Probability

Haley Boren

1. 15 students 8 questions, no more than 1 question per student

$$\underbrace{\frac{15}{15} * \frac{14}{15} * \frac{13}{15} * \frac{12}{15} * \frac{11}{15} * \frac{10}{15} * \frac{9}{15} * \frac{8}{15}}_{\text{8 questions, 15 students}} = \frac{259459200}{2562890625} = \boxed{0.1012}$$

2. 00000 - 99999

$$\underset{\text{odd}\ ①}{5} \cdot \underset{\text{odd}\ ②}{4} \cdot \underset{\text{left over } 4/5}{6 \cdot 7} \cdot \underset{\text{even}\ ③}{5} = 4200 \text{ possible combinations}$$

$$p(\text{win}) = \frac{4200}{10^5} = 0.042$$

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

$$p(\text{lose}) = 1 - p(\text{win}) = 0.958$$

generate 8 choose 5 disregard order

$$\boxed{\binom{8}{5} (0.042)^5 (0.958)^3 = 0.0000064}$$

win 5 lose 3

3. A = at least 2 dice show >= 4

B = all 3 dice show the same value $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{6^3}$

A & B are independent if $P(A \cap B) = P(A) \cdot P(B)$

$A = P(\text{Exactly 2 dice roll} >= 4) + P(\text{3 dice roll} >= 4)$

$$\binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) \qquad \binom{3}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$$

$$\frac{3}{8} + \frac{1}{8}$$

$\frac{3}{6}$ chance roll >=4 on any given roll:

1 2 3 | 4 5 6

$$\frac{4}{8} = \frac{1}{2}$$

does not matter what 1st roll is

$$B = \frac{\cancel{6}}{6} \cdot \frac{1}{\cancel{6}} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \boxed{\frac{1}{72}}$$

$$P(A \cap B) = \frac{1}{6^3} + \frac{1}{6^3} + \frac{1}{6^3}$$

all 4's
all 5's
all 6's

$$= \frac{3}{216} = \boxed{\frac{1}{72}}$$

$$\boxed{P(A \cap B) = P(A) \cdot P(B) = \frac{1}{72}}$$

A & B are independent

4. flush → 5 card hand same suit

possible flush combinations: $\underset{\text{suit}}{\binom{4}{1}} \underset{\text{Choose from that suit}}{\binom{13}{5}}$

$$P(\text{hand is a flush}) = \frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}} = 0.0019808$$

all possible 5 card hands

X = # of hands to play before getting a flush

by definition X is geometric

p (which is 0.0019808) is $0 \le p \le 1$ & we care about 1st success,

so $E(X) = \frac{1}{p} = \frac{1}{0.0019808}$

$$\boxed{= 504.84 \text{ hands}}$$

5. $P(\text{Win} \mid \text{Star}) = 0.70$

$P(\text{win} \mid !\text{Star}) = 0.50$

Set # of trials w/ set probs, binom

$X =$ wins out of last 5

$P(\text{Star plays series}) = 0.75$

$P(X=4 \mid \text{star}) = \binom{5}{4}(.7)^4(.3)^1 = 0.36015$

$P(X=4 \mid !\text{star}) = \binom{5}{4}(.5)^4(.5)^1 = 0.15625$

$P(X=4) = P(\text{star}) * P(X=4 \mid \text{star}) + P(\overline{\text{star}})(P(X=4) \mid !\text{star})$

$= (0.75)(0.36015) + (0.25)(0.15625)$

$= 0.3092$

X=4 indicates won 4 out of 5 games

$P(\text{star} \mid X=4) = \dfrac{P(X=4 \mid \text{star}) \cdot P(\text{star})}{P(X=4)} = \dfrac{(0.36105)(0.75)}{0.3072}$

$\boxed{= 0.874 \text{ chance the superstar played given won last 4 of 5 games}}$