

Bivariate Correlation

Spearman Correlation Coefficient $r = \frac{\text{covariance}}{\text{standard deviation}}$

$$\text{Covariance} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

Example 1:

The following table shows the student degree in statistics and science. Find out the Spearman Correlation Coefficient among them.

Student	1	2	3	4	5	6	7	8	9	10
Statistics	20	23	8	29	14	12	11	20	17	18
Science	20	25	11	24	23	16	12	21	22	26

	A	B	C	D	E	F	G	H	I
1	Student	statistics	science	x-xbar	y-ybar	(x-xbar)(y-ybar)		(x-xbar)^2	(y-ybar)^2
2	1	20	20	2.8	0	0		7.84	0
3	2	23	25	5.8	5	29		33.64	25
4	3	8	11	-9.2	-9	82.8		84.64	81
5	4	29	24	11.8	4	47.2		139.24	16
6	5	14	23	-3.2	3	-9.6		10.24	9
7	6	12	16	-5.2	-4	20.8		27.04	16
8	7	11	12	-6.2	-8	49.6		38.44	64
9	8	20	21	2.8	1	2.8		7.84	1
10	9	17	22	-0.2	2	-0.4		0.04	4
11	10	18	26	0.8	6	4.8		0.64	36
12	Mean=	17.2	20			227		349.6	252
13					covariance=	25.22		5.91	5.02
14							Standard Deviation=	29.68	
15						Spearman Correlation Coefficient r =	0.85		

Example 2:

Find the Spearman Correlation Coefficient between the Intelligence Ratio (IT) and Emotional Ration (ER) from the following data.

Student	1	2	3	4	5	6	7	8	9	10
IR	105	104	102	101	100	99	98	96	93	32
ER	101	103	100	98	95	96	104	98	97	94

	A	B	C	D	E	F	G	H	I
1	Student	IR	ER	x-xbar	y-ybar	(x-xbar)(y-ybar)		(x-xbar)^2	(y-ybar)^2
2	1	105	101	6	2.4	14.4		36	5.76
3	2	104	103	5	4.4	22		25	19.36
4	3	102	100	3	1.4	4.2		9	1.96
5	4	101	98	2	-0.6	-1.2		4	0.36
6	5	100	95	1	-3.6	-3.6		1	12.96
7	6	99	96	0	-2.6	0		0	6.76
8	7	98	104	-1	5.4	-5.4		1	29.16
9	8	96	98	-3	-0.6	1.8		9	0.36
10	9	93	97	-6	-1.6	9.6		36	2.56
11	10	92	94	-7	-4.6	32.2		49	21.16
12	Mean=	99	98.6			74		170	100.4
13					covariance=	8.22		4.12	3.17
14							Standard Deviation=	13.06	
15						Spearman Correlation Coefficient r =	0.63		

Example 3:

Calculate the Find Spearman Correlation Coefficient among x and y using the following data.

x	2	4	5	6	8	11
y	18	12	10	8	7	5

	A	B	C	D	E	F	G	H	I
1	x	y	ER	x-xbar	y-ybar	(x-xbar)(y-ybar)		(x-xbar)^2	(y-ybar)^2
2	2	18	101	8	2.1666667	17.33333333		64	4.694444
3	4	12	103	2	4.1666667	8.333333333		4	17.361111
4	5	10	100	0	1.1666667	0		0	1.361111
5	6	8	98	-2	-0.8333333	1.666666667		4	0.694444
6	8	7	95	-3	-3.8333333	11.5		9	14.69444
7	11	5	96	-5	-2.8333333	14.16666667		25	8.027778
8	Mean=	10	98.8333			53		106	46.83333
9					covariance=	10.60		4.20	2.79
10							Standard Deviation=	11.74	
11						Spearman Correlation Coefficient r =	0.90		

Example 4:

10 participants in a contest are ranked by two judges as follows. Find Spearman Correlation Coefficient.

x	1	3	7	5	4	6	2	10	9	8
y	3	1	4	5	6	9	7	8	10	2

	A	B	C	D	E	F	G	H
1	Judge 1(x)	Judge 2(y)	x-xbar	y-ybar	(x-xbar)(y-ybar)		(x-xbar)^2	(y-ybar)^2
2	1	3	-4.5	-2.5	11.25		20.25	6.25
3	3	1	-2.5	-4.5	11.25		6.25	20.25
4	7	4	1.5	-1.5	-2.25		2.25	2.25
5	5	5	-0.5	-0.5	0.25		0.25	0.25
6	4	6	-1.5	0.5	-0.75		2.25	0.25
7	6	9	0.5	3.5	1.75		0.25	12.25
8	2	7	-3.5	1.5	-5.25		12.25	2.25
9	10	8	4.5	2.5	11.25		20.25	6.25
10	9	10	3.5	4.5	15.75		12.25	20.25
11	8	2	2.5	-3.5	-8.75		6.25	12.25
12	5.5	5.5			34.5		82.5	82.5
13				covariance=	3.83		2.87	2.87
14						Standard Deviation=	8.25	
15					Spearman Correlation Coefficient r =	0.46		

Example 4:

Two judges in a beauty contest rank the 12 contestants as follows. Find Spearman Correlation Coefficient.

x	1	2	3	4	5	6	7	8	9	10	11	12
y	12	19	6	10	3	5	4	7	8	2	11	1

	A	B	C	D	E	F	G	H
1	Judge 1(x)	Judge 2(y)	x-xbar	y-ybar	(x-xbar)(y-ybar)		(x-xbar)^2	(y-ybar)^2
2	1	12	-5.50	4.67	-25.67		30.25	21.78
3	2	19	-4.50	11.67	-52.50		20.25	136.11
4	3	6	-3.50	-1.33	4.67		12.25	1.78
5	4	10	-2.50	2.67	-6.67		6.25	7.11
6	5	3	-1.50	-4.33	6.50		2.25	18.78
7	6	5	-0.50	-2.33	1.17		0.25	5.44
8	7	4	0.50	-3.33	-1.67		0.25	11.11
9	8	7	1.50	-0.33	-0.50		2.25	0.11
10	9	8	2.50	0.67	1.67		6.25	0.44
11	10	2	3.50	-5.33	-18.67		12.25	28.44
12	11	11	4.50	3.67	16.50		20.25	13.44
13	12	1	5.50	-6.33	-34.83		30.25	40.11
14	6.50	7.33			-110.00		143.00	284.67
15				covariance=	-10.00		3.45	4.87
16						Standard Deviation=	16.81	
17					Spearman Correlation Coefficient r =	-0.59		