

The simplest form of a standard 3-stage hierarchical Bayesian model in the context of modeling MEP size can be described as follows. Let there be N_P exchangeable sequences $\{(x_i^p, y_i^p)_{i=1}^{n(p)} \mid j = 1, 2 \dots N_P\}$ of MEP sizes $y_i^p \in \mathbb{R}^+$ recorded at stimulation intensity $x_i^p \in \mathbb{R}_0^+$ from p^{th} participant, for a total of N_P participants. Here $n(p)$ denotes the number of intensities tested for p^{th} participant.

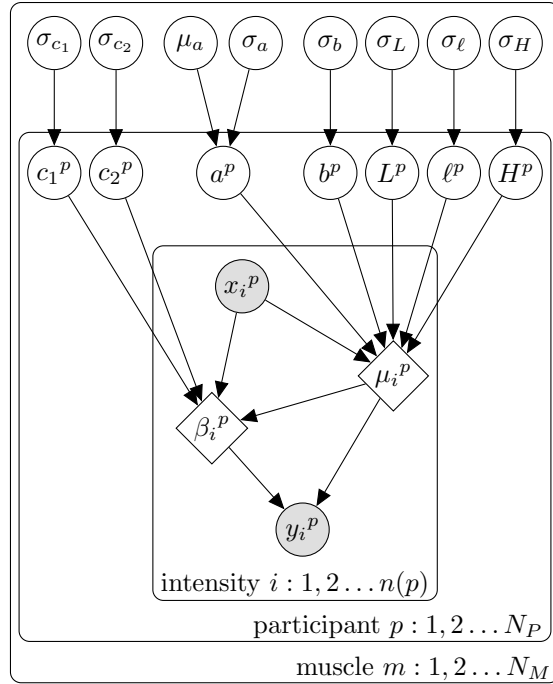
The first stage of hierarchy is the *participant-level* and specifies parametric model $P(y_i^p \mid x_i^p, \theta^p)$ for each of the N_P participants and models the MEP size y_i^p as a function of stimulus intensity x_i^p and participant-specific parameters θ^p . In the second stage, $\theta_1, \theta_2 \dots \theta^P$ are assumed to be exchangeable and generated from a common distribution $P(\theta^p \mid \gamma)$ with hyper-parameters γ . In the third stage, the hyper-parameters γ are assumed to be unknown and assigned *weakly-informative* prior, also called hyperprior-density $P(\gamma)$.

$$y_i^p \mid x_i^p, \theta^p, \gamma \sim P(y_i^p \mid x_i^p, \theta^p, \gamma) \quad (4.3.1)$$

$$\theta^p \mid \gamma \sim P(\theta^p \mid \gamma) \quad (4.3.2)$$

$$\gamma \sim P(\gamma) \quad (4.3.3)$$

Figure 4.3.1 shows the graphical representation of the default hierarchical model implemented by hbmeq for fitting recruitment-curves on TMS data [ref.] in [ref. to function comparison fig.].



Observation model

$$y_i^p \sim \text{Gamma}(\mu_i^p \cdot \beta_i^p, \beta_i^p)$$

$$\mu_i^p \leftarrow \mathcal{F}(x_i^p \mid a^p, \Omega^p)$$

$$\beta_i^p \leftarrow \frac{1}{c_1^p} + \frac{1}{c_2^p \cdot \mu_i^p}$$

Participant specific parameters

$$a^p \sim \text{TruncatedNormal}(\mu_a, \sigma_a)$$

$$\theta^p \sim \text{HalfNormal}(\sigma_\theta) \forall \theta^p \in \Omega^p$$

Priors

$$\mu_a \sim \text{TruncatedNormal}(50, 20)$$

$$\sigma_a \sim \text{HalfNormal}(30)$$

$$\sigma_L \sim \text{HalfNormal}(0.05)$$

$$\sigma_\theta \sim \text{HalfNormal}(5) \forall \sigma_\theta \in \Omega$$

$$\Omega^p \leftarrow \{b^p, L^p, \ell^p, H^p, c_1^p, c_2^p\}$$

$$\Omega \leftarrow \{\sigma_b, \sigma_\ell, \sigma_H, \sigma_{c_1}, \sigma_{c_2}\}$$