Counting

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1. unusual

order doesn't matter

$\{u, n, s, a, l\} \rightarrow \binom{4}{4} = 1$ unique set of letters where only 1 u

$\{u, u, \underbrace{\_, \_, \_}_{\{n,s,a,l\}}\} \rightarrow \binom{4}{3} = 4$ unique sets of letters when 2 u's

$\{u, u, u, \underbrace{\_, \_}_{\{n,s,a,l\}}\} \rightarrow \binom{4}{2} = 6$ unique sets of letters when 3 u's

order now matters

1 u: $\{u, \_, \_, \_, \_\}$

$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5! \times 1$

2 u's: $\{\overset{1}{u}, \overset{2}{u}, \_, \_, \_\} = \frac{5!}{2!} \times 4$

$\overset{1}{u}\,n\,\overset{2}{u}\,a\,l$
$\overset{2}{u}\,n\,\overset{1}{u}\,a\,l$
same string, diff combo
can interchange the 2 u's

3 u's: $\{\overset{1}{u}, \overset{2}{u}, \overset{3}{u}, \_, \_\}$ $\frac{5!}{3!} \times 6$

$\overset{1}{u}\overset{2}{u}\overset{3}{u}al$
$\overset{1}{u}\overset{3}{u}\overset{2}{u}al$
$\overset{2}{u}\overset{1}{u}\overset{3}{u}al$
...

1a) unique subset of 5 letters (of the 7)

$$\binom{4}{4} + \binom{4}{3} + \binom{4}{2}$$

1b) How many different strings made?

$$(5!)(1) + \left(\frac{5!}{2!}\right)(4) + \left(\frac{5!}{3!}\right)(6) = 480 \text{ strings}$$

2. 52 cards   5 card hand

↓

2 pairs + 1 other

13 possible card values to choose for pairs, pick 2 for the 2 pairs

$$\boxed{\underbrace{\binom{13}{2}\binom{4}{2}\binom{4}{2}}_{2 \text{ pairs}}\underbrace{\binom{11}{1}\binom{4}{1}}_{\text{5th non pair card must be different value as the pairs}} = 123,552}$$

3. 16 songs   7 couples

order does not matter

r-combinations of a set of n objects where repetition is allowed

n = # of couples, r = songs

↳ distribution of songs on set of n couples

$$\frac{n-1+r}{n-1}$$

2 cases

fighting couple gets 1 song + fighting couple gets zero songs

6 possible couples : n
15 songs = r

6 possible couples = n
16 songs = r

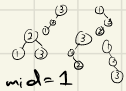
$$\binom{6-1+15}{6-1} + \binom{6-1+16}{6-1}$$

$$\boxed{\binom{20}{5} + \binom{21}{5}}$$

$$15504 + 20349 = \boxed{35853}$$

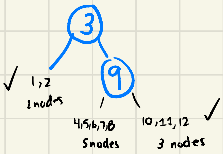
4.

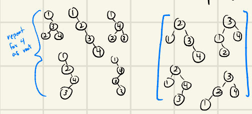
BST 2 Nodes = **2** smallest = 1 largest = 1 mid = 0



3 Nodes = **5** smallest = 2 largest = 2 mid = 1

4 nodes = 14 smallest = 5 largest = 5 mid = 4





root: 2 nodes, 1 node + root: 1 node, 2 nodes

2 ways + 2 ways = 4 ways

\+

root: 3 nodes + root: 3 nodes

5 ways + 5 ways = 10 ways

= 14 ways

$$\boxed{\underbrace{2}_{\text{2 nodes}} \cdot \underbrace{5}_{\text{3 nodes}} \cdot \underbrace{42}_{\text{5 nodes}} = 420 \text{ ways}}$$

5 nodes = smallest = 14 largest = 14 mid =

root (4 nodes) + root (4 nodes) + root (3 nodes, 1 node) + root (1 n, 3 n) + root (2 n, 2 n)

14 ways + 14 ways + 5 ways + 5 ways + 4 ways

= 42 ways

5. 4 identical nurses, order does not matter

Case 1	Case 2
4 nurses available	3 nurses available
1 1 1 7	1 1 8
1 1 2 6	1 2 7
1 1 3 5	1 3 6
1 1 4 4	1 4 5
1 2 2 5	2 2 6
1 2 3 4	2 3 5
2 2 2 4	3 3 4
2 2 3 3	4 4 2
3 3 3 1	
} 9	} 8

$\boxed{\text{17 combinations}}$