$$y \mid x \sim (1 - q_y) \cdot \text{Gamma}\left(\mu\beta, \beta\right) + q_y \cdot \text{HalfNormal}\left(\sigma_{\text{outlier}}\right) \tag{4.2.5}$$

$$q_y \sim \text{Bernoulli}\left(p_{\text{outlier}}\right) \tag{4.2.6}$$

$$p_{\text{outlier}} \sim \text{Uniform}\left(0, C_{p_{\text{outlier}}}\right) \tag{4.2.7}$$

$$\sigma_{\text{outlier}} \sim \text{HalfNormal}\left(C_{\sigma_{\text{outlier}}}\right) \tag{4.2.8}$$

where $C_{p_{\text{outlier}}}, C_{\sigma_{\text{outlier}}} > 0$ are constants and $C_{p_{\text{outlier}}} < 1$ is chosen to be small, usually in the range $0.01 - 0.02$. Intuitively, this means we expect there to be roughly $1\% - 2\%$ outliers which will be captured by the HalfNormal distribution.