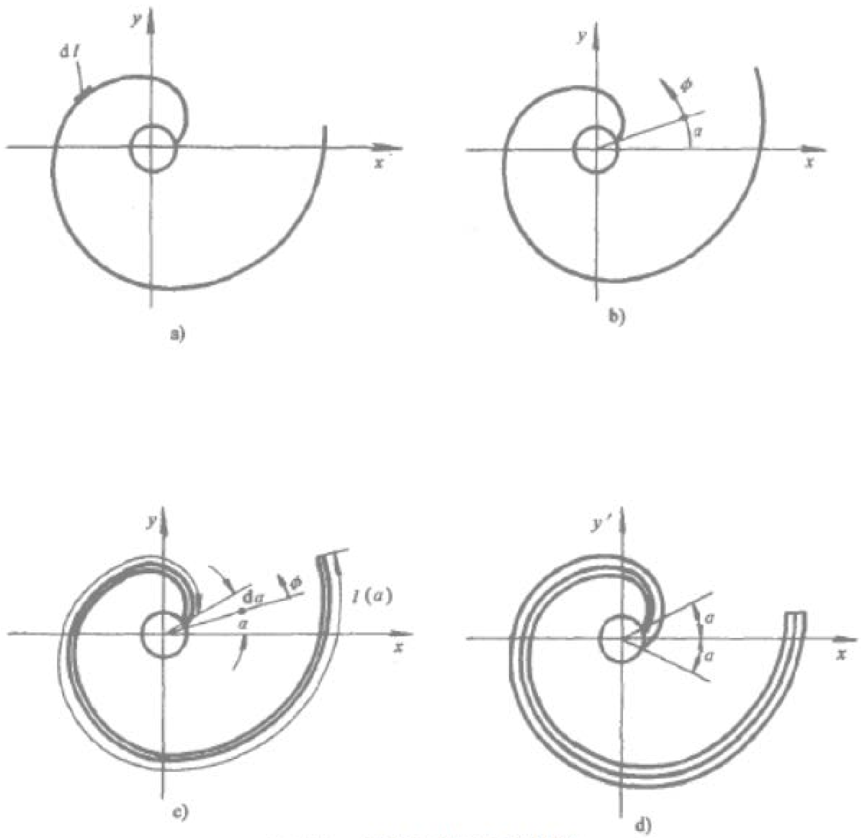


图 3-11 圆渐开线的重心计算

设图 3-11a 所示的圆渐开线重心是 (x_G, y_G)，令 l 为渐开线长度，在渐开线上取微元线

段 dl，则有 $\boxed{dl=a\varphi d\varphi}$，这里 $\boxed{a\text{ 为基圆半径}}$，故有

$$\begin{cases} x_G=\dfrac{\int x dl}{\int dl}=\dfrac{\int_0^{\phi} a(\cos\varphi+\varphi\sin\varphi)a\varphi\, d\varphi}{\int_0^{\phi} a\varphi\, d\varphi}=2a\left(-\cos\phi+\dfrac{3\sin\phi}{\phi}+3\,\dfrac{\cos\phi-1}{\phi^2}\right) \\ y_G=\dfrac{\int y dl}{\int dl}=\dfrac{\int_0^{\phi} a(\sin\varphi-\varphi\cos\varphi)a\varphi\, d\varphi}{\int_0^{\phi} a\varphi\, d\varphi}=2a\left(-\sin\phi-3\,\dfrac{\cos\phi}{\phi}+\dfrac{\sin\phi}{\phi^2}\right) \end{cases} \tag{3-21}$$

式中，ϕ 为渐开线终端渐开角。

对于图 3-11b 所示的渐开线，它的重心应是 α 的函数，即 $[x_G(\alpha), y_G(\alpha)]$。

$$x_G(\alpha)=\frac{\int_0^{\phi-\alpha} a[\cos(\varphi+\alpha)+\varphi\sin(\varphi+\alpha)]a\varphi d\varphi}{\int_0^{\phi-\alpha} a\varphi\, d\varphi}$$

$$=2a\left[-\cos\phi+3\,\frac{\sin\phi}{(\phi-\alpha)}+3\,\frac{(\cos\phi-\cos\alpha)}{(\phi-\alpha)^2}\right]$$

$$y_G(\alpha)=\frac{\int_0^{\phi-\alpha} a[\sin(\varphi+\alpha)-\varphi\cos(\varphi+\alpha)]a\varphi d\varphi}{\int_0^{\phi-\alpha} a\varphi\, d\varphi}$$

$$=2a\left[-\sin\phi-3\,\frac{\cos\phi}{\phi-\alpha}+3\,\frac{(\sin\phi-\sin\alpha)}{(\phi-\alpha)^2}\right]$$

注意这里的Φ和φ的含义不一样，前者代表展开结束角，后者表示从展开开始角α后转过的角度，有Φ=φ+α

在图 3-11c 中的 α 与 $\alpha+d\alpha$ 之间，微小宽度的渐开线的面积 $dS(\alpha)$ 为

$$dS(\alpha)=l(\alpha)ad\alpha=\frac{1}{2}a(\phi-\alpha)^2ad\alpha=\frac{1}{2}a^2(\phi-\alpha)^2d\alpha$$

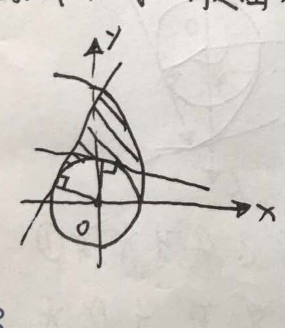
所以，图 3-11d 所示的单位宽度渐开线重心计算式为

注意：这里是微元的质心坐标

$$\begin{cases} x_G=\dfrac{\int_{-\alpha}^{\alpha}\boxed{x_G(\alpha)}dS(\alpha)}{\int_{-\alpha}^{\alpha}dS(\alpha)}=2a\left(-\cos\phi+9\,\dfrac{\phi}{3\phi^2+\alpha^2}\sin\phi+9\,\dfrac{\alpha\cos\phi-\sin\alpha}{3\alpha\phi^2+\alpha^2}\right) \\ y_G=\dfrac{\int_{-\alpha}^{\alpha}\boxed{y_G(\alpha)}dS(\alpha)}{\int_{-\alpha}^{\alpha}dS(\alpha)}=2a\left(-\sin\phi-\dfrac{9\phi}{3\phi^2+\alpha^2}\cos\phi+\dfrac{9\sin\phi}{3\varphi^2+\alpha^2}\right) \end{cases} \tag{3-22}$$

3) 圆渐开线一般面积积分

圆渐开线一般面积积分



利用三角形微元面积积分

$$dA=\frac{1}{2}l\cdot l\,d\varphi=\frac{1}{2}l^2d\varphi=\frac{1}{2}r_b^2(\varphi-\varphi_0)^2d\varphi$$

$$\therefore \int_{\varphi_1}^{\varphi_2}dA=\frac{r_b^2(\varphi-\varphi_0)^3}{6}\Bigg|_{\varphi_1}^{\varphi_2}$$

4) 圆渐开线面积质心求解

圆渐开线所围面积质心求解

$$dA = \frac{1}{2} l^2 d\phi = \frac{1}{2} r_b^2 (\phi - \phi_0)^2 d\phi$$

∵ 微元是三角形微元、处理上要在坐标上添加$\frac{2}{3}$的权重

$$x_G = \frac{\frac{2}{3}\int x\,dA}{\int dA} = \frac{\frac{2}{3}h_s \int [\cos\phi + (\phi-\phi_0)\sin\phi]\,\frac{1}{2} r_b^2 (\phi-\phi_0)^2\,d\phi}{V}$$

$$= \frac{r_b^2 h_s}{3V} \int [(\phi-\phi_0)^2 \cos\phi + (\phi-\phi_0)^3 \sin\phi]\,d\phi$$

$$= \frac{r_b^2 h_s}{3V} \Big[(\phi^2+\phi_0^2-2\phi_0\phi-2)\sin\phi + 2(\phi-\phi_0)\cos\phi + (3\phi^2+3\phi_0^2-6\phi_0\phi-6)\sin\phi + (-\phi^3+3\phi_0\phi^2-3\phi_0^2\phi+\phi_0^3+6\phi-6\phi_0)\cos\phi \Big]$$

$$= \frac{r_b^2 h_s}{3V} \left\{ 4(\phi^2+\phi_0^2-2\phi_0\phi-2)\sin\phi + [8(\phi-\phi_0) - (\phi-\phi_0)^3]\cos\phi \right\} \Big|_{\phi_1}^{\phi_2}$$

$$= \frac{r_b^2 h_s}{3V} \left\{ 4[(\phi-\phi_0)^2-2]\sin\phi - (\phi-\phi_0)[(\phi-\phi_0)^2-8]\cos\phi \right\} \Big|_{\phi_1}^{\phi_2}$$

同理可得

$$y_G = \frac{\frac{2}{3}\int y\,dA}{\int dA} = \frac{2h_s}{3V} \int [\sin\phi - (\phi-\phi_0)\cos\phi]\,\frac{1}{2} r_b^2 (\phi-\phi_0)^2\,d\phi$$

$$= \frac{r_b^2 h_s}{3V} \int [(\phi-\phi_0)^2 \sin\phi - (\phi-\phi_0)^3 \cos\phi]\,d\phi$$

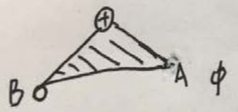
$$= \frac{r_b^2 h_s}{3V} \left\{ -4[(\phi-\phi_0)^2-2]\cos\phi - (\phi-\phi_0)[(\phi-\phi_0)^2-8]\sin\phi \right\} \Big|_{\phi_1}^{\phi_2}$$

综上 圆渐开线所围面积质心

$$\begin{cases} x_G = \frac{r_b^2 h_s}{3V} \left\{ 4[(\phi-\phi_0)^2-2]\sin\phi - (\phi-\phi_0)[(\phi-\phi_0)^2-8]\cos\phi \right\} \Big|_{\phi_1}^{\phi_2} \\ y_G = -\frac{r_b^2 h_s}{3V} \left\{ 4[(\phi-\phi_0)^2-2]\cos\phi + (\phi-\phi_0)[(\phi-\phi_0)^2-8]\sin\phi \right\} \Big|_{\phi_1}^{\phi_2} \end{cases}$$

5) 圆渐开线基圆三角形面积求解

对一个顶点位于静涡盘圆心，另一个顶点位于动盘盘基圆圆心，第三个顶点位于静涡盘壁面，展开角为 ϕ，其面积计算如下



$$A\left(r_b[\cos\phi+(\phi-\phi_0)\sin\phi],\ r_b[\sin\phi-(\phi-\phi_0)\cos\phi]\right)$$

$$B\left(r_o\cos(\phi_{ie}-\frac{\pi}{2}-\theta),\ r_o\sin(\phi_{ie}-\frac{\pi}{2}-\theta)\right)$$

$$\triangle BOA=\frac{1}{2}\vec{A}\times\vec{B}=\frac{1}{2}r_o r_b\begin{bmatrix}\cos(\phi_{ie}-\frac{\pi}{2}-\theta)[\sin\phi-(\phi-\phi_0)\cos\phi]\\ -\sin(\phi_{ie}-\frac{\pi}{2}-\theta)[\cos\phi+(\phi-\phi_0)\sin\phi]\end{bmatrix}$$

$$=\frac{1}{2}r_o r_b\begin{bmatrix}\sin(\phi_{ie}-\theta)[\sin\phi-(\phi-\phi_0)\cos\phi]\\ +\cos(\phi_{ie}-\theta)[\cos\phi+(\phi-\phi_0)\sin\phi]\end{bmatrix}$$

$$=\frac{r_o r_b}{2}\left[(\phi-\phi_0)\sin(\theta+\phi-\phi_{ie})+\cos(\theta+\phi-\phi_{ie})\right]$$

1.3 曲线单位法向量的推导

设曲线 $S(x(\phi),y(\phi))$

其切线斜率为 $k=\frac{dy}{dx}$

则其法线斜率为 $k'=\frac{-1}{k}$

即 $k'=-\frac{dx}{dy}$ $\vec{n}_x=-dx$ $\vec{n}_y=dy$

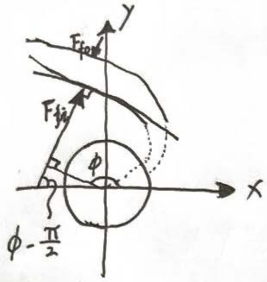
对 $\vec{n}_x$ 与 $\vec{n}_y$ 正则化

$$\mathbf{n}_x=\frac{\left(\frac{dy}{d\phi}\right)}{\sqrt{\left(\frac{dx}{d\phi}\right)^2+\left(\frac{dy}{d\phi}\right)^2}}$$

$$\mathbf{n}_y=\frac{-\left(\frac{dx}{d\phi}\right)}{\sqrt{\left(\frac{dx}{d\phi}\right)^2+\left(\frac{dy}{d\phi}\right)^2}}.$$

1.4 单位气体力方向的推导

对于动静涡盘气体力方向的推导



静涡盘内壁面

$$\frac{\vec{F_{fi}}}{|\vec{F_{fi}}|} = \cos(\phi-\frac{\pi}{2})\vec{i} + \sin(\phi-\frac{\pi}{2})\vec{j} = \sin\phi\vec{i} - \cos\phi\vec{j}$$

静涡盘外壁面，与内壁面方向相反

$$\frac{\vec{F_{fo}}}{|\vec{F_{fo}}|} = -\frac{\vec{F_{fi}}}{|\vec{F_{fi}}|} = -\sin\phi\vec{i} + \cos\phi\vec{j}$$

动涡盘与静涡盘坐标系相位相差π，而静涡盘外壁面与动涡盘内壁面是一对相反力，

故 $$\frac{\vec{F_{oi}}}{|F_{oi}|} = -(-\frac{\vec{F_{fo}}}{|F_{fo}|}) = -\sin\phi\vec{i} + \cos\phi\vec{j}$$

同理，对于动涡盘外壁面与静涡盘内壁面

$$\frac{\vec{F_{oo}}}{|F_{oo}|} = -(-\frac{\vec{F_{fi}}}{|F_{fi}|}) = \sin\phi\vec{i} - \cos\phi\vec{j}$$

$$\therefore \begin{cases} \vec{n_{oi}} = -\sin\phi\vec{i} + \cos\phi\vec{j} \\ \vec{n_{oo}} = \sin\phi\vec{i} - \cos\phi\vec{j} \end{cases} \qquad \begin{cases} \vec{n_{fi}} = \sin\phi\vec{i} - \cos\phi\vec{j} \\ \vec{n_{fo}} = -\sin\phi\vec{i} + \cos\phi\vec{j} \end{cases}$$

1.5 基本三角函数基本公式

$$\int \sin\phi \, d\phi = -\cos\phi \ , \quad \int \cos\phi \, d\phi = \sin\phi$$

$$\int \phi \sin\phi \, d\phi = -\phi\cos\phi + \sin\phi$$

$$\int \phi \cos\phi \, d\phi = \phi \sin\phi + \cos\phi$$

$$\int \phi^2 \sin\phi \, d\phi = -\phi^2\cos\phi + 2\phi\sin\phi + 2\cos\phi$$

$$\int \phi^2 \cos\phi \, d\phi = \phi^2\sin\phi + 2\phi\cos\phi - 2\sin\phi$$

二、基本量

2.1 动静涡盘点的坐标

静涡盘各点坐标

$$x_{fi} = r_b \left(\cos\phi + (\phi - \phi_{i0}) \sin\phi\right)$$

$$y_{fi} = r_b \left(\sin\phi - (\phi - \phi_{i0}) \cos\phi\right)$$

$$x_{fo} = r_b \left(\cos\phi + (\phi - \phi_{o0}) \sin\phi\right)$$

$$y_{fo} = r_b \left(\sin\phi - (\phi - \phi_{o0}) \cos\phi\right)$$

$$\begin{cases} x_1 = x_2\cos\alpha_1 + y_2\cos\alpha_2 + z_2\cos\alpha_3 + a_x \\ y_1 = x_2\cos\beta_1 + y_2\cos\beta_2 + z_2\cos\beta_3 + a_y \\ z_1 = x_2\cos\gamma_1 + y_2\cos\gamma_2 + z_2\cos\gamma_3 + a_z \end{cases} \quad (2\text{-}5)$$

对涡旋压缩机来说，垂直于轴线剖分涡旋体得到的任一投影面，具有相同的涡旋型线及其组合。因此，空间型面的共轭问题，可转换成平面曲线的啮合问题。平面直角坐标系 $O_1x_1y_1$ 与 $O_2x_2y_2$ 各坐标轴之间的夹角（对涡旋压缩机而言）见表 2-2。

参见图 2-3，O_2 在坐标系 $O_1x_1y_1$ 中的坐标 a_x，a_y）满足下式

$$\begin{cases} a_x = r\cos\theta \\ a_y = -r\sin\theta \end{cases} \quad (2\text{-}6)$$

式中，r 表示 O_1 与 O_2 之间的距离，即主轴偏心量或转动半径。

表 2-2

	O_{x_2}	O_{y_2}
O_{x_1}	α_1 (π)	$\alpha_2\left(\frac{\pi}{2}\right)$
O_{y_1}	$\beta_1\left(\frac{\pi}{2}\right)$	β_2 (π)

将关系式（2-6）和表 2-2 中的有关数据代入式（2-5）中，并去掉涉及 z 坐标项，可得涡旋型线在平面直角坐标系之间的一般变换公式

$$\begin{cases} x_1 = -x_2 + r\cos\theta \\ y_1 = -y_2 - r\sin\theta \end{cases} \quad (2\text{-}7)$$

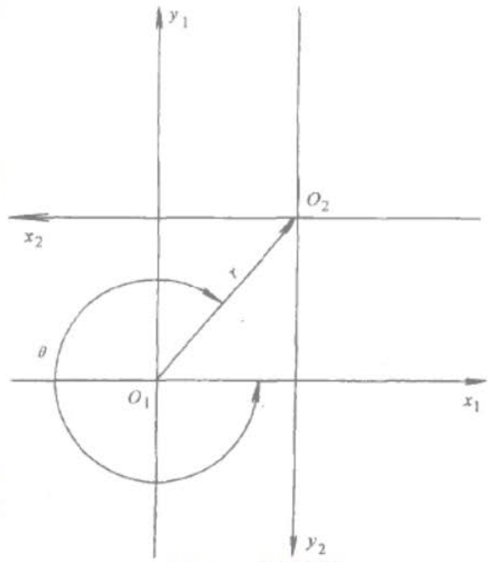


图 2-3 坐标变换

- 展开角逆时针为正
- 所以从表达式中我们可以看到动涡盘的第一项都是负的，至于为什么第二项还有一个Δθ呢？这是因为保证在一开始装配的时候就能保持啮合的状态

动涡盘各点坐标

$$x_{oi} = -r_b\left(\cos\phi + (\phi - \phi_{i0})\sin\phi\right) + r_o\cos\left(\theta_\Delta - \theta\right)$$

$$y_{oi} = -r_b\left(\sin\phi - (\phi - \phi_{i0})\cos\phi\right) + r_o\sin\left(\theta_\Delta - \theta\right)$$

$$x_{oo} = -r_b\left(\cos\phi + (\phi - \phi_{o0})\sin\phi\right) + r_o\cos\left(\theta_\Delta - \theta\right)$$

$$y_{oo} = -r_b\left(\sin\phi - (\phi - \phi_{o0})\cos\phi\right) + r_o\sin\left(\theta_\Delta - \theta\right)$$

2.2 求解关于初始装配条件下待定系数后的动涡盘等效曲柄转角

1) 求解求Δθ

- 所以应该则么求Δθ呢？可以在初始状态的时候，使θ=0，动静涡盘在φie下啮合

动静涡盘的啮合角相差180°

$$x:\begin{cases} r_b\left(\cos\left(\phi_e-\theta\right)+\left(\phi_e-\theta-\phi_{i0}\right)\sin\left(\phi_e-\theta\right)\right) \\ =-r_b\left(\cos\left(\phi_e-\theta\boxed{-\pi}\right)+\left(\phi_e-\theta\boxed{-\pi}-\phi_{o0}\right)\sin\left(\phi_e-\theta\boxed{-\pi}\right)\right)+r_o\cos\left(\theta_\Delta-\theta\right) \end{cases}$$
$$y:\begin{cases} r_b\left(\sin\left(\phi_e-\theta\right)-\left(\phi_e-\theta-\phi_{i0}\right)\cos\left(\phi_e-\theta\right)\right) \\ =-r_b\left(\sin\left(\phi_e-\theta\boxed{-\pi}\right)-\left(\phi_e-\theta\boxed{-\pi}-\phi_{o0}\right)\cos\left(\phi_e-\theta\boxed{-\pi}\right)\right)+r_o\sin\left(\theta_\Delta-\theta\right) \end{cases}. \tag{4.11}$$

Setting θ equal to zero and solving for θ_Δ yields

$$\boxed{\theta_\Delta=\phi_{ie}-\frac{\pi}{2}.} \tag{4.12}$$

Thus, the coordinates of the involutes for the orbiting scroll are given by

$$\begin{aligned} x_{oi}&=-r_b\left(\cos\phi+\left(\phi-\phi_{i0}\right)\sin\phi\right)+r_o\cos\left(\phi_{ie}-\tfrac{\pi}{2}-\theta\right) \\ y_{oi}&=-r_b\left(\sin\phi-\left(\phi-\phi_{i0}\right)\cos\phi\right)+r_o\sin\left(\phi_{ie}-\tfrac{\pi}{2}-\theta\right) \\ x_{oo}&=-r_b\left(\cos\phi+\left(\phi-\phi_{o0}\right)\sin\phi\right)+r_o\cos\left(\phi_{ie}-\tfrac{\pi}{2}-\theta\right) \\ y_{oo}&=-r_b\left(\sin\phi-\left(\phi-\phi_{o0}\right)\cos\phi\right)+r_o\sin\left(\phi_{ie}-\tfrac{\pi}{2}-\theta\right). \end{aligned} \tag{4.13}$$

- 求解关于初始装配条件下待定系数后的动涡盘等效曲柄转角
  theta_m = geo.phi_fie - theta + 3.0*pi/2.0 = geo.phi_fie - theta - pi/2.0

2.3 啮合角求解

1) 啮合角求解

各啮合点展开角计算

$$\phi_{k,fi}=\phi_{fie}-\theta-(k-1)\,2\pi$$

$$\phi_{k,oo}=\phi_{fie}-\theta-(k-1)\,2\pi-\pi.$$

$$\phi_{k,fo}=\phi_{fie}-\theta-(k-1)\,2\pi-\pi$$

$$\phi_{k,oi}=\phi_{fie}-\theta-(k-1)\,2\pi.$$

2.4 排气角

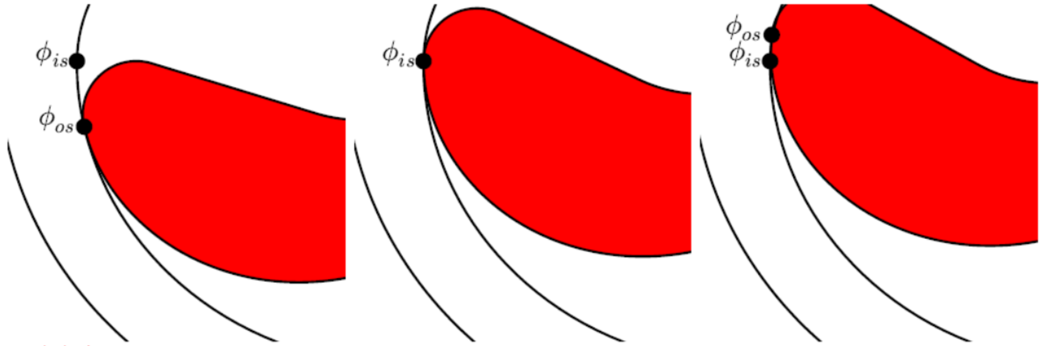
1) 排气角不充分条件和最大腔室对数

- 其实本质上来讲排气角的条件是啮合角不得小于展开开始角，用这个原则来判断，静涡盘内壁面啮合角φfic=φe-θ-

2πNc;动涡盘外壁面啮合角φfic=φe-π-θ-2πNc;

- 如果采用静涡盘内壁面啮合角φfic = φe-θ-2πNc，则由于φfic > φfis，即φfis < φe-θ-2πNc
- 如果采用动涡盘内壁面啮合角φoic = φe-π-θ-2πNc，则由于φoic > φois，即φois < φe-π-θ-2πNc

加减π是为了把动静涡盘的啮合点转移到相同的局部坐标系中，这里是为了转移到动涡盘的局部坐标中
展开角逆时针为负，所以当φos不断减少的时候，说明转动正在进行，直到啮合点与开始角相等



啮合点 把静涡盘坐标转移到动涡盘

(a) $\phi_{os} > \phi_{is} - \pi$ (b) $\phi_{os} = \phi_{is} - \pi$ (c) $\phi_{os} < \phi_{is} - \pi$ (Collision)

$$\phi_{os} = \phi_{ie} - \theta - 2\pi N_c - \pi$$

这里的os代表的是动涡盘 这里的ie代表的是静涡盘 Nc代表涡旋展开了多少圈
或者是多少对腔室

2) 然后利用该不充分条件和最大腔室对数计算排气角

- floor()函数代表向下取整

$$N_{c,max} = floor\left(\frac{\phi_{ie} - \phi_{os} - \pi}{2\pi}\right) \tag{4.16}$$

向下取整

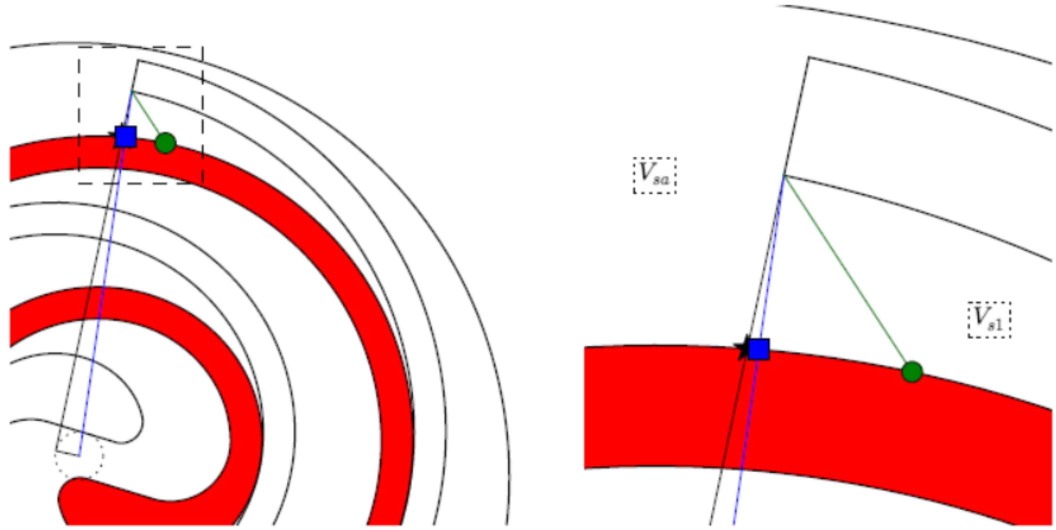
where the $\boxed{\textit{floor()} \text{ function rounds down the argument to the nearest integer}}$. Thus, the $\boxed{\text{discharge angle}}$ of the compressor can determined from

$$\theta_d = \phi_{ie} - \phi_{os} - 2\pi N_{c,max} - \pi. \tag{4.17}$$

2.5 吸气破入点

1) 吸气破入点的三种定义

- 第一种定义：固定在动涡盘外壁面，当地展开角固定为为φie-π
- 第二种定义：固定在动涡盘外壁面，该点在静涡盘展开结束角的法线与动涡盘外壁面的交点处
- 第三种定义：固定在动涡盘外壁面，该点在静涡盘内壁面展开结束角的终点跟静涡盘基圆圆心的连线，与动涡盘外壁面的交点处



(a) Overview ($\theta = \pi/2$)　　(b) Inset from Overview

第一种定义 $\phi_{s,sa} = \phi_{ie} - \pi.$ 固定点

第二种定义 $\phi_{s,sa} = \phi_e - \pi + \frac{r_o/r_b}{\phi_{ie} - \pi - \phi_{o0}} \sin\theta$ 切线

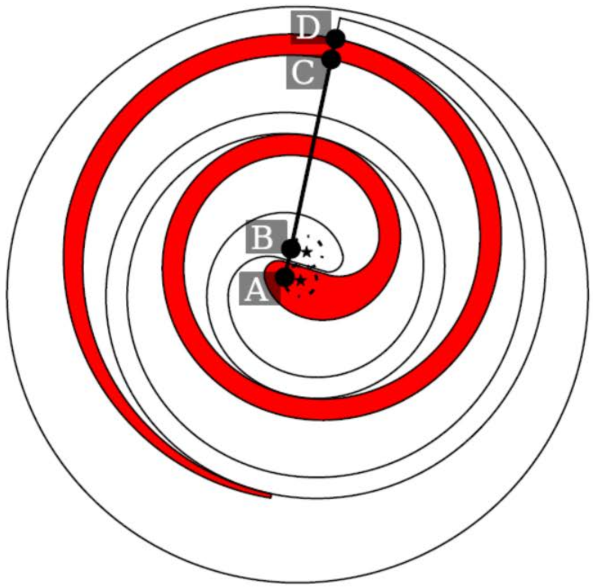
第三种定义 圆心

$$\begin{aligned}
\phi_{s,sa} &= \phi_e - \pi + B \\
B &= \frac{1}{2}\left((\phi_{o0} - \phi_{ie} + \pi) + \sqrt{(\phi_{o0} - \phi_{ie} + \pi)^2 - 4D}\right) \\
D &= \frac{r_o}{r_b}\frac{(\phi_{i0} - \phi_{ie})\sin\theta - \cos\theta + 1}{\phi_{ie} - \phi_{i0}}
\end{aligned}$$

2.6 公转半径和壁厚

1) ro orbiting radius 公转半径

- 公转半径只是涡盘基圆半径和内外表面发生角的函数



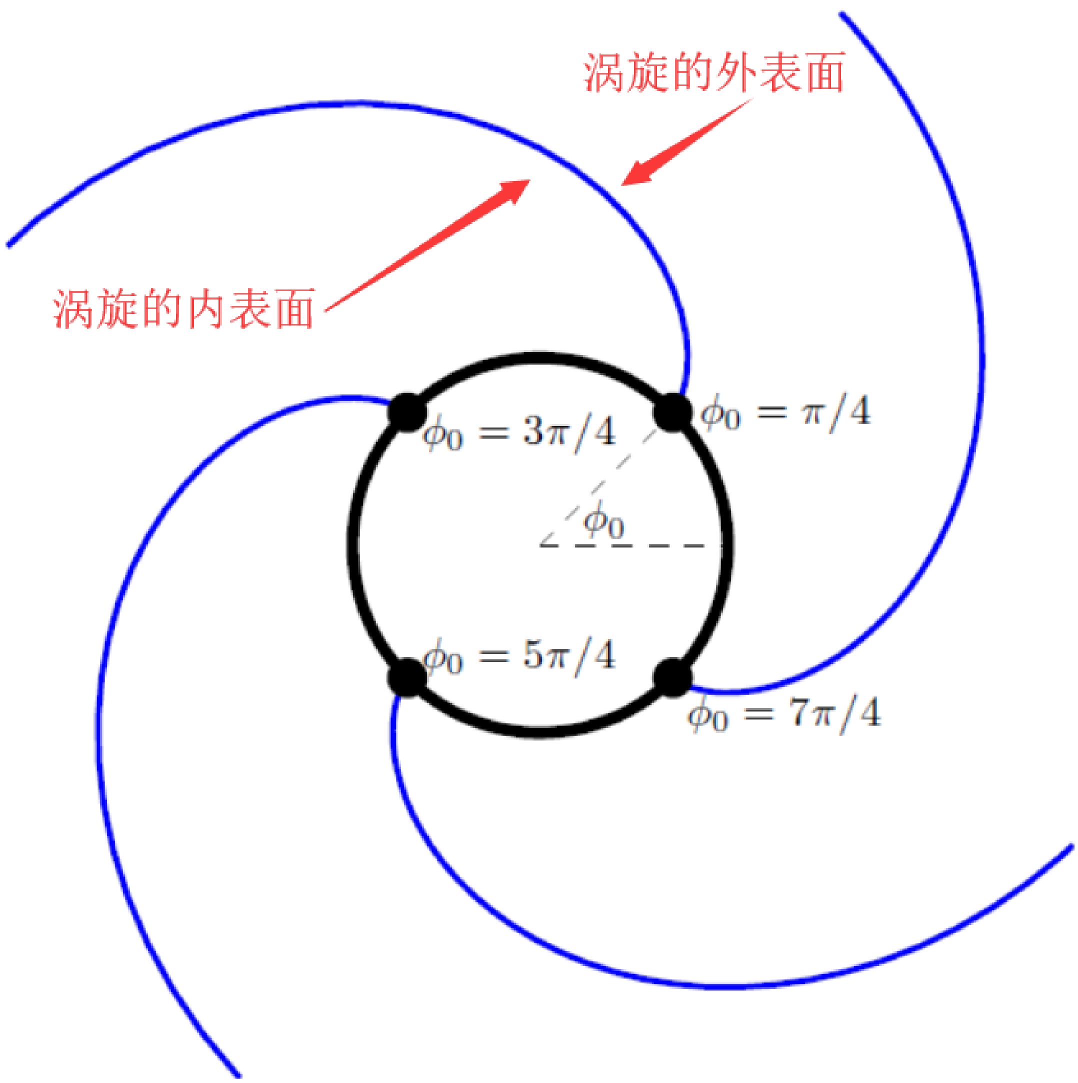
$$r_o = (\overline{AD} - \overline{BC}) - \overline{CD}$$

$$r_o = (r_b\left[\phi_{ie} - \phi_{i0}\right] - r_b\left[(\phi_{ie} - \pi) - \phi_{i0}\right]) - t_s$$

$$r_o = r_b\pi - t_s = r_b(\pi - \phi_{i0} + \phi_{o0}).$$

1) t 壁厚

- t The thickness of the scroll 涡旋的厚度 t=ts = rb*(phi_i0 - phi_o0)，涡旋盘厚度通常为5毫米量级。



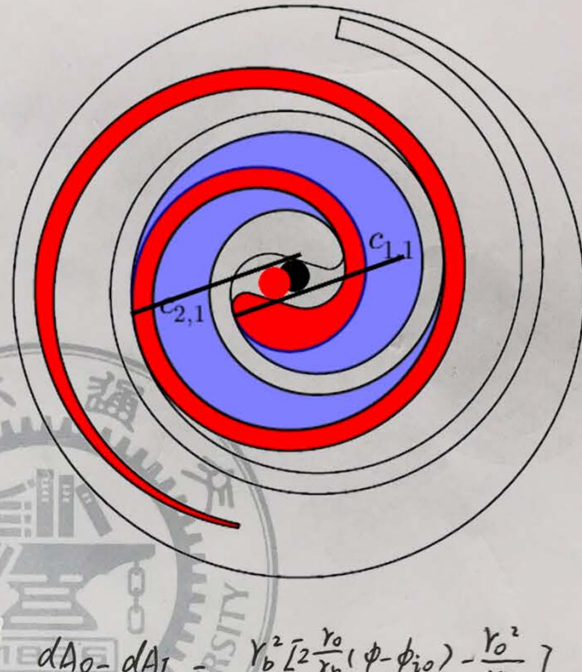
2.7 工作腔面积的计算

1) 利用刘老师书上的方法，但是壁厚这里我采用了文献的方法

瞬时工作腔的面积、结合文献与刘振全老师书上的做法——平移法

$$dA_o = \frac{r_b^2(\phi-\phi_{io})^2}{2}d\phi$$

$$dA_I = \frac{r_b^2(\phi-\phi_{io}')^2}{2}d\phi$$



其中关于 ϕ_{io}' 的求法如下，
由于 ϕ_{io}' 是动涡盘经过平移与静涡盘基圆重合后的等效起始角
则根据壁厚的计算式，有

$$r_o = r_b(\phi_{io}' - \phi_{io})$$

$$\therefore \phi_{io}' = \frac{r_o}{r_b} + \phi_{io}$$

$$\therefore dA_I = \frac{r_b^2(\phi-\phi_{io}-\frac{r_o}{r_b})^2}{2}d\phi$$

$$dA_o - dA_I = \frac{r_b^2[2\frac{r_o}{r_b}(\phi-\phi_{io}) - \frac{r_o^2}{r_b^2}]}{2}d\phi = [r_o r_b(\phi-\phi_{io}) - r_o^2/2]d\phi$$

此时动涡盘跟就转化为静涡盘的坐标了。

∴工作腔的面积

$$A = \int_{\theta_1}^{\theta_2}[r_o r_b(\phi-\phi_{io}) - \frac{r_o^2}{2}]d\phi$$

$$= \frac{1}{2}[r_o r_b\phi^2 - (r_o r_b\phi_{io} + r_o^2)\phi]\Big|_{\theta_1}^{\theta_2}$$

$$= -\frac{\pi r_o r_b}{2}[-\frac{\theta_2^2-\theta_1^2}{\pi} + \frac{\phi_{io}+r_o^2}{\pi}(\theta_2-\theta_1)]$$

$$= -\pi r_o r_b\frac{\theta_2-\theta_1}{2\pi}[-(\theta_1+\theta_2) + \phi_{io} + \frac{r_o}{r_b}]$$

由于工作腔一般是成对出现的.

$$\therefore A_{pair} = 2A$$

$$= -2\pi r_o r_b\frac{\theta_2-\theta_1}{2\pi}[-(\theta_1+\theta_2) + \phi_{io} + \frac{r_o}{r_b}]$$

$$体积, V = h_s A_{pair}$$

$$= -2\pi r_o r_b h_s\frac{\theta_2-\theta_1}{2\pi}[-(\theta_1+\theta_2) + \phi_{io} + \frac{r_o}{r_b}]$$

三、吸入腔

3.1 吸入腔外部面积体积质心计算

1) 吸入腔外部面积体积计算

Outer Surface

$$\because x_{fi} = r_b[\cos\phi + (\phi - \phi_{io})\sin\phi]$$

$$y_{fi} = r_b[\sin\phi - (\phi - \phi_{io})\cos\phi]$$

$\longrightarrow$ 引理 $dA = \frac{1}{2}\left(x\frac{dy}{d\phi} - y\frac{dx}{d\phi}\right)d\phi$ (Kreyszig, 2006)

$$\frac{dy}{d\phi} = r_b\{\cos\phi - [\cos\phi + (-\sin\phi)(\phi - \phi_{io})]\} = r_b(\phi - \phi_{io})\sin\phi$$

$$\frac{dx}{d\phi} = r_b\{-\sin\phi + \sin\phi + (\phi - \phi_{io})\cos\phi\} = r_b(\phi - \phi_{io})\cos\phi$$

$$dA = \frac{1}{2}\left(x\frac{dy}{d\phi} - y\frac{dx}{d\phi}\right)d\phi = \frac{r_b(\phi - \phi_{io})}{2}(x\sin\phi - y\cos\phi)\,d\phi$$

$$= \frac{r_b^2(\phi - \phi_{io})}{2}\{[\cos\phi\sin\phi + (\phi - \phi_{io})\sin^2\phi] - [\sin\phi\cos\phi - (\phi - \phi_{io})\cos^2\phi]\}d\phi$$

$$= \frac{1}{2}r_b^2(\phi - \phi_{io})^2\,d\phi$$

$$\therefore A_{o,s_1} = \left.\frac{r_b^2(\phi - \phi_o)^3}{6}\right|_{\phi_1}^{\phi_2}$$

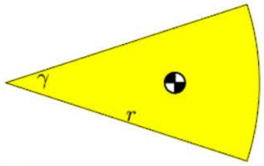
$\phi_{ie} - \theta$

而对于吸入腔的静涡盘内部壁面，从最外围啮合角到最终展开角ϕ_{ie}

$$\text{则体积}\ V_{o,s_1} = \left.\frac{h_s r_b^2(\phi - \phi_{io})^3}{6}\right|_{\phi_{ie} - \theta}^{\phi_{ie}} = \frac{h_s r_b^2[(\phi_{ie} - \phi_{io})^3 - (\phi_{ie} - \theta - \phi_{io})^3]}{6}$$

$$\frac{dV_{o,s_1}}{d\theta} = \frac{h_s r_b^2}{2}(\phi_{ie} - \theta - \phi_{io})^2$$

2) 吸入腔静涡旋质心计算



$$dA = d\,\tfrac{1}{2}\theta r^2 = \theta r\,dr$$

$$\tfrac{1}{2}\theta r^2$$

$$dA = \tfrac{1}{2} r\,d\theta \cdot r = \frac{r^2}{2}d\theta = \tfrac{1}{2}r^2 d\theta$$

$$c_x = \frac{\int_{\phi_1}^{\phi_2}\left(\frac{2}{3}x + \frac{1}{3}x_p\right)dA}{\int_{\phi_1}^{\phi_2} dA}$$

$$c_y = \frac{\int_{\phi_1}^{\phi_2}\left(\frac{2}{3}y + \frac{1}{3}y_p\right)dA}{\int_{\phi_1}^{\phi_2} dA}.$$

$$\theta_m = \frac{\int_{\theta_1}^{\theta_2}\frac{1}{2}r^2\cdot\theta\,d\theta}{\int_{\theta_1}^{\theta_2}\frac{1}{2}r^2 d\theta} = \frac{\int_{\theta_1}^{\theta_2}\theta\,d\theta}{\int_{\theta_1}^{\theta_2}d\theta} = \frac{\frac{1}{2}\theta^2\Big|_{\theta_1}^{\theta_2}}{\theta\Big|_{\theta_1}^{\theta_2}} = \frac{\frac{1}{2}\,\theta_2^2-\theta_1^2}{\theta_2-\theta_1} = \frac{1}{2}(\theta_1+\theta_2)$$

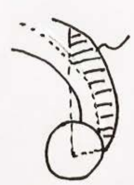
$$r_m = \frac{\int_{r_1}^{r_2}\theta r^2 dr}{\int_{r_1}^{r_2}\theta r\,dr} = \frac{\frac{1}{3}r^3\Big|_{r_1}^{r_2}}{\frac{1}{2}r^2\Big|_{r_1}^{r_2}} = \frac{2}{3}\,\frac{r_2^3-r_1^3}{r_2^2-r_1^2} = \frac{2}{3}\,\frac{r_1^2+r_1r_2+r_2^2}{r_1+r_2}$$

当 $r_1=0$. $\theta_1=0$ 时. $\theta_m = \frac{1}{2}\theta_2$ $r_m = \frac{2}{3}r_2$

- 密度偏移，要注意的一点是求质心坐标的时候，分子里面的坐标要求是微元的重心坐标，对于矩形微元而言重心坐标即在其中心，但是在扇形微元的时候，重心在其半径的2/3处的地方，造成了所谓的“密度偏移”

质心坐标、

$$\begin{cases} C_x = \dfrac{\int_{\phi_1}^{\phi_2} x_G\,dA}{\int_{\phi_1}^{\phi_2} dA} = \dfrac{\int_{\phi_1}^{\phi_2}\frac{2}{3}x\,dA}{\int_{\phi_1}^{\phi_2} dA} \\ C_y = \dfrac{\int_{\phi_1}^{\phi_2} y_G\,dA}{\int_{\phi_1}^{\phi_2} dA} = \dfrac{\int_{\phi_1}^{\phi_2}\frac{2}{3}y\,dA}{\int_{\phi_1}^{\phi_2} dA} \end{cases}$$



微元 $dA(\phi) = A(\phi+d\phi) - A(\phi)$

每个微元 $= A_\phi\,d\phi$

的质心在 $\frac{2}{3}$ 处的位置 $r_G = \frac{2}{3}r$

相当于 $\begin{cases} x_G = \frac{2}{3}x \\ y_G = \frac{2}{3}y \end{cases}$

即密度偏移

$$\frac{2}{3}x\,dA = \frac{r_b^3}{3}(\phi-\phi_0)^2(\cos(\phi) + (\phi-\phi_0)\sin\phi)$$

$$\frac{2}{3}y\,dA = \frac{r_b^3}{3}(\phi-\phi_0)^2(\sin(\phi) - (\phi-\phi_0)\cos\phi).$$

$$f_{xA}(\phi,\phi_0) = \frac{r_b^3}{3}\left[4\left((\phi-\phi_0)^2-2\right)\sin\phi + (\phi_0-\phi)\left((\phi-\phi_0)^2-8\right)\cos\phi\right]\Big|_{\varphi 1}^{\varphi 2}$$

$$f_{yA}(\phi,\phi_0) = \frac{r_b^3}{3}\left[(\phi_0-\phi)\left((\phi-\phi_0)^2-8\right)\sin\phi - 4\left((\phi-\phi_0)^2-2\right)\cos\phi\right]\Big|_{\varphi 1}^{\varphi 2}$$

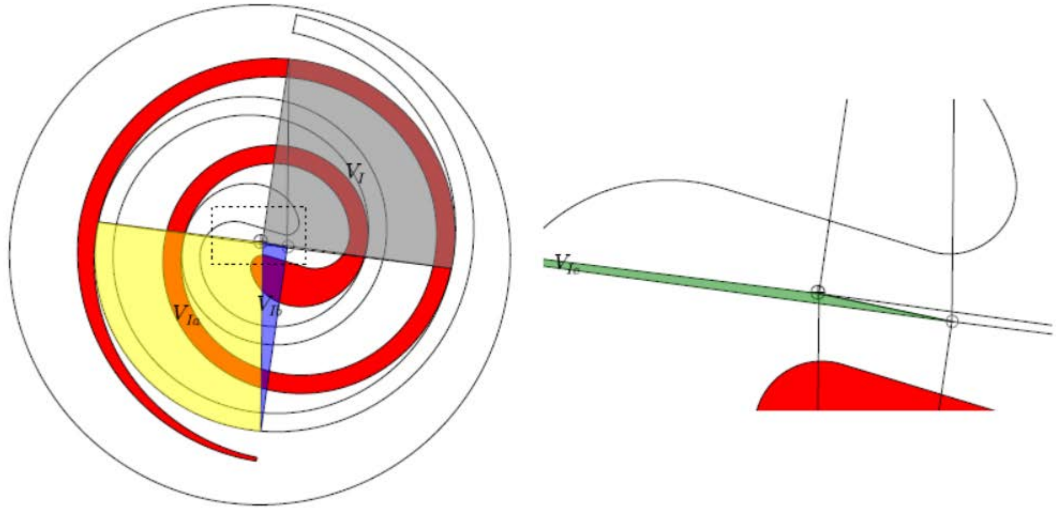
$$c_{x,O,s1} = \frac{h_s}{V_{O,s1}}\left[f_{xA}(\phi_{ie},\phi_{i0}) - f_{xA}(\phi_{ie}-\theta,\phi_{i0})\right]$$

$$c_{y,O,s1} = \frac{h_s}{V_{O,s1}}\left[f_{yA}(\phi_{ie},\phi_{i0}) - f_{yA}(\phi_{ie}-\theta,\phi_{i0})\right].$$

3.2 吸入腔内部面积体积质心计算

1) 吸入腔内部面积体积质心计算

- 吸入腔内部涡旋面积体积质心计算



(a) Decomposed $V_{I\,s1}$ showing volumes $V_{Ia,s1}$ and $V_{Ib,s1}$

(b) Inset from decomposition showing volume $V_{Ic,s1}$

$$V_{I,s1} = V_{Ia,s1} + V_{Ib,s1} - V_{Ic,s1}$$

with the derivative with respect to the crank angle of

$$\frac{dV_{I,s1}}{d\theta} = \frac{dV_{Ia,s1}}{d\theta} + \frac{dV_{Ib,s1}}{d\theta} - \frac{dV_{Ic,s1}}{d\theta}.$$

别忘了加上这两项

$$c_{x,I,s1} = -\frac{c_{x,Ia,s1}V_{Ia,s1} + c_{x,Ib,s1}V_{Ib,s1} - c_{x,Ic,s1}V_{Ic,s1}}{V_{Ia,s1} + V_{Ib,s1} - V_{Ic,s1}} + \boxed{r_o\cos(\phi_{ie} - \pi/2 - \theta)}$$

$$c_{y,I,s1} = -\frac{c_{y,Ia,s1}V_{Ia,s1} + c_{y,Ib,s1}V_{Ib,s1} - c_{y,Ic,s1}V_{Ic,s1}}{V_{Ia,s1} + V_{Ib,s1} - V_{Ic,s1}} + \boxed{r_o\sin(\phi_{ie} - \pi/2 - \theta)}$$

2) 吸入腔内部面积体积质心计算**PartA**

$$V_{Ia,s1} = \frac{h_s r_b^2}{6}\left[(\boxed{\phi_{ie} - \pi + B} - \phi_{o0})^3 - (\boxed{\phi_{ie} - \pi - \theta} - \phi_{o0})^3\right]$$

开始角

过程角

where the derivative of the volume V_{Ia} is given by

$$\frac{dV_{Ia,s1}}{d\theta} = \frac{h_s r_b^2}{2}\left[(\boxed{\phi_{ie} - \pi + B} - \phi_{o0})^2 B' + (\boxed{\phi_{ie} - \pi - \theta} - \phi_{o0})^2\right]$$

质心坐标

$$c_{x,Ia,s1} = \frac{h_s}{V_{Ia,s1}}\left[f_{xA}(\phi_{ie} - \pi + B, \phi_{o0}) - f_{xA}(\phi_{ie} - \pi - \theta, \phi_{o0})\right]$$

$$c_{y,Ia,s1} = \frac{h_s}{V_{Ia,s1}}\left[f_{yA}(\phi_{ie} - \pi + B, \phi_{o0}) - f_{yA}(\phi_{ie} - \pi - \theta, \phi_{o0})\right]$$

3) 吸入腔内部涡旋涡旋面积体积质心计算**PartB**

$$c_{x,Ib,s1} = \frac{-r_b(B-\phi_{o0}+\phi_e-\pi)\sin(B+\phi_e) - r_b\cos(B+\phi_e) - r_o\sin(\theta-\phi_e)}{3}$$

$$c_{y,Ib,s1} = \frac{-r_b\sin(B+\phi_e) + r_b(B-\phi_{o0}+\phi_e-\pi)\cos(B+\phi_e) - r_o\cos(\theta-\phi_e)}{3}.$$

$$V_{Ib,s1} = \frac{h_s r_b r_o}{2}\left[(\phi_{ie}-\pi+B-\phi_{o0})\sin(B+\theta) + \cos(B+\theta)\right]$$

with the derivative of volume $V_{Ib,s1}$ with respect to the crank angle of

$$\frac{dV_{Ib,s1}}{d\theta} = \frac{h_s r_b r_o(B'+1)}{2}\left[(\phi_{ie}-\pi+B-\phi_{o0})\cos(B+\theta) - \sin(B+\theta)\right].$$

质心计算

$$x_{s-sa} = r_b\left(\cos(\phi_{ie}-\pi+B) + (\phi_{ie}-\pi+B-\phi_{o0})\sin(\phi_{ie}-\pi+B)\right)$$

$$y_{s-sa} = r_b\left(\sin(\phi_{ie}-\pi+B) - (\phi_{ie}-\pi+B-\phi_{o0})\cos(\phi_{ie}-\pi+B)\right).$$

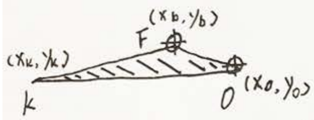
4) 吸入腔内部涡旋涡旋面积体积质心计算PartC

$$V_{Ic,s1} = \frac{h_s r_b r_o}{2}$$

where the derivative of the volume $V_{Ic,s1}$ with respect to the crank angle is

$$\frac{dV_{Ic,s1}}{d\theta} = 0.$$

求 $A_{Ic,s1}$　　已知 $(x_b, y_b) = (0, 0)$



$$\therefore A_{Ic,s1} = \frac{1}{2}\vec{FK} \times \vec{FO} = \frac{1}{2}(x_k, y_k) \times (x_o, y_o) = \frac{x_k y_o - y_k x_o}{2}$$

又
$$\begin{cases} x_k = r_b[\cos(\phi_{ie} - \pi - \theta) + (\phi_{ie} - \pi - \theta - \phi_{o0})\sin(\phi_{ie} - \pi - \theta)] \\ y_k = r_b[\sin(\phi_{ie} - \pi - \theta) - (\phi_{ie} - \pi - \theta - \phi_{o0})\cos(\phi_{ie} - \pi - \theta)] \end{cases}$$

$$\begin{cases} x_o = r_o \cos(\phi_{ie} - \frac{\pi}{2} - \theta) \\ y_o = r_o \sin(\phi_{ie} - \frac{\pi}{2} - \theta) \end{cases}$$

设 $\alpha = \cos\phi_{ie} - \pi - \theta$　则 $\phi_{ie} - \frac{\pi}{2} - \theta = \alpha + \frac{\pi}{2}$

得
$$\begin{cases} x_k = r_b[\cos\alpha + (\alpha - \phi_{o0})\sin\alpha] \\ y_k = r_b[\sin\alpha - (\alpha - \phi_{o0})\cos\alpha] \end{cases}$$

$$\begin{cases} x_o = r_o \cos(\alpha + \frac{\pi}{2}) \\ y_o = r_o \sin(\alpha + \frac{\pi}{2}) \end{cases}$$

$$\therefore A_{Ic,s1} = \frac{x_k y_o - y_k x_o}{2} = \frac{r_b r_o}{2}\left[\begin{array}{l}[\cos\alpha + (\alpha - \phi_{o0})\sin\alpha]\sin(\alpha + \frac{\pi}{2}) \\ -[\sin\alpha - (\alpha - \phi_{o0})\cos\alpha]\cos(\alpha + \frac{\pi}{2})\end{array}\right]$$

$$= \frac{1}{2} r_b r_o [\cos^2\alpha + (\alpha - \phi_{o0})\sin\alpha\cos\alpha + \sin^2\alpha - (\alpha - \phi_{o0})\cos\alpha\sin\alpha]$$

$$= \frac{1}{2} r_b r_o$$

$$\therefore V_{Ic,s1} = A_{Ic,s1} h_s = \frac{h_s r_b r_o}{2}$$

$$c_{x,Ic,s1} = \frac{r_b\left(-\theta - \phi_{o0} + \phi_e - \pi\right)\sin\left(\theta - \phi_e\right) - r_o \sin\left(\theta - \phi_e\right) - r_b \cos\left(\theta - \phi_e\right)}{3}$$

对于求三角形的重心，直接3个顶点坐标求平均即可

$$c_{y,Ic,s1} = \frac{r_b \sin\left(\theta - \phi_e\right) + r_b\left(-\theta - \phi_{o0} + \phi_e - \pi\right)\cos\left(\theta - \phi_e\right) - r_o \cos\left(\theta - \phi_e\right)}{3}.$$

5) 吸入腔总的面积体积质心和排量的计算

- 吸入腔总的面积体积质心计算
- 压缩机排量定义为动涡盘转一圈后的瞬间吸入腔的总体积，但是这个我没有推过，不知道排量的公式对还是错

Overall Calculations

Combining the constituent volumes which form the suction chamber s_1 yields

$$V_{s1} = V_{O,s1} - V_{I,s1} \tag{4.84}$$

and the derivative of this volume with respect to the crank angle is given by

$$\frac{dV_{s1}}{d\theta} = \frac{dV_{O,s1}}{d\theta} - \frac{dV_{I,s1}}{d\theta}. \tag{4.85}$$

质心计算

$$c_{x,s1} = \frac{c_{x,O,s1}V_{O,s1} - c_{x,I,s1}V_{I,s1}}{V_{s1}}$$

$$c_{y,s1} = \frac{c_{y,O,s1}V_{O,s1} - c_{y,I,s1}V_{I,s1}}{V_{s1}}.$$

对称的另一半吸入腔的计算

$$V_{s2} = V_{s1}$$

$$\frac{dV_{s2}}{d\theta} = \frac{dV_{s1}}{d\theta}$$

while the centroid of s_2 is the centroid of s_1 mirrored through the point of symmetry of the scroll set $\left(\frac{r_o}{2}\cos(\phi_{ie} - \pi/2 - \theta), \frac{r_o}{2}\sin(\phi_{ie} - \pi/2 - \theta)\right)$ yielding

质心关于曲柄销圆心的中心对称

$$c_{x,s2} = -c_{x,s1} + r_o\cos(\phi_{ie} - \pi/2 - \theta) \tag{4.90}$$

$$c_{y,s2} = -c_{y,s1} + r_o\sin(\phi_{ie} - \pi/2 - \theta). \tag{4.91}$$

压缩机排量定义为动涡盘转一圈后的瞬间吸入腔的总体积

$$V_{disp} = -2\pi h_s r_b r_o (3\pi - 2\phi_{ie} + \phi_{i0} + \phi_{o0}).$$

3.3 气体力与力矩

1) 气体力和力矩的推导

$$\begin{aligned} d\mathbf{F}_{oi} &= -pdA\sin(\phi)\hat{i} + pdA\cos(\phi)\hat{j} \\ d\mathbf{F}_{oo} &= pdA\sin(\phi)\hat{i} - pdA\cos(\phi)\hat{j} \end{aligned}$$

涡盘齿高

$$dA = h_s r_b(\phi - \phi_0)$$

$$d\mathbf{F}_{oi} = -ph_s r_b\left[(\phi - \phi_{i0})\sin(\phi)\hat{i} - (\phi - \phi_{i0})\cos(\phi)\hat{j}\right]$$

2) 气体力矩方向的推导

- 中间是叉乘，rO is a vector from (xO, yO)，φie-pi/2之前也讲过，是动涡盘和曲柄销的安装角度

$$x_{\bigcirc} = r_o \cos(\phi_{ie} - \pi/2 - \theta) \qquad d\mathbf{M}_{\bigcirc,oi} = \mathbf{r}_{\bigcirc,oi} \times d\mathbf{F}_{oi}$$

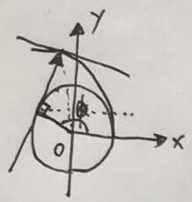
$$y_{\bigcirc} = r_o \sin(\phi_{ie} - \pi/2 - \theta). \qquad d\mathbf{M}_{\bigcirc,oo} = \mathbf{r}_{\bigcirc,oo} \times d\mathbf{F}_{oo}$$

3) 证明过程

- 吸气腔水平面力和力矩的计算 但是在计算力矩的时候有点奇怪，按道理来讲，应该有一个因子是关于回转半径ro的，但是这里没有，只有rb平方。

求力F在吸入腔 S_1, S_2. S_1为动外静内 S_2为动内静外.

①先求压力P在壁面上的分量方向



对于静涡盘

$$\vec{f}_{fi}(\phi) = \vec{i}\cos\left[\frac{\pi}{2}-(\pi-\phi)\right] + \vec{j}\sin\left(\frac{\pi}{2}-(\pi-\phi)\right)$$
$$= \vec{i}\sin\phi - \vec{j}\cos\phi$$
$$\vec{f}_{fo} = \vec{f}_{fi}(\phi+\pi) = -\vec{i}\sin\phi + \vec{j}\cos\phi$$

对于动涡盘，方向上与静涡盘坐标相差π相位. 而与其相对壁面所受力互为相反力.

$$\therefore \begin{cases} S_2: \vec{f}_{oi}(\phi) = \vec{f}_{fo}(\phi+\pi+\pi) = -\vec{i}\sin\phi + \vec{j}\cos\phi \\ S_1: \vec{f}_{oo}(\phi) = \vec{f}_{fi}(\phi+\pi+\pi) = \vec{i}\sin\phi - \vec{j}\cos\phi \end{cases}$$

$$对于\begin{cases} S_1: d\vec{F}_{oo}(\phi) = \vec{P}_{s1}dA = h_s P_{s1}(\vec{i}\sin\phi - \vec{j}\cos\phi)dl \\ S_2: d\vec{F}_{oi}(\phi) = \vec{P}_{s2}dA = h_s P_{s2}(-\vec{i}\sin\phi + \vec{j}\cos\phi)dl \end{cases}$$

$$而\ dl = \underbrace{r_b(\phi-\phi_0)}_{半径}\underbrace{d\phi}_{微分弧度}$$

$$\begin{cases} S_1: \dfrac{\vec{F}_{oo}}{P_{s1}} = h_s r_b(\phi-\phi_0)\displaystyle\int_{\phi_e-\pi-\theta}^{\phi_e-\pi+B}(\vec{i}\sin\phi - \vec{j}\cos\phi)d\phi \\ S_2: \dfrac{\vec{F}_{oi}}{P_{s2}} = h_s r_b(\phi-\phi_0)\displaystyle\int_{\phi_e-\theta}^{\phi_e}(-\vec{i}\sin\phi + \vec{j}\cos\phi)d\phi \end{cases}$$

$$而力臂\ \vec{rO} = r_o\cos(\phi_{ia}-\pi/2-\theta)\vec{i} + r_o\sin(\phi_{ia}-\pi/2-\theta)\vec{j}$$

$$\begin{cases} MO_{s1} = \displaystyle\int_{\phi_e-\pi-\theta}^{\phi_e-\pi+B}\left[\vec{rO}\times\frac{d\vec{F}_{oo}}{P_{s1}}\right]d\phi \\ MO_{s2} = \displaystyle\int_{\phi_e-\theta}^{\phi_e}\left[\vec{rO}\times\frac{dF_{oi}}{P_{s2}}\right]d\phi \end{cases}$$

which yields the moments

有点奇怪，按道理应该出现曲轴半径r0，但是这里没有出现

$$\frac{\mathbf{M}_{\bigcirc,s1}}{p_{s1}} = \frac{r_b^2 h_s\,(B-\theta-2\,\phi_{o0}+2\,\phi_e-2\,\pi)\,(B+\theta)}{2}\hat{k} \tag{4.99}$$

$$\frac{\mathbf{M}_{\bigcirc,s2}}{p_{s2}} = \frac{r_b^2 h_s\,\theta\,(\theta+2\,\phi_{i0}-2\,\phi_e)}{2}\hat{k}. \tag{4.100}$$

4) 吸气腔轴向力的计算

吸气腔气体压力，各处相等

$$\frac{\mathbf{F}_{z,s1}}{p_{s1}-p_{shell}}=\frac{V_{s1}}{h_s}$$

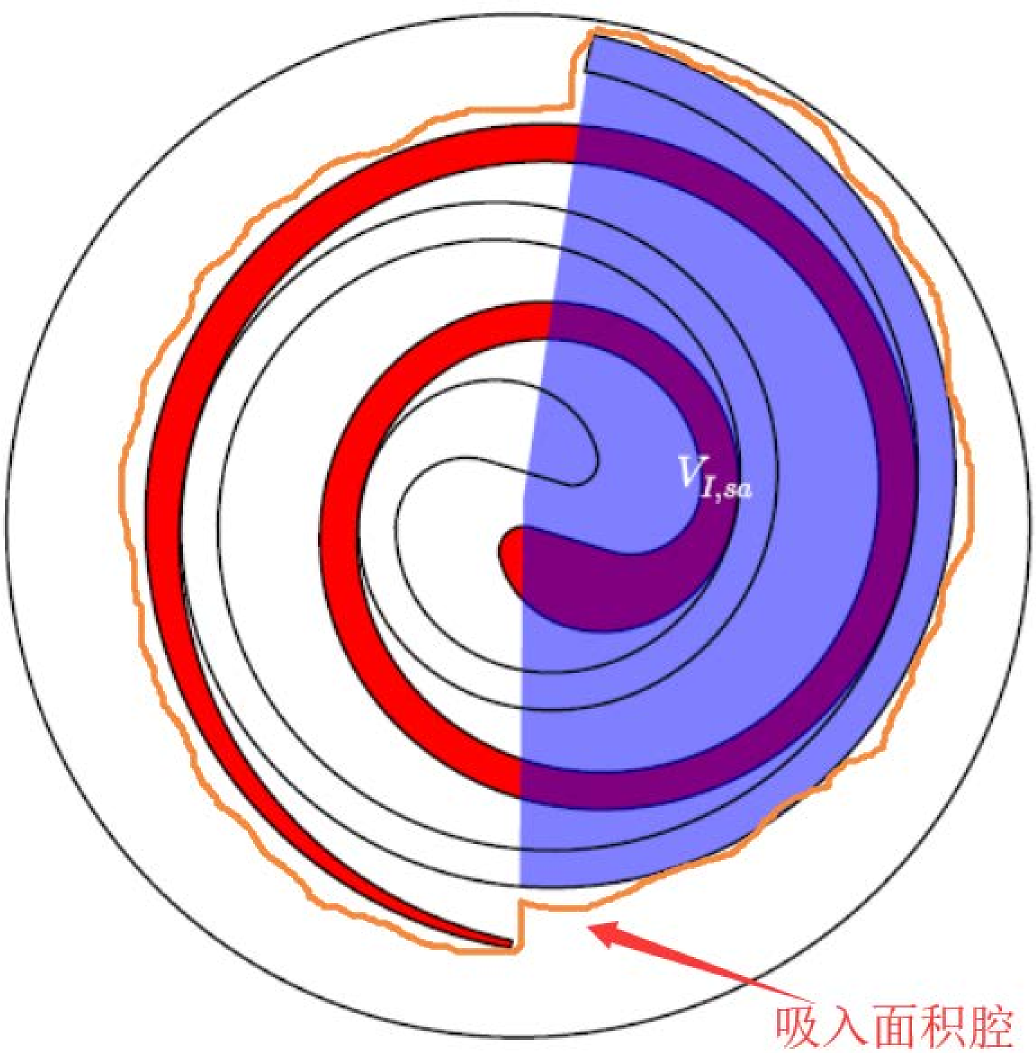
背压

$$\frac{\mathbf{F}_{z,s2}}{p_{s2}-p_{shell}}=\frac{V_{s2}}{h_s}$$

3.4 吸气面积腔

1) 定义

- 其实就是在花篮中涡旋盘以外的区域
- 吸气面积腔，其实这个腔并不是很重要，因为以来这里是用来引导进气，二来这个区域一般有有其他用于加工过程的金属占据着空间
- 可以大概认为可以使用对称面积*2的方法，但是这个方法不是绝对精确的，因为吸入破入角对于静涡盘和动涡盘来讲是不相同的



2) 求解面积体积力和力矩

$$V_{I,sa} = \frac{h_s r_b^2}{6}\left[(\phi_{ie} - \phi_{o0})^3 - (\phi_{ie} - \pi + B - \phi_{o0})^3\right]$$

$$\frac{dV_{I,sa}}{d\theta} = -\frac{h_s r_b^2}{2}(\phi_{ie} - \pi + B - \phi_{o0})^2 B'$$

花篮内壁半径

$$V_{sa} = \pi h_s \frac{D_{wall}^2}{4} - 2V_{I,sa}$$

$$\frac{dV_{sa}}{d\theta} = -2\frac{dV_{I,sa}}{d\theta} \tag{4.104}$$

where D_{wall} is the inner diameter of the shell into which the scroll set is placed.

Forces

The forces on the orbiting scroll from the *sa* chamber are given by

$$\frac{\mathbf{F}_{sa}}{p_{sa}} = h_s r_b \int_{\phi_e-\pi+B}^{\phi_e} \left[(\phi - \phi_{o0})\sin(\phi)\hat{i} - (\phi - \phi_{o0})\cos(\phi)\hat{j}\right] d\phi \tag{4.105}$$

which when integrated yield the net forces for the suction chambers given in Appendix B.4. The moment around the crank pin generated by the portion of the involute for the *sa* chamber is equal to

$$\frac{\mathbf{M}_{\bigcirc,sa}}{p_{sa}} = \int_{\phi_e-\pi+B}^{\phi_e} [\mathbf{r}_{\bigcirc,sa} \times \frac{d\mathbf{F}_{sa}}{p_{sa}}]d\phi \tag{4.106}$$

where the moments are taken around the point from Eqn. (4.40) which yields

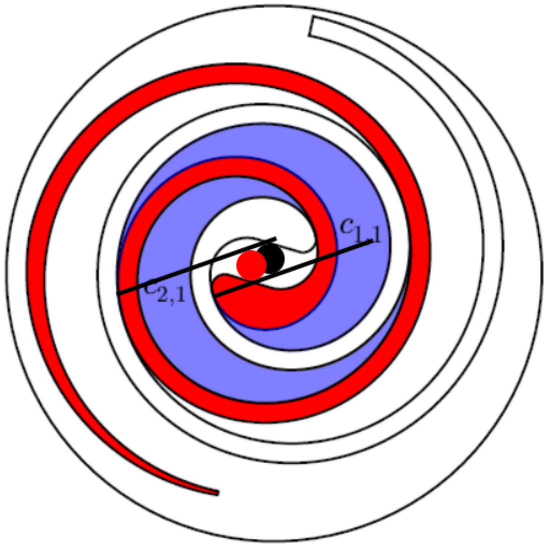
$$\frac{\mathbf{M}_{\bigcirc,sa}}{p_{sa}} = -\frac{r_b^2 h_s\,(B-\pi)\,(B - 2\,\phi_{o0} + 2\,\phi_e - \pi)}{2}\hat{k}.$$

这个就更加莫名奇妙了，不知道怎么推出来的

四、压缩腔

4.1 压缩腔面积体积计算

1) 文献中的方法，我对内部面积的求解方法心存怀疑



外部 $dA_O = \frac{r_b^2(\phi-\phi_{i0})^2}{2}d\phi$

内部多余的面积=

两根黑线和两个基圆围成的面积-动涡盘基圆的面积

内部 $dA_I = \frac{r_b^2(\phi-\phi_{o0})^2 - \boxed{r_b r_o(\phi-\phi_{o0})\cos(\theta-\phi_{ie}+\phi)}}{2}d\phi.$

For the α-th compression chamber, the volume is therefore given by

$$V_{c1,\alpha} = h_s \left[\int_{\phi_{ie}-\theta-2\pi\alpha}^{\phi_{ie}-\theta-(\alpha-1)2\pi} \frac{dA_O}{d\phi}d\phi - \int_{\phi_{ie}-\theta-2\pi\alpha-\pi}^{\phi_{ie}-\theta-(\alpha-1)2\pi-\pi} \frac{dA_I}{d\phi}d\phi \right]. \quad (4.111)$$

After integration and simplification, the volume of the α-th compression chamber is determined to be

$$V_{c1,\alpha} = -\pi h_s r_b r_o (2\theta + 4\pi\alpha - 2\phi_{ie} - \pi + \phi_{i0} + \phi_{o0}) \quad (4.112)$$

and the derivative of the volume of the compression chamber is given as

$$\frac{dV_{c1,\alpha}}{d\theta} = -2\pi h_s r_b r_o \quad (4.113)$$

2) 刘振全老师的方法，我对与直接在内部区域的展开角减**π**的做法心存怀疑

在公转中心位置处和工作位置处的任一时刻的工作腔，如图 3-17 所示，图为公转中心位置处的单个工作腔，b 图为工作位置处的单个工作腔。

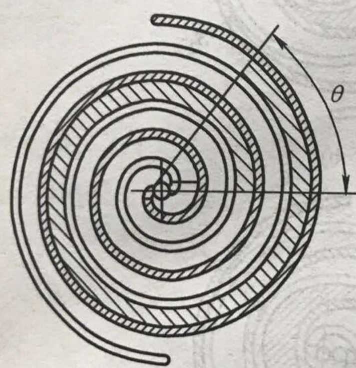


图 3-16　任意时刻的工作腔的轴向投影面积

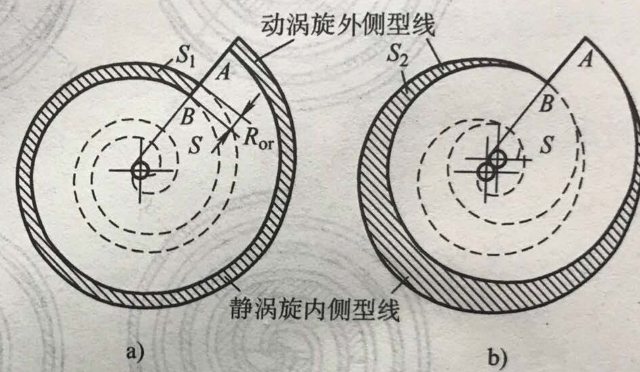


图 3-17　任意时刻的工作腔

任意时刻工作腔容积为，当 $\theta=\theta_0$时为进气容积，当 $\theta=\theta'+\theta_0$时为排气腔容积。任意时刻的工作腔容积为

$$V(\theta)=S(\phi_e-\theta,\ \phi_e-2\pi-\theta,\ \alpha,\ \pi-\alpha)\ 2h,\ 0\leqslant\theta\leqslant\theta' \tag{3-40}$$

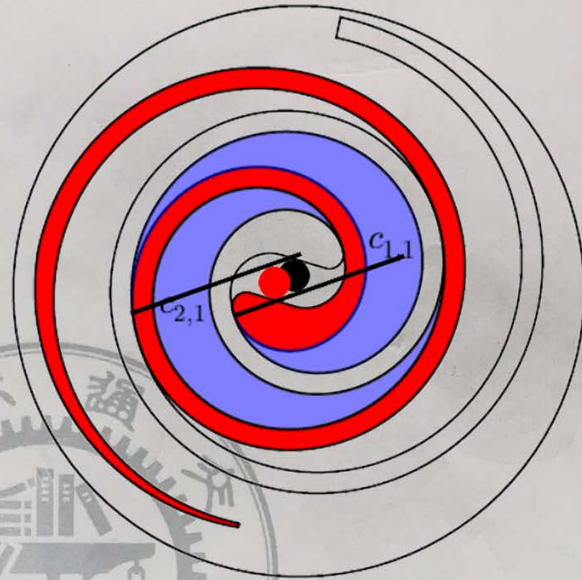
$$V(\theta)=2h\left[\frac{1}{6}R_b^2(\phi_e-\theta-\alpha)^3-\frac{1}{6}R_b^2(\phi_e-\theta-\pi+\alpha)^3\right]-\left[\frac{1}{6}R_b^2(\phi_e-2\pi-\theta-\alpha)^3-\frac{1}{6}R_b^2(\phi_e-3\pi-\theta+\alpha)^3\right] \tag{3-41}$$

3) 我把前面两种方法结合在一起后，做出的结果跟文献中的形式基本相同

瞬时工作腔的面积、结合文献与刘振全老师书上的做法—平移法

$$dA_O = \frac{r_b^2(\phi-\phi_{io})^2}{2}\,d\phi$$

$$dA_I = \frac{r_b^2(\phi-\phi'_{io})^2}{2}\,d\phi$$



其中关于 ϕ'_{io} 的求法如下，

由于 ϕ'_{io} 是动涡盘经过平移与静涡盘基圆重合后的等效起始角

则根据壁厚的计算式，有

$$r_o = r_b(\phi'_{io} - \phi_{io})$$

$$\therefore \phi'_{io} = \frac{r_o}{r_b} + \phi_{io}$$

$$\therefore dA_I = \frac{r_b^2(\phi - \phi_{io} - \frac{r_o}{r_b})^2}{2}\,d\phi$$

$$dA_O - dA_I = \frac{r_b^2\left[2\frac{r_o}{r_b}(\phi-\phi_{io}) - \frac{r_o^2}{r_b^2}\right]}{2}\,d\phi = \left[r_o r_b(\phi-\phi_{io}) - r_o^2/2\right]d\phi$$

此时动涡盘跟就转化为静涡盘的坐标了。

$$V_{c1,\alpha} = h_s\int_{\phi_{ie}-\theta-2\pi\alpha}^{\phi_{ie}-\theta-(\alpha-1)2\pi} \frac{dA_O - dA_I}{d\phi}\,d\phi$$

$$= h_s\int_{\phi_{ie}-\theta-2\pi\alpha}^{\phi_{ie}-\theta-(\alpha-1)2\pi}\left[r_o r_b(\phi-\phi_{io}) - r_o^2/2\right]d\phi$$

$$= -\pi h_s r_b r_o\left[2\theta + 4\pi\alpha - 2\phi_{ie} - \left(2\pi - \frac{r_o}{r_b}\right) + 2\phi_{io}\right]$$

$$\frac{dV_{c1,\alpha}}{d\theta} = -2\pi h_s r_b r_o$$

4.2 压缩比

- 文献中的方法，我试过是正确的

$$V_{c1,d} = -\pi h_s r_b r_o \left(-2\phi_{os} - 3\pi + \phi_{i0} + \phi_{o0}\right) \tag{4.114}$$

and thus, the built-in volume ratio, an important design parameter, is given by

$$V_{ratio} = \frac{V_{disp}}{2V_{c1,d}} = \frac{3\pi - 2\phi_{ie} + \phi_{i0} + \phi_{o0}}{-2\phi_{os} - 3\pi + \phi_{i0} + \phi_{o0}}. \tag{4.115}$$

- 我的方法，形式跟文献的基本一致

对于最里面压缩腔的体积

由展开角下限为 $\phi_{os} = \phi_e - \pi - 2\pi N_{c,max} - \theta$

这里 $N_{c,max}$ 用 α 压缩腔室指数表示

则 $\alpha = N_{c,max} = \dfrac{\phi_e - \pi - \theta - \phi_{os}}{2\pi}$ 代入 $V_{c,\alpha} = \int_{\phi_{ie}-\theta-2\pi\alpha}^{\phi_{ie}-\theta-2\pi(\alpha-1)} \left[r_o r_b(\phi - \phi_{io}) - \frac{r_o^2}{2}\right] d\phi$

得最里面压缩腔体积

$$V_{c1,d} = h_s \int_{\phi_{os}+\pi}^{\phi_{os}+3\pi} \left[r_o r_b(\phi - \phi_{io}) - \frac{r_o^2}{2}\right] d\phi$$

$$= -\pi r_o r_b h_s \left[-2\phi_{os} - \left(4\pi - \frac{r_o}{r_b}\right) + 2\phi_{io}\right]$$

对于吸入腔的体积

展开角从 $(\phi_{ie} - 2\pi) \longrightarrow \phi_{ie}$

$$V_{disp} = h_s A = h_s \int_{\phi_{ie}-2\pi}^{\phi_{ie}} r_o r_b(\phi - \phi_{io}) - \frac{r_o^2}{2}] d\phi$$

$$= -2\pi r_o r_b h_s \left[-(2\phi_{ie} - 2\pi) + 2\phi_{io} + \frac{r_o}{r_b}\right]$$

$$= -2\pi r_o r_b h_s \left[\left(2\pi + \frac{r_o}{r_b}\right) - 2\phi_{ie} + 2\phi_{io}\right.$$

∴ 压缩比

$$V_{ratio} = \frac{V_{disp}}{2V_{c1,d}} = \frac{\left(2\pi + \frac{r_o}{r_b}\right) - 2\phi_{ie} + 2\phi_{io}}{-2\phi_{os} - \left(4\pi - \frac{r_o}{r_b}\right) + 2\phi_{io}}$$

4.3 压缩腔的质心

- 关于压缩腔的质心的证明实在太麻烦了，这里我没有去验证

$$c_{x,c1,\alpha} = \frac{2h_s}{3} \frac{\int_{\phi_e-\theta-2\pi\alpha}^{\phi_e-\theta-(\alpha-1)2\pi} x_{oi} \frac{dA_I}{d\phi} d\phi - \int_{\phi_e-\theta-2\pi\alpha-\pi}^{\phi_e-\theta-(\alpha-1)2\pi-\pi} x_{oo} \frac{dA_O}{d\phi} d\phi}{V_{c1,\alpha}} \quad (4.116)$$

$$c_{y,c1,\alpha} = \frac{2h_s}{3} \frac{\int_{\phi_e-\theta-2\pi\alpha}^{\phi_e-\theta-(\alpha-1)2\pi} y_{oi} \frac{dA_I}{d\phi} d\phi - \int_{\phi_e-\theta-2\pi\alpha-\pi}^{\phi_e-\theta-(\alpha-1)2\pi-\pi} y_{oo} \frac{dA_O}{d\phi} d\phi}{V_{c1,\alpha}}. \quad (4.117)$$

which yields the solution

关于质心的推导实在太麻烦了，
这里我没有去做验证

$$c_{x,c1,\alpha} = -2r_b \cos(\theta - \phi_{ie}) - \Psi \sin(\theta - \phi_{ie}) \quad (4.118)$$

$$c_{y,c1,\alpha} = 2r_b \sin(\theta - \phi_{ie}) - \Psi \cos(\theta - \phi_{ie}) \quad (4.119)$$

where the term Ψ is defined for compactness by

$$\Psi = \frac{r_b}{3} \frac{3(\theta + \phi_{o0})^2 + \pi^2 - 15 + (\theta + \phi_{o0})(12\pi\alpha - 6\phi_{ie}) + 3\phi_{ie}^2 + 12\pi\alpha(\pi\alpha - \phi_{ie})}{2\theta + \phi_{o0} + \phi_{i0} - 2\phi_{ie} + 4\pi\alpha| - \pi}. \quad (4.120)$$

The volume and derivative of chamber volume for the $c_{2,\alpha}$ chamber is given by

$$V_{c2,\alpha} = V_{c1,\alpha} \quad (4.121)$$

$$\frac{dV_{c2,\alpha}}{d\theta} = \frac{dV_{c1,\alpha}}{d\theta} \quad (4.122)$$

by symmetry. This yields the centroid for the $c_{2,\alpha}$ chamber of

$$c_{x,c2,\alpha} = -c_{x,c1,\alpha} + \boxed{r_o \cos(\phi_{ie} - \pi/2 - \theta)} \quad (4.123)$$

$$c_{y,c2,\alpha} = -c_{y,c1,\alpha} + \boxed{r_o \sin(\phi_{ie} - \pi/2 - \theta).} \quad (4.124)$$

4.4 压缩腔的力和力矩

- 这个没什么好说的，跟吸入腔的计算方法类似

Forces

The gas forces on the orbiting scroll from the compression chambers are calculated by a similar means to that for the suction chambers. The axial forces are given by

其实跟吸入腔的
计算方法类似

$$\frac{\mathbf{F}_{z,c1,\alpha}}{p_{c1,\alpha}-p_{shell}} = \frac{V_{c1,\alpha}}{h_s} \tag{4.125}$$

$$\frac{\mathbf{F}_{z,c2,\alpha}}{p_{c2,\alpha}-p_{shell}} = \frac{V_{c2,\alpha}}{h_s} \tag{4.126}$$

and the in-plane forces given by

曲线的法向量分量我们之前有讨论过

$$\frac{\mathbf{F}_{c1,\alpha}}{p_{c1,\alpha}} = h_s r_b \int_{\phi_e-\theta-2\pi\alpha-\pi}^{\phi_e-\theta-2\pi(\alpha-1)-\pi} \left[\boxed{(\phi-\phi_{o0})\sin(\phi)\hat{i}} \boxed{-(\phi-\phi_{o0})\cos(\phi)\hat{j}}\right] d\phi \tag{4.127}$$

$$\frac{\mathbf{F}_{c2,\alpha}}{p_{c2,\alpha}} = -h_s r_b \int_{\phi_e-\theta-2\pi\alpha}^{\phi_e-\theta-2\pi(\alpha-1)} \left[\boxed{(\phi-\phi_{i0})\sin(\phi)\hat{i}} + \boxed{(\phi-\phi_{i0})\cos(\phi)\hat{j}}\right] d\phi \tag{4.128}$$

and the solutions are found in Appendix B.4. The moments around the crank pin are given by

这个力臂的分量
于曲柄销圆心的

$$\frac{\mathbf{M}_{\bigcirc,c1,\alpha}}{p_{c1,\alpha}} = \int_{\phi_e-\theta-2\pi\alpha-\pi}^{\phi_e-\theta-2\pi(\alpha-1)-\pi} [\mathbf{r}_{\bigcirc,c1,\alpha} \times \frac{d\mathbf{F}_{c1,\alpha}}{p_{c1,\alpha}}] d\phi \tag{4.129}$$

$$\frac{\mathbf{M}_{\bigcirc,c2,\alpha}}{p_{c2,\alpha}} = \int_{\phi_e-\theta-2\pi\alpha}^{\phi_e-\theta-2\pi(\alpha-1)} [\mathbf{r}_{\bigcirc,c2,\alpha} \times \frac{d\mathbf{F}_{c2,\alpha}}{p_{c2,\alpha}}] d\phi \tag{4.130}$$

where the moments are taken around the point from Eqn. (4.40) which yields the moments around the crank pin

$$\frac{\mathbf{M}_{\bigcirc,c1,\alpha}}{p_{c1,\alpha}} = -2\,\pi\, r_b^2\, h_s\, (\theta+\phi_{o0}-\phi_e+2\,\pi\,\alpha-\pi)\,\hat{k} \tag{4.131}$$

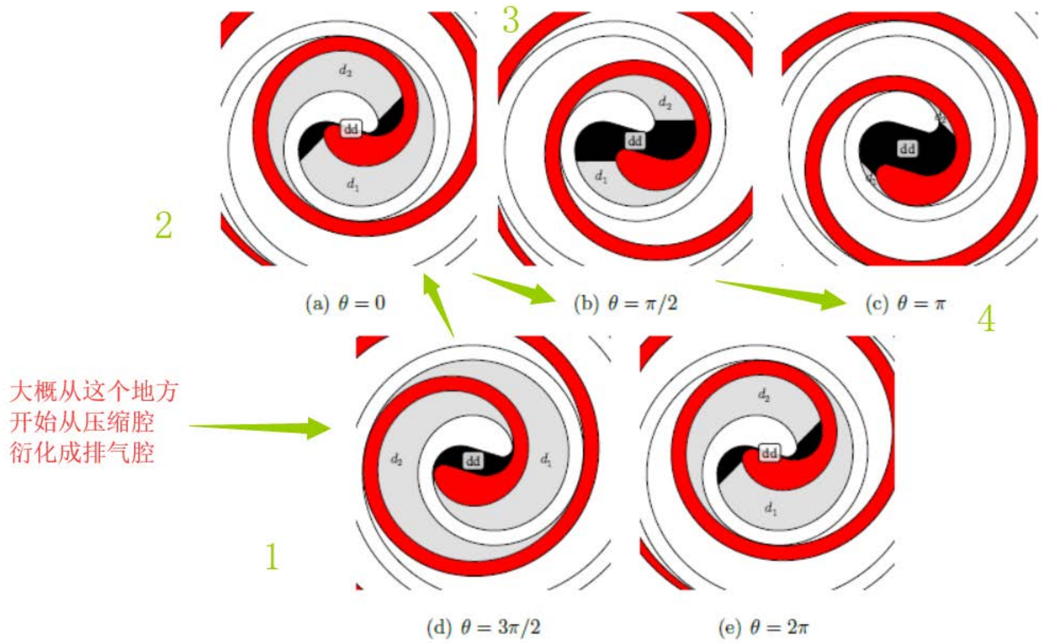
$$\frac{\mathbf{M}_{\bigcirc,c2,\alpha}}{p_{c2,\alpha}} = \frac{\pi\, r_b^2\, h_s\, (2\,\theta+2\,\phi_{i0}-2\,\phi_e+4\,\pi\,\alpha+\pi)}{2}\hat{k}. \tag{4.132}$$

五、排气腔

5.1 排气腔的演化

1) 排气腔的演化

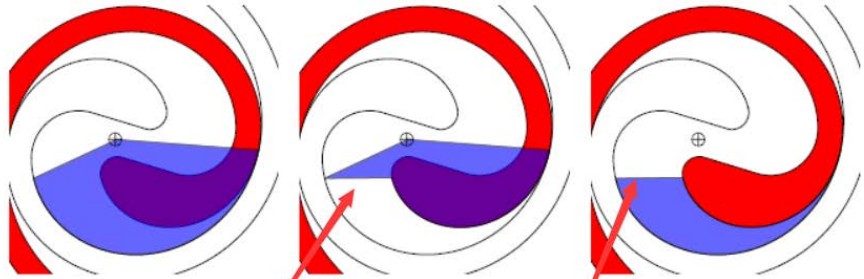
- 一般过了排气角π/2之后才使得d1和dd压力相等



5.2 d1腔体积的定义和面积体积质心求解

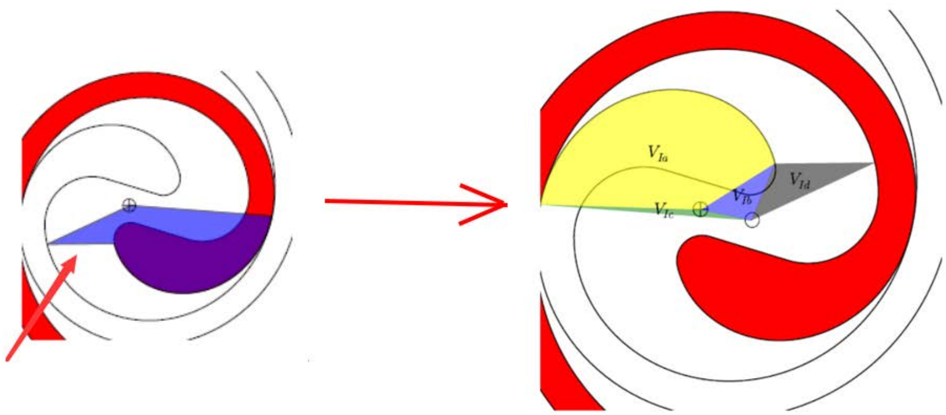
1) d1腔体积的定义和面积体积质心求解

- d1和dd腔的分界线是由动盘的φo和静盘的φos+π连线。目的是保证在排气角处分界线的长度为0
- 面积求解的套路一般都是把动涡盘的参考系转化为静涡盘的参考系



(a) To outer surface $V_{O,d1}$ (b) To inner surface $V_{I,d1}$ (c) Difference forms the d_1 chamber

d1和dd腔的分界线是由动盘的φo和静盘的φos+π连线。目的是保证在排气角处分界线的长度为0



- Ad1 = AO - A1
- A1 = A1a + A1b + A1c + A1d A1将关于动涡盘的面积转化到静涡盘上
- Ad1 = AO - (A1a + A1b + A1c + A1d)

2) A0的面积体积质心求解

- 转化为静涡盘的参考系，排气角看起来为什么要加上π，但是你从动涡盘的角度去看，转化为动涡盘的参考系展开角是需要减去π的，此时就剩下φos - φo0了

从静涡盘的参考系去看

$$V_{O,d1} = \frac{h_s r_b^2 \left[(\phi_{ie} - \theta - 2\pi N_c - \phi_{i0})^3 - (\phi_{os} + \pi - \phi_{i0})^3\right]}{6}$$

啮合角

排气角，从静涡盘的参考系看不出来，把它转化为动涡盘的参考系就容易看出来了，展开角减去π变成了Φos

and the derivative of $V_{O,d1}$ with respect to the crank angle is

$$\frac{dV_{O,d1}}{d\theta} = -\frac{h_s r_b^2}{2}(\phi_{ie} - \theta - 2\pi N_c - \phi_{i0})^2$$

The centroids can also be calculated by the method proposed in Section 4.6.2, which yields

$$c_{x,O,d1} = \frac{h_s}{V_{O,d1}}\left[f_{xA}(\phi_{ie} - \theta - 2\pi N_c, \phi_{i0}) - f_{xA}(\phi_{os} + \pi, \phi_{i0})\right] \tag{4.135}$$

$$c_{y,O,d1} = \frac{h_s}{V_{O,d1}}\left[f_{yA}(\phi_{ie} - \theta - 2\pi N_c, \phi_{i0}) - f_{yA}(\phi_{os} + \pi, \phi_{i0})\right].$$

3) A1a的面积体积质心求解

- 动涡盘上的面积通过旋转和平移搬到静涡盘上，展开角不变

$\phi_2 = \phi_{ie} - \theta - 2\pi N_c - \pi$ where N_c is defined from Eqn. (4.108). Applying the same analysis as for the suction chamber yields the volume of $V_{Ia,d1}$ of

$$V_{Ia,d1} = \frac{h_s r_b^2 \left[(\phi_{ie} - \theta - 2\pi N_c - \pi - \phi_{o0})^3 - (\phi_{os} - \phi_{o0})^3\right]}{6}$$

动涡盘上的面积通过旋转和平移搬到静涡盘上，展开角不变

and the derivative of $V_{Ia,d1}$ with respect to the crank angle is

$$\frac{dV_{Ia,d1}}{d\theta} = -\frac{h_s r_b^2}{2}(\phi_{ie} - \theta - 2\pi N_c - \pi - \phi_{o0})^2.$$

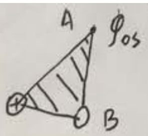
Applying the analysis of Section 4.6.2 for the centroid of area enclosed between an involute curve and the fixed origin yields

$$c_{x,Ia,d1} = \frac{h_s}{V_{Ia,d1}}\left[f_{xA}(\phi_{ie} - \theta - 2\pi N_c - \pi, \phi_{o0}) - f_{xA}(\phi_{os}, \phi_{o0})\right]$$

$$c_{y,Ia,d1} = \frac{h_s}{V_{Ia,d1}}\left[f_{yA}(\phi_{ie} - \theta - 2\pi N_c - \pi, \phi_{o0}) - f_{yA}(\phi_{os}, \phi_{o0})\right].$$

4) A1b的面积体积质心求解

- 动涡盘上的面积通过旋转和平移搬到静涡盘上，展开角不变，运用圆渐开线基圆三角形面积求解的结论可以直接得到答案



$$A\left(r_b[\cos\phi_{os}+(\phi_{os}-\phi_{o0})\sin\phi_{os}],\ r_b[\sin\phi_{os}-(\phi_{os}-\phi_{o0})\cos\phi_{os}]\right)$$

$$B\left(r_o\cos(\phi_{ie}-\tfrac{\pi}{2}-\theta),\ r_o\sin(\phi_{ie}-\tfrac{\pi}{2}-\theta)\right)$$

$$\Delta BOA=\frac{1}{2}\vec{A}\times\vec{B}=\frac{1}{2}r_o r_b\begin{bmatrix}\cos(\phi_{ie}-\frac{\pi}{2}-\theta)[\sin\phi_{os}-(\phi_{os}-\phi_{o0})\cos\phi_{os}]\\-\sin(\phi_{ie}-\frac{\pi}{2}-\theta)[\cos\phi_{os}+(\phi_{os}-\phi_{o0})\sin\phi_{os}]\end{bmatrix}$$

$$=\frac{1}{2}r_o r_b\begin{bmatrix}\sin(\phi_{ie}-\theta)[\sin\phi_{os}-(\phi_{os}-\phi_{o0})\cos\phi_{os}]\\+\cos(\phi_{ie}-\theta)[\cos\phi_{os}+(\phi_{os}-\phi_{o0})\sin\phi_{os}]\end{bmatrix}$$

$$=\frac{1}{2}r_o r_b\left[(\phi_{os}-\phi_{o0})\sin(\phi_{os}-\phi_{ie}+\theta)+\cos(\phi_{ie}-\theta-\phi_{os})\right]$$

$$=\frac{r_o r_b}{2}\left[(\phi_{os}-\phi_{o0})\sin(\theta+\phi_{os}-\phi_{ie})+\cos(\theta+\phi_{os}-\phi_{ie})\right]$$

$$V_{Ib,d1}=\frac{h_s r_b r_o}{2}\left[(\phi_{os}-\phi_{o0})\sin(\theta+\phi_{os}-\phi_{ie})+\cos(\theta+\phi_{os}-\phi_{ie})\right] \tag{4.141}$$

$$\frac{dV_{Ib,d1}}{d\theta}=\frac{h_s r_b r_o}{2}\left[(\phi_{os}-\phi_{o0})\cos(\theta+\phi_{os}-\phi_{ie})-\sin(\theta+\phi_{os}-\phi_{ie})\right] \tag{4.142}$$

$$c_{x,Ib,d1}=\frac{-r_o\sin(\theta-\phi_{ie})+r_b(\phi_{os}-\phi_{o0})\sin(\phi_{os})+r_b\cos(\phi_{os})}{3} \tag{4.143}$$

$$c_{y,Ib,d1}=\frac{-r_o\cos(\theta-\phi_{ie})-r_b(\phi_{os}-\phi_{o0})\cos(\phi_{os})+r_b\sin(\phi_{os})}{3}. \tag{4.144}$$

The volume $V_{Ic,d1}$ is also formed by a triangle, in this case with vertices at the origin, the innermost conjugate point on the outer involute of the fixed scroll, and the orbiting scroll origin. Thus, the governing equations for this triangle are

$$V_{Ic,d1}=\frac{h_s r_b r_o}{2} \tag{4.145}$$

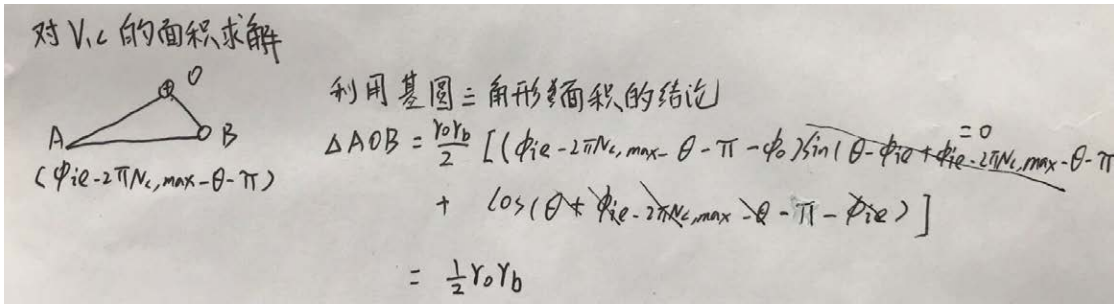
$$\frac{dV_{Ic,d1}}{d\theta}=0 \tag{4.146}$$

$$c_{x,Ic,d1}=\frac{[r_b(-\theta+\phi_{ie}-\phi_{o0}-2\pi N_c-\pi)-r_o]\sin(\theta-\phi_{ie})-r_b\cos(\theta-\phi_{ie})}{3} \tag{4.147}$$

$$c_{y,Ic,d1}=\frac{[r_b(-\theta+\phi_{ie}-\phi_{o0}-2\pi N_c-\pi)-r_o]\cos(\theta-\phi_{ie})+r_b\sin(\theta-\phi_{ie})}{3}. \tag{4.148}$$

5) A1c的面积体积质心求解

- 运用圆渐开线基圆三角形面积求解的结论可以直接得到答案



6) A1d的面积体积质心求解

- V1d,d1这个三角形比较麻烦，因为它的一个顶点在静涡盘上，另一个顶点在动涡盘上，算起来比较麻烦，这里我就不推导了

$$V_{Id,d1}=\frac{h_s r_b r_o}{2}\left[(\phi_{os}-\phi_{i0}+\pi)\sin(\theta+\phi_{os}-\phi_{ie})+\cos(\theta+\phi_{os}-\phi_{ie})+1\right]$$

$$\frac{dV_{Id,d1}}{d\theta}=\frac{h_s r_b r_o}{2}\left[(\phi_{os}-\phi_{i0}+\pi)\cos(\theta+\phi_{os}-\phi_{ie})-\sin(\theta+\phi_{os}-\phi_{ie})\right]$$

$$c_{x,Id,d1}=\frac{r_b\,(2\,\phi_{os}-\phi_{o0}-\phi_{i0}+\pi)\,\sin(\phi_{os})-2\,(r_o\sin(\theta-\phi_{ie})-r_b\cos(\phi_{os}))}{3}$$

V1d, d1这个三角形比较麻烦，因为它的一个顶点在静涡盘上，另一个顶点在动涡盘上，算起来比较麻烦，这里我就不推导了

$$c_{y,Id,d1}=\frac{-2\,(r_o\cos(\theta-\phi_{ie})-r_b\sin(\phi_{os}))-r_b\,(2\,\phi_{os}-\phi_{o0}-\phi_{i0}+\pi)\,\cos(\phi_{os})}{3}.$$

7) A1的面积体积质心求解汇总

- A1 = A1a + A1b + A1c + A1d

$$c_{x,I,d1}=-\frac{c_{x,Ia,d1}V_{Ia,d1}+c_{x,Ib,d1}V_{Ib,d1}+c_{x,Ic,d1}V_{Ic,d1}+c_{x,Id,d1}V_{Id,d1}}{V_{Ia,d1}+V_{Ib,d1}+V_{Ic,d1}+V_{Id,d1}}+\boxed{r_o\cos(\theta_m)} \quad (4.153)$$

$$c_{y,I,d1}=-\frac{c_{y,Ia,s1}V_{Ia,d1}+c_{y,Ib,d1}V_{Ib,d1}+c_{y,Ic,d1}V_{Ic,d1}+c_{y,Id,d1}V_{Id,d1}}{V_{Ia,d1}+V_{Ib,d1}+V_{Ic,d1}+V_{Id,d1}}+\boxed{r_o\sin(\theta_m)} \quad (4.154)$$

and the volume of the inner segment is equal to

别忘了这个随动项

$$V_{I,d1}=V_{Ia,d1}+V_{Ib,d1}+V_{Ic,d1}+V_{Id,d1} \quad (4.155)$$

$$\frac{dV_{I,d1}}{d\theta}=\frac{dV_{Ia,d1}}{d\theta}+\frac{dV_{Ib,d1}}{d\theta}+\frac{dV_{Ic,d1}}{d\theta}+\frac{dV_{Id,d1}}{d\theta}. \quad (4.156)$$

8) Ad1的面积体积质心求解汇总

- Ad1 = AO - A1

The total volume for the d_1 chamber can therefore be obtained from

$$V_{d1} = V_{O,d1} - V_{I,d1} \tag{4.157}$$

$$\frac{dV_{d1}}{d\theta} = \frac{dV_{O,d1}}{d\theta} - \frac{dV_{I,d1}}{d\theta} \tag{4.158}$$

and the centroid of the d_1 chamber is given by

$$c_{x,d1} = \frac{c_{x,O,d1}V_{O,d1} - c_{x,I,d1}V_{I,d1}}{V_{d1}} \tag{4.159}$$

$$c_{y,d1} = \frac{c_{y,O,d1}V_{O,d1} - c_{y,I,d1}V_{I,d1}}{V_{d1}} \tag{4.160}$$

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By symmetry, the centroid for the d_2 chamber is

$$c_{x,d2} = -c_{x,d1} + r_o\cos(\phi_{ie} - \pi/2 - \theta) \tag{4.161}$$

$$c_{y,d2} = -c_{y,d1} + r_o\sin(\phi_{ie} - \pi/2 - \theta). \tag{4.162}$$

and the volume and derivative of volume for the d_2 chamber are given by

$$V_{d2} = V_{d1} \tag{4.163}$$

$$\frac{dV_{d2}}{d\theta} = \frac{dV_{d1}}{d\theta}. \tag{4.164}$$

9) Ad1的力和力矩求解

- 还是以前的套路

Forces

The gas forces on the orbiting scroll from the discharge chambers are calculated by a similar means to that for the other chambers. The axial forces are given by

$$\frac{\mathbf{F}_{z,d1}}{p_{d1}-p_{shell}}=\frac{V_{d1}}{h_s} \tag{4.165}$$

$$\frac{\mathbf{F}_{z,d2}}{p_{d2}-p_{shell}}=\frac{V_{d2}}{h_s} \tag{4.166}$$

and the in-plane forces given by

$$\frac{\mathbf{F}_{d1}}{p_{d1}}=h_s r_b\int_{\phi_{os}}^{\phi_e-\theta-2\pi N_c-\pi}\left[(\phi-\phi_{o0})\sin(\phi)\hat{i}-(\phi-\phi_{o0})\cos(\phi)\hat{j}\right]d\phi \tag{4.167}$$

$$\frac{\mathbf{F}_{d2}}{p_{d2}}=-h_s r_b\int_{\phi_{os}+\pi}^{\phi_e-\theta-2\pi N_c}\left[(\phi-\phi_{i0})\sin(\phi)\hat{i}+(\phi-\phi_{i0})\cos(\phi)\hat{j}\right]d\phi \tag{4.168}$$

where solutions for the force terms are in Appendix B.4. The moments around the crank pin are given by

$$\frac{\mathbf{M}_{\bigcirc,d1}}{p_{d1}}=\int_{\phi_{os}}^{\phi_e-\theta-2\pi N_c-\pi}[\mathbf{r}_{\bigcirc,d1}\times\frac{d\mathbf{F}_{d1}}{p_{d1}}]d\phi \tag{4.169}$$

$$\frac{\mathbf{M}_{\bigcirc,d2}}{p_{d2}}=\int_{\phi_{os}+\pi}^{\phi_e-\theta-2\pi N_c}[\mathbf{r}_{\bigcirc,d2}\times\frac{d\mathbf{F}_{d2}}{p_{d2}}]d\phi \tag{4.170}$$

which yields the torques

$$\frac{\mathbf{M}_{\bigcirc,d1}}{p_{d1}}=\frac{r_b^2\,h_s\,(\theta-\phi_{os}+2\,\phi_{o0}-\phi_e+2\,\pi\,N_c+\pi)\,(\theta+\phi_{os}-\phi_e+2\,\pi\,N_c+\pi)}{2}\hat{k} \tag{4.171}$$

$$\frac{\mathbf{M}_{\bigcirc,d2}}{p_{d2}}=-\frac{r_b^2\,h_s\,(\theta-\phi_{os}+2\,\phi_{i0}-\phi_e+2\,\pi\,N_c-\pi)\,(\theta+\phi_{os}-\phi_e+2\,\pi\,N_c+\pi)}{2}\hat{k}. \tag{4.172}$$

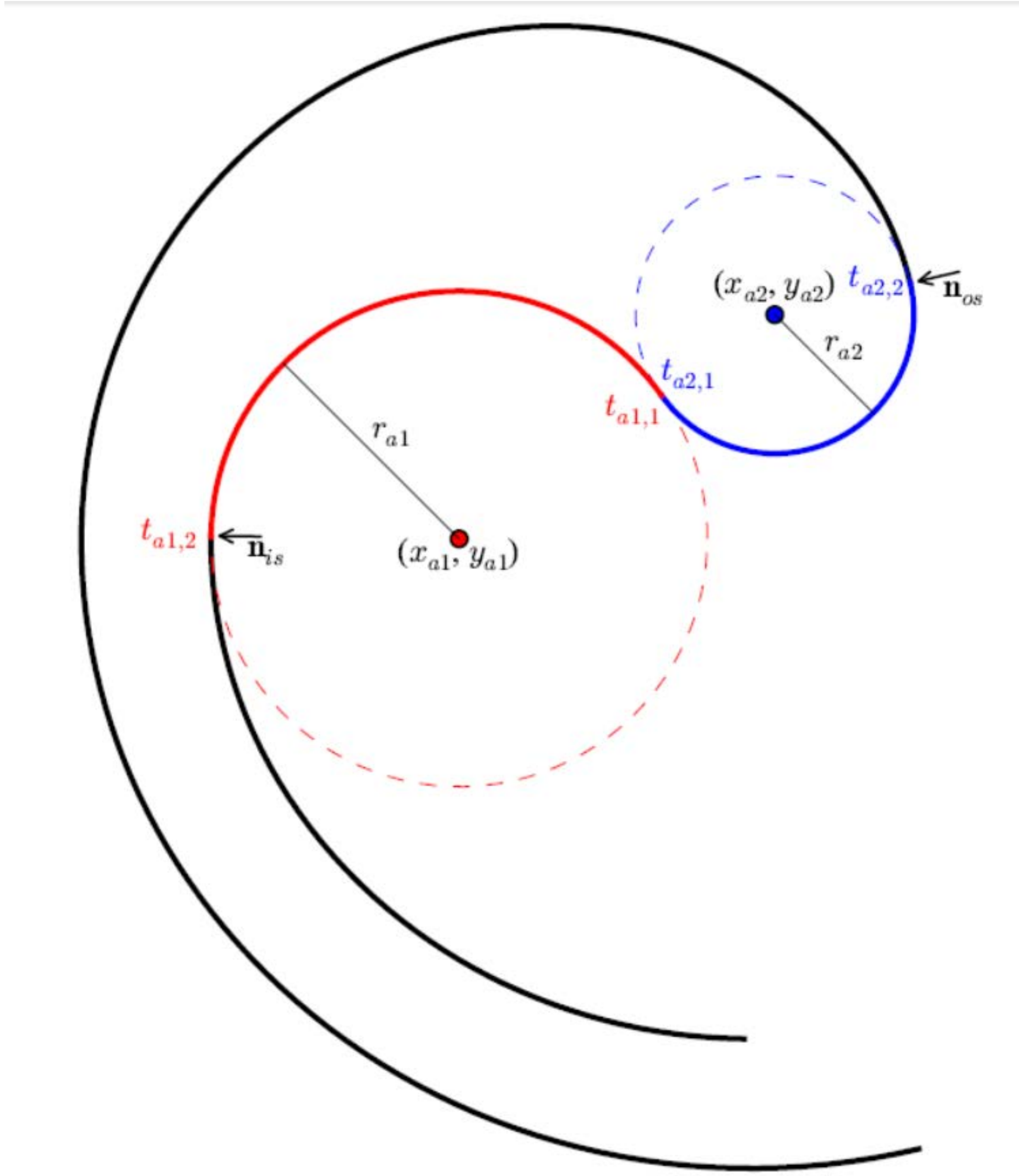
5.3 dd腔

1) dd的形成曲线

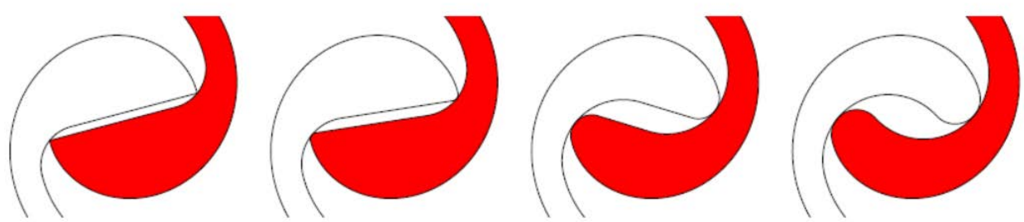
- dd腔的定义比较麻烦，因为它的形成曲线有不同的类型
- 从φ0~φs这一段涡旋是用来形成排气区域的组合曲线的
- 组合曲线满足四个条件
  - 1，曲线必须在φis处跟涡旋内壁相切;
  - 2，曲线必须在φos处跟涡旋外壁相切;

3，曲线在运行过程中不会使得动静涡盘相碰；

- 4，曲线必须经过涡旋的内外φs点

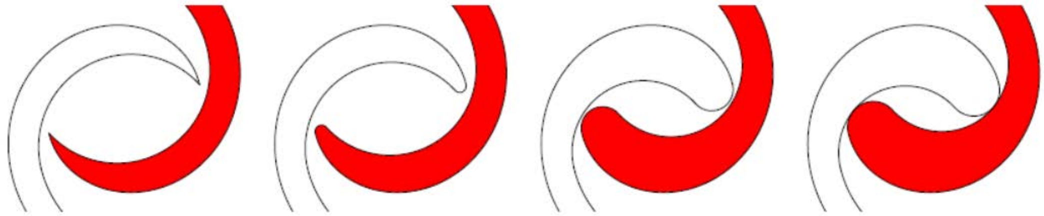


- 曲线类型
  - 第一种组合曲线类型——弧线-直线-弧线 ra2是连接外涡旋壁面的圆弧



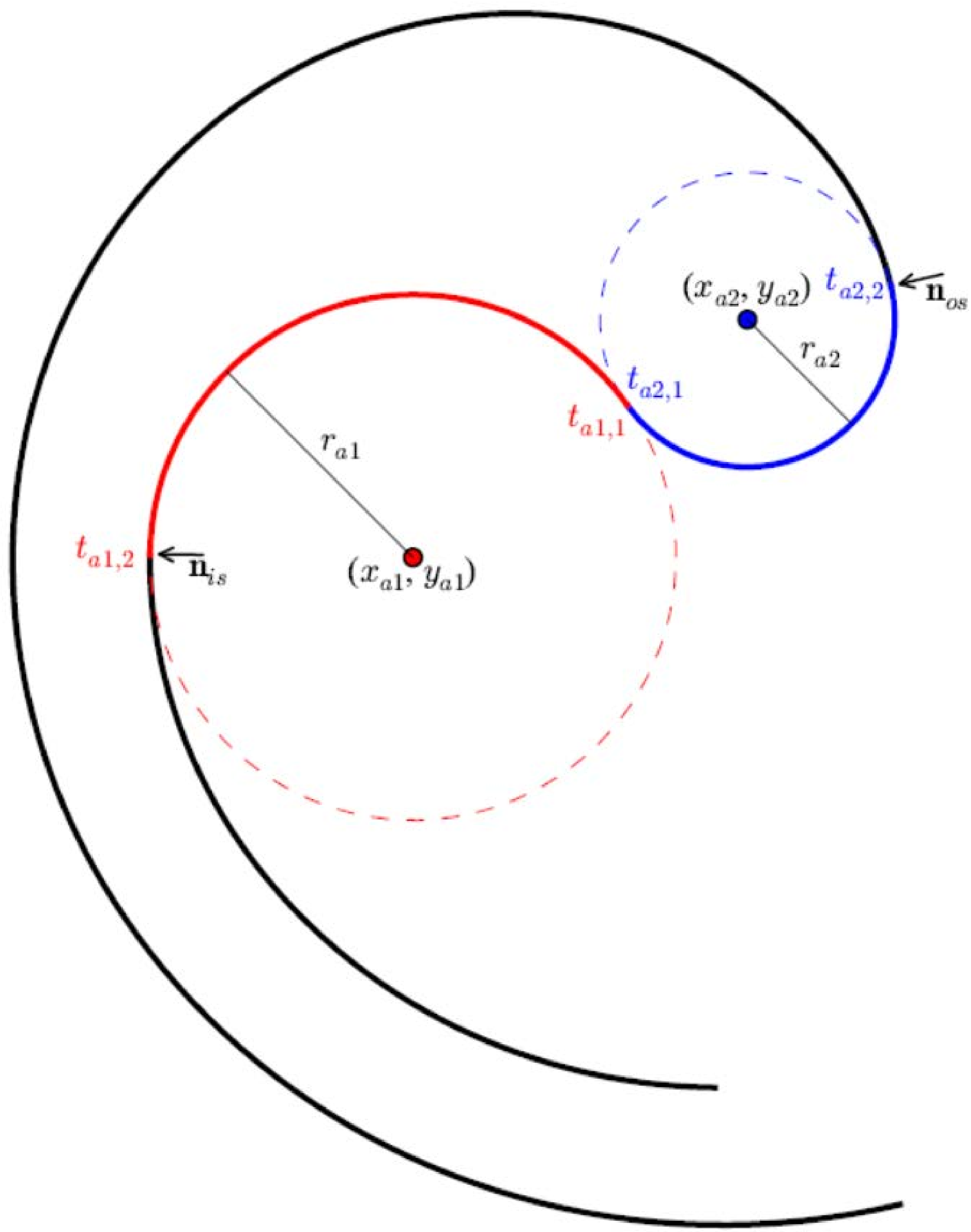
(a) r_{a2}=0 (b) r_{a2}=$r_{a2,max}/4$ (c) r_{a2}=$r_{a2,max}/1.2$ (d) r_{a2}=$r_{a2,max}$

  - 第二种组合曲线类型——弧线-弧线-弧线

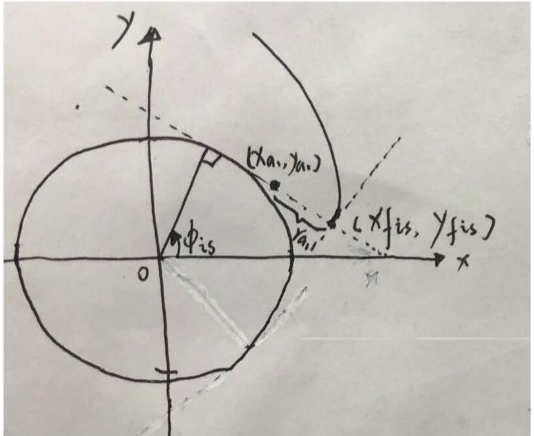


(a) r_{a2}=0 (b) $r_{a2}=r_{a2,max}/4$ (c) $r_{a2}=r_{a2,max}/1.2$ (d) $r_{a2}=r_{a2,max}$

2) 两段弧线组合曲线



- 组合曲线圆心坐标



$$x_{a1} = x_{fis} - \sin(\phi_{is}) r_{a1}$$

$$y_{a1} = y_{fis} + \cos(\phi_{is}) r_{a1}$$

$$x_{a2} = x_{fos} - \sin(\phi_{os}) r_{a2}$$

$$y_{a2} = y_{fos} + \cos(\phi_{os}) r_{a2}.$$

$x_{fis} - x_{a1} = r_{a1}\cos(\frac{\pi}{2} - \phi_{is})$, $y_{a1} - y_{fis} = r_{a1}\sin(\frac{\pi}{2} - \phi_{is})$

即 $x_{a1} = x_{fis} - r_{a1}\sin\phi_{is}$

同理 $y_{a1} = y_{fis} + r_{a1}\cos\phi_{is}$

$x_{a2} = x_{fos} - r_{a2}\sin\phi_{os}$

$y_{a2} = y_{fos} + r_{a2}\cos\phi_{os}$

- 组合曲线圆心半径

两圆相切的条件 $$\sqrt{(x_{a1}-x_{a2})^2+(y_{a1}-y_{a2})^2}=r_{a1}+r_{a2}. \tag{4.177}$$

After simplification and grouping of terms, the result for the radius of arc 1 is

$$r_{a1}=\frac{\frac{1}{2}(\Delta y)^2+\frac{1}{2}(\Delta x)^2+r_{a2}\Delta x\sin(\phi_{os})-r_{a2}\Delta y\cos(\phi_{os})}{r_{a2}\cos(\phi_{os}-\phi_{is})+\Delta x\sin(\phi_{is})-\Delta y\cos(\phi_{is})+r_{a2}} \tag{4.178}$$

with $\boxed{\Delta x=x_{fis}-x_{fos}}$ and $\boxed{\Delta y=y_{fis}-y_{fos}}$. The degenerate case of this set of curves occurs when $\boxed{r_{a2}=0}$, which yields a single arc with radius

$$\begin{cases} x_{a1} = x_{fis} - r_{a1}\sin\phi_{is} \\ y_{a1} = y_{fis} + r_{a1}\cos\phi_{is} \\ x_{a2} = x_{fos} - r_{a2}\sin\phi_{os} \\ y_{a2} = y_{fos} + r_{a2}\cos\phi_{os} \\ \underbrace{\sqrt{(x_{a1}-x_{a2})^2+(y_{a1}-y_{a2})^2} = r_{a1}+r_{a2}}_{\text{两圆相切条件}} \end{cases}$$

$$\left[(x_{fis}-x_{fos})-(r_{a1}\sin\phi_{is}-r_{a2}\sin\phi_{os})\right]^2+\left[(y_{fis}-y_{fos})+(r_{a1}\cos\phi_{is}-r_{a2}\cos\phi_{os})\right]^2=(r_{a1}+r_{a2})^2$$

令 $\Delta x = x_{fis}-x_{fos}$　$\Delta y = y_{fis}-y_{fos}$

则 $$\Delta x^2 + r_{a1}^2\sin^2\phi_{is} + r_{a2}^2\sin^2\phi_{os} - 2r_{a1}r_{a2}\sin\phi_{is}\sin\phi_{os} - 2\Delta x(r_{a1}\sin\phi_{is}-r_{a2}\sin\phi_{os}) + \Delta y^2 + r_{a1}^2\cos^2\phi_{is} + r_{a2}^2\cos^2\phi_{os} - 2r_{a1}r_{a2}\cos\phi_{is}\cos\phi_{os} + 2\Delta y(r_{a1}\cos\phi_{is}-r_{a2}\cos\phi_{os}) = (r_{a1}+r_{a2})^2$$

$$\Delta x^2+\Delta y^2+r_{a1}^2+r_{a2}^2-2r_{a1}r_{a2}(\cos\phi_{is}\cos\phi_{os}+\sin\phi_{is}\sin\phi_{os}) - 2(\Delta x\sin\phi_{is}-\Delta y\cos\phi_{is})r_{a1}+2(\Delta x\sin\phi_{os}-\Delta y\cos\phi_{os})r_{a2} = r_{a1}^2+r_{a2}^2+2r_{a1}r_{a2}$$

$$\underbrace{\tfrac{1}{2}(\Delta x^2+\Delta y^2)}_{A} - r_{a1}r_{a2}\underbrace{\left[\cos(\phi_{is}-\phi_{os})+1\right]}_{B} = \underbrace{(\Delta x\sin\phi_{is}-\Delta y\cos\phi_{is})}_{C}r_{a1} - \underbrace{(\Delta x\sin\phi_{os}-\Delta y\cos\phi_{os})}_{D}r_{a2}$$

$$A - Br_{a1}r_{a2} = Cr_{a1} - Dr_{a2} \qquad A = (C+Br_{a2})r_{a1} - Dr_{a2}$$

$$r_{a1} = \frac{A+Dr_{a2}}{C+Br_{a2}} = \frac{\frac{1}{2}(\Delta x^2+\Delta y^2)+(\Delta x\sin\phi_{os}-\Delta y\cos\phi_{os})r_{a2}}{\Delta x\sin\phi_{is}-\Delta y\cos\phi_{is}+[1+\cos(\phi_{is}-\phi_{os})]r_{a2}}$$

当 $r_{a2}=0$ 时

$$r_{a1} = \frac{\Delta x^2+\Delta y^2}{2(\Delta x\sin\phi_{is}-\Delta y\cos\phi_{is})}$$

3) 弧线**-**直线**-**弧线组合曲线

4) dd腔体积的定义和面积体积质心力和力矩求解

5.4 泄漏的流动面积和主流动路径

六、PDSim.scroll package

6.1 PDSim.scroll.common_scroll_geo module

1) Class：PDSim.scroll.common_scroll_geo.CVInvolute

> - 包含展开角度的简单结构
> - phi: 展开角φ
> - theta: 曲轴转角
> - double phi_max: 沿渐开线的最大展开角
> - double phi_min: 沿渐开线的最小展开角
> - double phi_0：沿着这个渐开线发生角
> - double dphi_max_dtheta: 沿渐开线的最大展开角对曲轴转角θ的导数
> - double dphi_min_dtheta: 沿渐开线的最小展开角对曲轴转角θ的导数
> - involute_index(类名) involute: 渐开线的渐开线指数

```
 //一种通用的涡旋角度结构体
cdef class CVInvolute:
def __init__(self):
    pass

// 渐开线类别
cdef enum involute_index:
INVOLUTE_FI
INVOLUTE_FO
INVOLUTE_OI
INVOLUTE_OO

// 渐开线类别返回字符串
cpdef bytes involute_index_to_key(int index):
"""
Return the string associated with a given index from the common_scroll_geo.involute_index enumeration
"""

if index == INVOLUTE_FI:
    return bytes('fi')
elif index == INVOLUTE_FO:
    return bytes('fo')
elif index == INVOLUTE_OI:
    return bytes('oi')
elif index == INVOLUTE_OO:
    return bytes('oo')
else:
    return bytes('')
```

2) Class： PDSim.scroll.common_scroll_geo.CVInvolutes

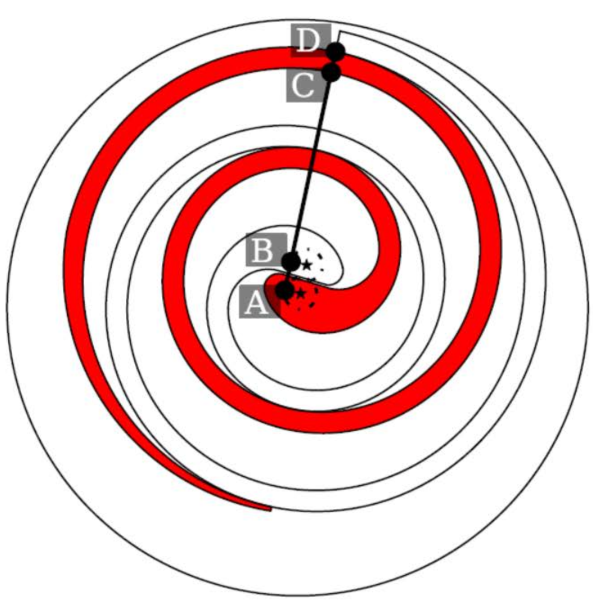
-
- CVInvolute Inner: 该腔室的内部渐开线
- CVInvolute Outer: 该腔室的外部渐开线
- Boolean has_line_1: 是否存在line #1
- Boolean has_line_2: 是否存在line #2

```
 //申明内部渐开线和外部渐开线两种角度结构
cdef class CVInvolutes:
def __init__(self):
    self.Inner = CVInvolute.__new__(CVInvolute)
    self.Outer = CVInvolute.__new__(CVInvolute)

//打印用
def __repr__(self):
    s = ''
    s += "Outer.involute = {i:s}\n".format(i=involute_index_to_key(self.Outer.involute))
    s += "Outer.phi_0 = {i:g}\n".format(i=self.Outer.phi_0)
    s += "Outer.phi_max = {i:g}\n".format(i=self.Outer.phi_max)
    s += "Outer.phi_min = {i:g}\n".format(i=self.Outer.phi_min)
    s += "Inner.involute = {i:s}\n".format(i=involute_index_to_key(self.Inner.involute))
    s += "Inner.phi_0 = {i:g}\n".format(i=self.Inner.phi_0)
    s += "Inner.phi_max = {i:g}\n".format(i=self.Inner.phi_max)
    s += "Inner.phi_min = {i:g}\n".format(i=self.Inner.phi_min)
    s += "has_line_1 = {i:g}\n".format(i=self.has_line_1)
    s += "has_line_2 = {i:g}".format(i=self.has_line_2)
    return s
```

2) Class： PDSim.scroll.common_scroll_geo.geoVals

- h 齿高
- ro orbiting radius 公转半径 公转半径只是涡盘基圆半径和内外表面发生角的函数

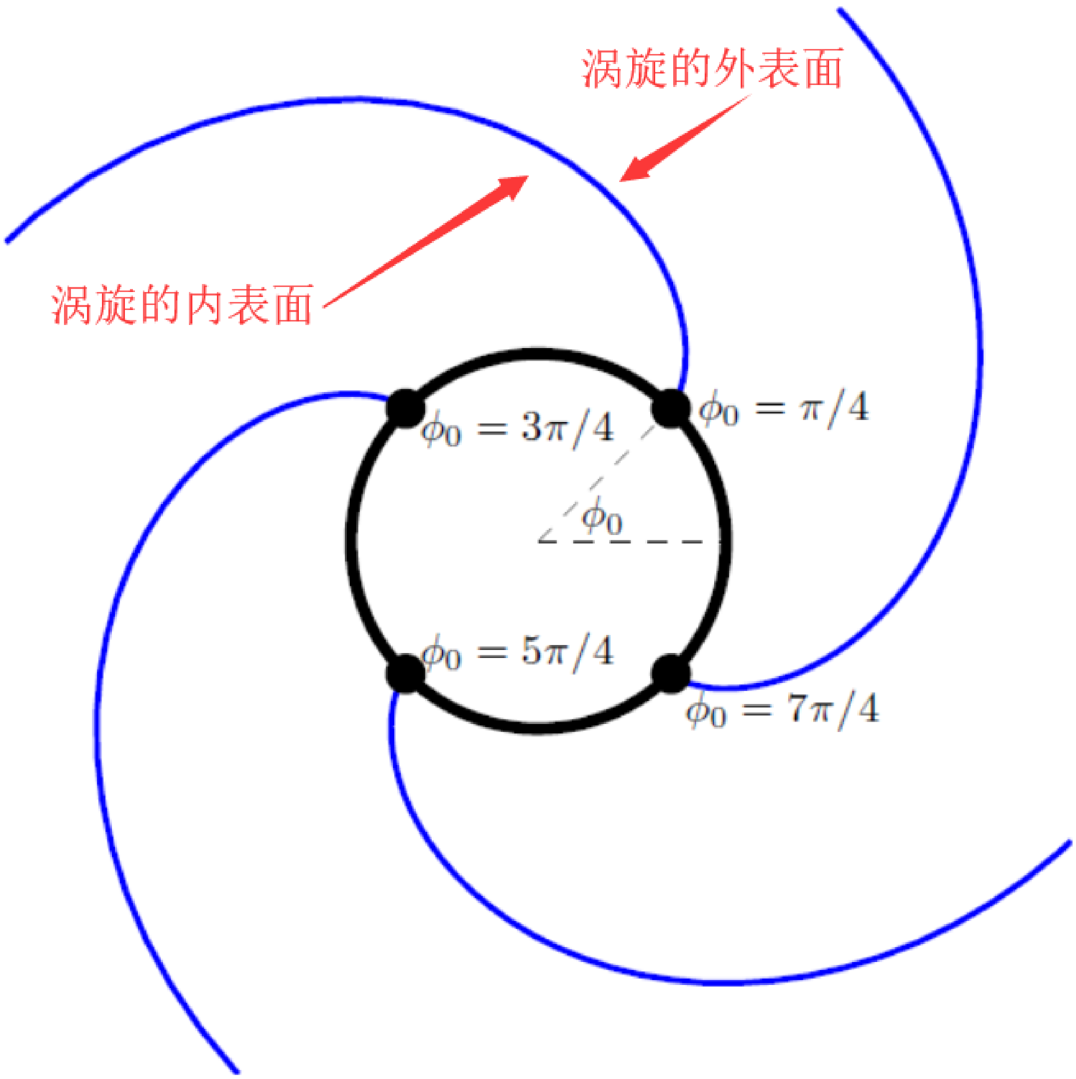


$$r_o = \left(\overline{AD} - \overline{BC}\right) - \overline{CD}$$

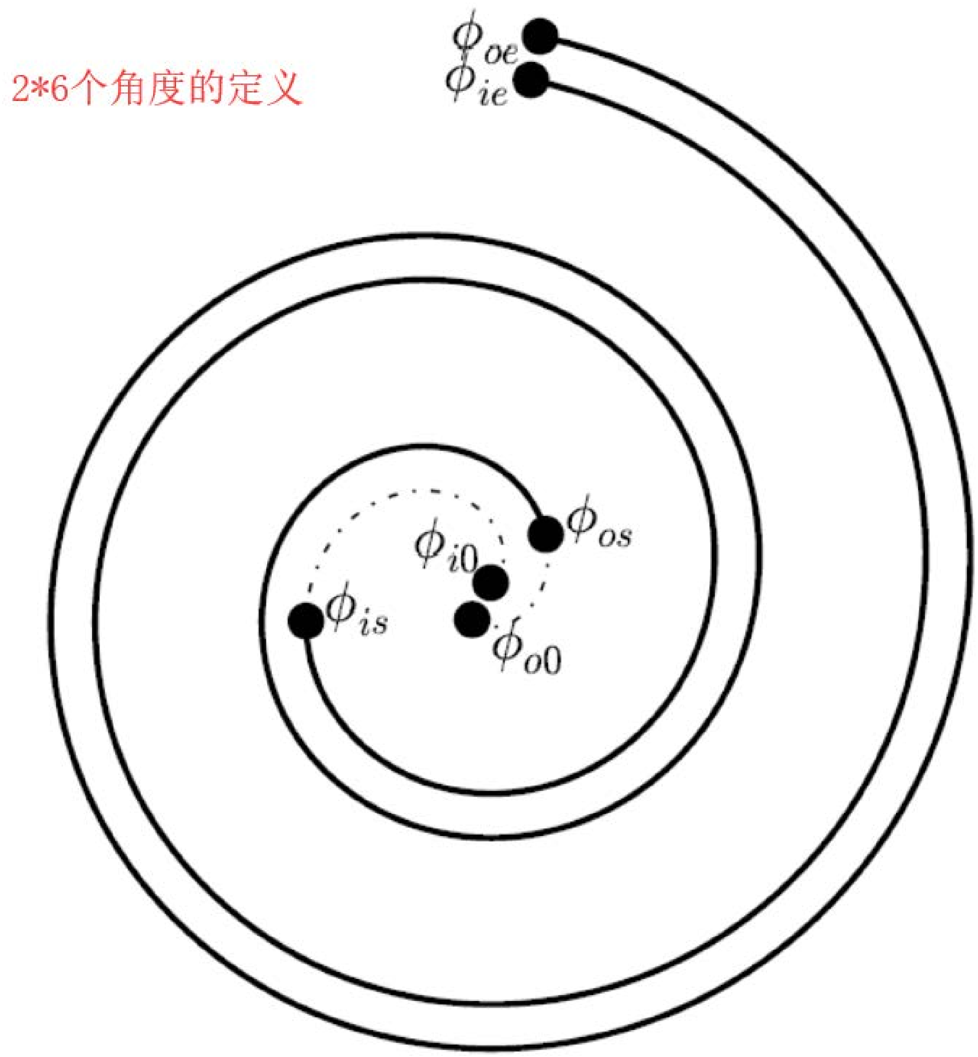
$$r_o = \left(r_b\left[\phi_{ie} - \phi_{i0}\right] - r_b\left[(\phi_{ie} - \pi) - \phi_{i0}\right]\right) - t_s$$

$$r_o = r_b\pi - t_s = r_b(\pi - \phi_{i0} + \phi_{o0}).$$

- rb the radius of the base circle 基圆半径
- t The thickness of the scroll 涡旋的厚度 t=ts = rb*(phi_i0 - phi_o0)，涡旋盘厚度通常为5毫米量级。



- phi_fie Inner Ending Angle 静涡盘内部渐开线结束角
- phi_fis Inner Starting Angle 静涡盘内部渐开线开始角
- phi_fi0 Inner Initial Angle 静涡盘内部渐开线发生角
- phi_foe Inner Ending Angle 静涡盘外部渐开线结束角
- phi_fos Inner Starting Angle 静涡盘外部渐开线开始角
- phi_fo0 Inner Initial Angle 静涡盘外部渐开线发生角
- phi_oie Inner Ending Angle 动涡盘内部渐开线结束角
- phi_ois Inner Starting Angle 动涡盘内部渐开线开始角
- phi_oi0 Inner Initial Angle 动涡盘内部渐开线发生角
- phi_ooe Inner Ending Angle 动涡盘外部渐开线结束角
- phi_oos Inner Starting Angle 动涡盘外部渐开线开始角
- phi_oo0 Inner Initial Angle 动涡盘外部渐开线发生角
- phi_ie_offset 初始值为0



- copy_inplace(self, geoVals target) 结构性复制
- is_symmetric(self) → bool 动外部渐开线所有角度都相等的时候返回true
- val_if_symmetric(self, double val) → double 如果is_symmetric则返回数值，否则返回数值错误
- xvec_disc_port 属于numpy.ndarray类型
- yvec_disc_port 属于numpy.ndarray类型
- rebuild_geoVals
- setattr(x, 'y', v) is equivalent to ``x.y = v''
- getattr(x, 'y') is equivalent to x.y
- **repr** 打印所有属性

```
 #This is a list of all the members in geoVals
geoValsvarlist=['h','ro','rb','t',
                 'phi_fi0','phi_fis','phi_fie',
                 'phi_fo0','phi_fos','phi_foe',
                 'phi_oi0','phi_ois','phi_oie',
                 'phi_oo0','phi_oos','phi_ooe',
                 'xa_arc1','ya_arc1','ra_arc1','t1_arc1','t2_arc1',
                 'xa_arc2','ya_arc2','ra_arc2','t1_arc2','t2_arc2',
                 'b_line', 't1_line', 't2_line', 'm_line',
                 'x0_wall','y0_wall','r_wall',
```

```
                'delta_radial', 'delta_flank',
                'phi_ie_offset','delta_suction_offset',
                'cx_scroll','cy_scroll','V_scroll','Vremove']

//外部渐开线所有角度都相等的时候返回true
cpdef bint is_symmetric(self):
    """
    Returns true if all the angles for the fixed scroll are the same as for the orbiting scroll
    """
    return (abs(self.phi_fi0-self.phi_oi0) < 1e-14
            and abs(self.phi_fis-self.phi_ois) < 1e-14
            and abs(self.phi_fie-self.phi_oie) < 1e-14
            and abs(self.phi_fo0-self.phi_oo0) < 1e-14
            and abs(self.phi_fos-self.phi_oos) < 1e-14
            and abs(self.phi_foe-self.phi_ooe) < 1e-14
            )
```

3) Class：PDSim.scroll.common_scroll_geo.Gr

- 面积的不定积分项

```
 cpdef double Gr(double phi, geoVals geo, double theta, int inv):
"""
The antiderivative of the area integration term, where

.. math::

    Gr \\equiv \\int\\left[\\left(-y\\frac{dx(\\phi)}{d\\phi}+x\\frac{dy(\\phi)}{d\\phi}\\right)d\\phi\\right]
"""

//求解关于初始装配条件下待定系数后的动涡盘等效曲柄转角
theta_m = geo.phi_fie - theta + 3.0*pi/2.0
if inv == INVOLUTE_FI:
    return phi*geo.rb**2*(phi**2 - 3*phi*geo.phi_fi0 + 3*geo.phi_fi0**2)/3
elif inv == INVOLUTE_FO:
    return phi*geo.rb**2*(phi**2 - 3*phi*geo.phi_fo0 + 3*geo.phi_fo0**2)/3
elif inv == INVOLUTE_OI:
    return geo.rb*(phi**3*geo.rb - 3*phi**2*geo.phi_oi0*geo.rb
                   + 3*phi*geo.phi_oi0**2*geo.rb
                   + 3*(phi-geo.phi_oi0)*geo.ro*cos(phi - theta_m)
                   - 3*geo.ro*sin(phi - theta_m))/3
elif inv == INVOLUTE_OO:
    return geo.rb*(phi**3*geo.rb - 3*phi**2*geo.phi_oo0*geo.rb
                   + 3*phi*geo.phi_oo0**2*geo.rb
                   + 3*(phi-geo.phi_oo0)*geo.ro*cos(phi - theta_m)
                   - 3*geo.ro*sin(phi - theta_m))/3
```

- 面积积分的对展开角φ的偏导数

```
 cpdef double dGr_dphi(double phi, geoVals geo, double theta, int inv):
```

```
"""
The partial derivative of Gr with respect to phi with theta held constant
"""

THETA = geo.phi_fie - theta - pi/2.0

if inv == INVOLUTE_FI:
    return geo.rb**2*(phi - geo.phi_fi0)**2
elif inv == INVOLUTE_FO:
    return geo.rb**2*(phi - geo.phi_fo0)**2
elif inv == INVOLUTE_OI:
    return geo.rb*(geo.rb*(phi - geo.phi_oi0)**2 + (phi- geo.phi_oi0)*geo.ro*sin(THETA - phi))
elif inv == INVOLUTE_OO:
    return geo.rb*(geo.rb*(phi - geo.phi_oo0)**2 + (phi- geo.phi_oo0)*geo.ro*sin(THETA - phi))
```

> - 面积积分的对曲轴转角θ的偏导数

```
 cpdef double dGr_dtheta(double phi, geoVals geo, double theta, int inv):
"""
The partial derivative of Gr with respect to theta with phi held constant
"""

THETA = geo.phi_fie - theta - pi/2.0

if inv == INVOLUTE_FI or inv == INVOLUTE_FO:
    return 0.0
elif inv == INVOLUTE_OI:
    return geo.rb*geo.ro*((phi - geo.phi_oi0)*sin(THETA - phi) - cos(THETA - phi))
elif inv == INVOLUTE_OO:
    return geo.rb*geo.ro*((phi - geo.phi_oo0)*sin(THETA - phi) - cos(THETA - phi))
```

4) class PDSim.scroll.common_scroll_geo.HTAnglesClass

> - 与涡旋压缩机腔室传热计算所需计算有关的角度的结构
> - double phi_1_i 内部渐开线外壁最大展开角
> - double phi_2_i 内部渐开线外壁最小展开角
> - double phi_1_o 外部渐开线内壁最大展开角
> - double phi_2_o 外部渐开线内壁最小展开角
> - double phi_i0 内部渐开线外壁原始展开角
> - double phi_o0 外部渐开线内壁原始展开角

```
 cdef class HTAnglesClass:
def __repr__(self):
    s = ''
    for k in ['phi_1_i','phi_2_i','phi_1_o','phi_2_o','phi_i0','phi_o0']:
        s += k + ' : ' + str(getattr(self,k)) + '\n'
    return s
```

5) PDSim.scroll.common_scroll_geo.VdV

> - 以一种通用的方式评估V和dV/dtheta[m^3/radian]
> - cpdef VdVstruct VdV(double theta, geoVals geo, CVInvolutes inv)

```
 cpdef VdVstruct VdV(double theta, geoVals geo, CVInvolutes inv):
"""
Evaluate V and dV/dtheta in a generalized manner for a chamber
"""
cdef double A_i, A_o, A_line_1 = 0, A_line_2 = 0, x_1, x_2, y_1, y_2, dx_1_dphi=0, dy_1_dphi=0, dx_2_dphi=0,
dy_2_dphi=0
cdef double dA_line_1_dtheta = 0, dA_line_2_dtheta = 0, dx_1_dtheta, dy_1_dtheta, dx_2_dtheta, dy_2_dtheta

## ------------------------ VOLUME -------------------------------

A_i = 0.5*(Gr(inv.Outer.phi_max, geo, theta, inv.Outer.involute) - Gr(inv.Outer.phi_min, geo, theta,
inv.Outer.involute))
if inv.has_line_1:
    _coords_inv_d_int(inv.Outer.phi_max, geo, theta, inv.Outer.involute, &x_1, &y_1)
    _coords_inv_d_int(inv.Inner.phi_max, geo, theta, inv.Inner.involute, &x_2, &y_2)
    A_line_1 = 0.5*(x_1*y_2 - x_2*y_1)
A_o = 0.5*(Gr(inv.Inner.phi_min, geo, theta, inv.Inner.involute) - Gr(inv.Inner.phi_max, geo, theta,
inv.Inner.involute))
if inv.has_line_2:
    _coords_inv_d_int(inv.Inner.phi_min, geo, theta, inv.Inner.involute, &x_1, &y_1)
    _coords_inv_d_int(inv.Outer.phi_min, geo, theta, inv.Outer.involute, &x_2, &y_2)
    A_line_2 = 0.5*(x_1*y_2 - x_2*y_1)

V = geo.h*(A_i + A_line_1 + A_o + A_line_2)

## ------------------------ DERIVATIVE -------------------------------

dA_i_dtheta = 0.5*(dGr_dphi(inv.Outer.phi_max, geo, theta, inv.Outer.involute)*inv.Outer.dphi_max_dtheta
                  +dGr_dtheta(inv.Outer.phi_max, geo, theta, inv.Outer.involute)
                  -dGr_dphi(inv.Outer.phi_min, geo, theta, inv.Outer.involute)*inv.Outer.dphi_min_dtheta
                  -dGr_dtheta(inv.Outer.phi_min, geo, theta, inv.Outer.involute)
                  )

if inv.has_line_1:
    coords_inv_dtheta(inv.Outer.phi_max, geo, theta, inv.Outer.involute, &dx_1_dtheta, &dy_1_dtheta)
    coords_inv_dtheta(inv.Inner.phi_max, geo, theta, inv.Inner.involute, &dx_2_dtheta, &dy_2_dtheta)
    _dcoords_inv_dphi_int(inv.Outer.phi_max, geo, theta, inv.Outer.involute, &dx_1_dphi, &dy_1_dphi)
    dx_1_dtheta += dx_1_dphi*inv.Outer.dphi_max_dtheta
    dy_1_dtheta += dy_1_dphi*inv.Outer.dphi_max_dtheta
    _dcoords_inv_dphi_int(inv.Inner.phi_max, geo, theta, inv.Inner.involute, &dx_2_dphi, &dy_2_dphi)
    dx_2_dtheta += dx_2_dphi*inv.Inner.dphi_max_dtheta
    dy_2_dtheta += dy_2_dphi*inv.Inner.dphi_max_dtheta
    dA_line_1_dtheta = 0.5*(x_1*dy_2_dtheta + y_2*dx_1_dtheta - x_2*dy_1_dtheta - y_1*dx_2_dtheta)

dA_o_dtheta = 0.5*(dGr_dphi(inv.Inner.phi_min, geo, theta, inv.Inner.involute)*inv.Inner.dphi_min_dtheta
                   +dGr_dtheta(inv.Inner.phi_min, geo, theta, inv.Inner.involute)
                   -dGr_dphi(inv.Inner.phi_max, geo, theta, inv.Inner.involute)*inv.Inner.dphi_max_dtheta
                   -dGr_dtheta(inv.Inner.phi_max, geo, theta, inv.Inner.involute)
                   )
```

```
if inv.has_line_2:
    coords_inv_dtheta(inv.Inner.phi_min, geo, theta, inv.Inner.involute, &dx_1_dtheta, &dy_1_dtheta)
    coords_inv_dtheta(inv.Outer.phi_min, geo, theta, inv.Outer.involute, &dx_2_dtheta, &dy_2_dtheta)

    _dcoords_inv_dphi_int(inv.Inner.phi_min, geo, theta, inv.Inner.involute, &dx_1_dphi, &dy_1_dphi)
    dx_1_dtheta += dx_1_dphi*inv.Inner.dphi_min_dtheta
    dy_1_dtheta += dy_1_dphi*inv.Inner.dphi_min_dtheta
    _dcoords_inv_dphi_int(inv.Outer.phi_min, geo, theta, inv.Outer.involute, &dx_2_dphi, &dy_2_dphi)
    dx_2_dtheta += dx_2_dphi*inv.Outer.dphi_min_dtheta
    dy_2_dtheta += dy_2_dphi*inv.Outer.dphi_min_dtheta

    dA_line_2_dtheta = 0.5*(x_1*dy_2_dtheta + y_2*dx_1_dtheta - x_2*dy_1_dtheta - y_1*dx_2_dtheta)

dV = geo.h*(dA_i_dtheta + dA_line_1_dtheta + dA_o_dtheta + dA_line_2_dtheta)

cdef VdVstruct VdV = VdVstruct.__new__(VdVstruct)
VdV.V = V
VdV.dV = dV
return VdV
```

6) class PDSim.scroll.common_scroll_geo.coords_inv

> - 对应于沿着渐开线（外部渐开线盘内部[fi]，外部渐开线盘外部渐开线[fo]，内部渐开线外部渐开线[oo]和内部渐开线内部渐开线[oi]）的点的渐开线角度
> - coords_inv(phi, geoVals geo, double theta, flag='fi') → tuple
> - phi_vec – 渐开线角度的一维数字数组或双精度矢量
> - geo – geoVals class scroll compressor geometry
> - theta – 单精度曲轴转角θ 0~2π
> - flag – 渐开线字符串flag： ‘fi’,’fo’,’oi’,’oo’
> - return - 涡旋盘坐标元组(数组) 如：(x,y)

```
 def coords_inv(phi,geo,theta,flag="fi")
if type(phi) is np.ndarray:
    return _coords_inv_np(phi,geo,theta,flag)
else:
    return _coords_inv_d(phi,geo,theta,flag)

//
cpdef tuple _coords_inv_np(np.ndarray[np.float_t] phi, geoVals geo,double theta, flag=""):
"""
Internal function that does the calculation if phi is definitely a 1D numpy vector
"""
rb = geo.rb
ro = rb*(pi - geo.phi_fi0 + geo.phi_oo0)
om = geo.phi_fie - theta + 3.0*pi/2.0

if flag=="fi":
    x = rb*np.cos(phi)+rb*(phi-geo.phi_fi0)*np.sin(phi)
    y = rb*np.sin(phi)-rb*(phi-geo.phi_fi0)*np.cos(phi)
elif flag=="fo":
```

```
    x = rb*np.cos(phi)+rb*(phi-geo.phi_fo0)*np.sin(phi)
    y = rb*np.sin(phi)-rb*(phi-geo.phi_fo0)*np.cos(phi)
elif flag=="oi":
    x = -rb*np.cos(phi)-rb*(phi-geo.phi_oi0)*np.sin(phi)+ro*np.cos(om)
    y = -rb*np.sin(phi)+rb*(phi-geo.phi_oi0)*np.cos(phi)+ro*np.sin(om)
elif flag=="oo":
    x = -rb*np.cos(phi)-rb*(phi-geo.phi_oo0)*np.sin(phi)+ro*np.cos(om)
    y = -rb*np.sin(phi)+rb*(phi-geo.phi_oo0)*np.cos(phi)+ro*np.sin(om)
else:
    raise ValueError('flag not valid')
return (x,y)

//
cpdef tuple _coords_inv_d(double phi, geoVals geo,double theta, flag=""):
"""
Internal function that does the calculation if phi is a double variable
"""

rb = geo.rb
ro = rb*(pi - geo.phi_fi0 + geo.phi_oo0)
om = geo.phi_fie - theta + 3.0*pi/2.0

if flag=="fi":
    x = rb*cos(phi)+rb*(phi-geo.phi_fi0)*sin(phi)
    y = rb*sin(phi)-rb*(phi-geo.phi_fi0)*cos(phi)
elif flag=="fo":
    x = rb*cos(phi)+rb*(phi-geo.phi_fo0)*sin(phi)
    y = rb*sin(phi)-rb*(phi-geo.phi_fo0)*cos(phi)
elif flag=="oi":
    x = -rb*cos(phi)-rb*(phi-geo.phi_oi0)*sin(phi)+ro*cos(om)
    y = -rb*sin(phi)+rb*(phi-geo.phi_oi0)*cos(phi)+ro*sin(om)
elif flag=="oo":
    x = -rb*cos(phi)-rb*(phi-geo.phi_oo0)*sin(phi)+ro*cos(om)
    y = -rb*sin(phi)+rb*(phi-geo.phi_oo0)*cos(phi)+ro*sin(om)
else:
    raise ValueError('flag not valid')
return (x,y)
```