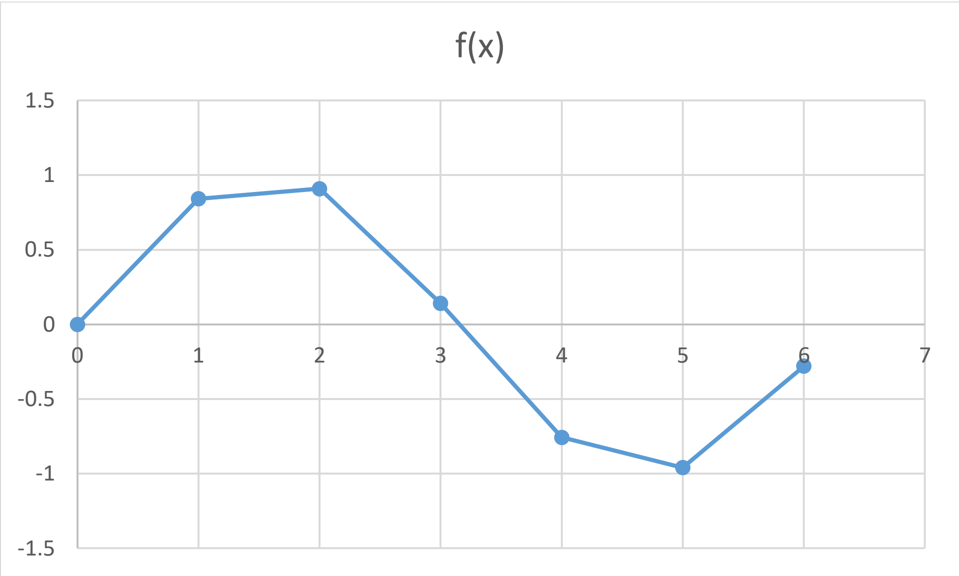
Interpolation: Interpolation is a type of estimation, a method of constructing (finding) new data points based on the range of a discrete set of known data points.

For example, for an unknown function f(x), few data points are given as under.

x	**y=f(x)**
Argument	**Entry**
0	0
1	0.8415
2	0.9093
3	0.1411
4	-0.7568
5	-0.9589
6	-0.2794

Find/estimate/interpolate the value of y=f(x) for x=2.5.
If I plot a graph for the above mentioned data point, it would look something like:



Linear interpolation: One of the simplest methods is linear interpolation (sometimes known as lerp). Consider the above example of estimating f(2.5). Since 2.5 is midway between 2 and 3, it is reasonable to take f(2.5) midway between f(2) = 0.9093 and f(3) = 0.1411, which yields **0.5252**.

Generally, linear interpolation takes two data points, say (x_a,y_a) and (x_b,y_b), and the interpolant is given by:
$y = y_a + (y_b – y_a) [(x-x_a) / (x_b-x_a)]$

Let us try to evaluate the value of y for x=2.5 with (x_a,y_a)=(2,0.9093) and (x_b,y_b)=(3,0.1411).
y = 0.9093 + (0.1411 – 0.9093) [(2.5-2) / (3-2)] = **0.5252**
Its quick and easy but not precise.

Interpolation Methods

Method Name	Condition	Remark
Newton Forward Interpolation Method	Subsequent values of x are at **equal** distance	If the value of x lies in the forward of the table
Newton Backward Interpolation Method	Subsequent values of x are at **equal** distance	If the value of x lies in the backward of the table
Binomial Interpolation Method		Used for finding the missing terms in the table of X
LaGrange's Interpolation Method	Subsequent values of x are at **unequal** distance	

Newton Forward Interpolation Method

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)(u-2)\ldots(u-n+1)}{n!}\Delta^n y_0$$

Example 1:

x		y=f(x)	
x_0	3	y_0	180
x_1	5	y_1	150
x_2	7	y_2	120
x_3	9	y_3	90

Find/estimate/interpolate the value of y=f(x) for x=4.

Can we apply Newton Forward Interpolation Method? Yes.
Why? Because $x_1-x_0 = x_2-x_1 = x_3-x_2 = x_n - x_{n-1} = h = 2$

Now, $u = \frac{x-x0}{h} = \frac{4-3}{2} = 0.5$

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)(u-2)\ldots(u-n+1)}{n!}\Delta^n y_0$$

	A	B	C	D	E	F	G
1	x=	4					
2	h=	2					
3	u=	0.5					
4	x		y=f(x)				
5	x_0	3	y_0	180	Δy	$\Delta^2 y$	$\Delta^3 y$
6				$\Delta y_0 =$	-30		
7	x_1	5	y_1	150	$\Delta^2 y_0 =$	0	
8					-30	$\Delta^3 y_0 =$	0
9	x_2	7	y_2	120		0	
10					-30		
11	x_3	9	y_3	90			
12							
13	y=	165					

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)(u-2)\ldots(u-n+1)}{n!}\Delta^n y_0$$

y = 165

Example 2:

x		y=f(x)	
x_0	2	y_0	14.5
x_1	3	y_1	16.3
x_2	4	y_2	17.5
x_3	5	y_3	18

Find/estimate/interpolate the value of y=f(x) for x=2.5.

Can we apply Newton Forward Interpolation Method? Yes.
Why? Because x_1-x_0 = x_2-x_1 = x_3-x_2 = x_n –x_{n-1}=h=1

Now, u = $\frac{x-x0}{h}$ = $\frac{2.5-2}{1}$ = 0.5

	A	B	C	D	E	F	G
1	x=	2.5					
2	h=	1					
3	u=	0.5					
4	x		y=f(x)				
5	x_0	2	y_0	14.5	Δ y	Δ^2 y	Δ^3 y
6				$\Delta\, y_0$ =	1.8		
7	x_1	3	y_1	16.3	$\Delta^2 y_0$ =	-0.6	
8					1.2	$\Delta^3 y_0$ =	-0.1
9	x_2	4	y_2	17.5		-0.7	
10					0.5		
11	x_3	5	y_3	18			
12							
13	y=	15.4688					

$$y = y_0 + \frac{u}{1!}\Delta\, y_0 + \frac{u\,(u-1)}{2!}\Delta^2\, y_0 + \frac{u\,(u-1)(u-2)}{3!}\Delta^3\, y_0 + \ldots.. + \frac{u\,(u-1)(u-2)\ldots(u-n+1)}{n!}\Delta^n\, y_0$$

y = 15.4688

Example 3:

x		y=f(x)	
x_0	0	y_0	0
x_1	1	y_1	0.8415
x_2	2	y_2	0.9093
x_3	3	y_3	0.1411
x_4	4	y_4	-0.7568
x_5	5	y_5	-0.9589
X_6	6	y_6	-0.2794

Find/estimate/interpolate the value of y=f(x) for x=2.5.

Can we apply Newton Forward Interpolation Method? Yes.

Why? Because x_1-x_0 = x_2-x_1 = x_3-x_2 = x_n –x_{n-1}=h=1

Now, u = $\frac{x-x0}{h}$ = $\frac{2.5-0}{1}$ = 2.5

	A	B	C	D	E	F	G	H	I	J
1	x=	2.5								
2	h=	1								
3	u=	2.5								
4	x		y=f(x)							
5	x_0	0	y_0	0	Δ y	Δ^2 y	Δ^3 y	Δ^4 y	Δ^5 y	Δ^6 y
6				$\Delta y_0 =$	0.8415					
7	x_1	1	y_1	0.8415	$\Delta^2 y_0 =$	-0.7737				
8					0.0678	$\Delta^3 y_0 =$	-0.0623			
9	x_2	2	y_2	0.9093		-0.836	$\Delta^4 y_0 =$	0.7686		
10					-0.7682		0.7063	$\Delta^5 y_0 =$	-0.6494	
11	x_3	3	y_3	0.1411		-0.1297		0.1192	$\Delta^6 y_0 =$	-0.1095
12					-0.8979		0.8255		-0.7589	
13	x_4	4	y_4	-0.7568		0.6958		-0.6397		
14					-0.2021		0.1858			
15	x_5	5	y_5	-0.9589		0.8816				
16					0.6795					
17	X_6	6	y_6	-0.2794						
18										
19	y=	0.45946								

$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + ... + \frac{u(u-1)(u-2)...(u-n+1)}{n!}\Delta^n y_0$

y = 0.45946

Example 4:

Marks		Number of Students	
x_0	30-40	y_0	31
x_1	40-50	y_1	42
x_2	50-60	y_2	51
x_3	60-70	y_3	35
X_4	70-80	Y_4	31

Find/estimate/interpolate the number of students who obtain marks between 40 to 45 from the above table.

h = length of the interval = 10

x = we would first find the values below 45 (i.e. x=45) (and then subtract 31 [below 40] from it to get the answer for range 40 to 45)

x_0 = 40 (below 40)

Now, $u = \frac{x-x0}{h} = \frac{45-40}{10} = 0.5$

	A	B	C	D	E	F	G	H
1			x=	45	between 40 to 45			
2			h=	10				
3			u=	0.5				
4	x		y=f(x)					
5	x_0= below 40	40	y_0	31	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
6				Δy_0 =	42			
7	x_1=below 50	50	y_1	73	$\Delta^2 y_0$ =	9		
8					51	$\Delta^3 y_0$ =	-25	
9	x_2=below 60	60	y_2	124		-16	$\Delta^4 y_0$ =	37
10					35		12	
11	x_3=below 70	70	y_3	159		-4		
12					31			
13	x_4=below 80	80	y_4	190				
14								
15			y'=	47.86719	48			
16			y=	16.86719	17			

$y' = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)(u-2)\ldots(u-n+1)}{n!}\Delta^n y_0$

y' = 48

Answer y = 48 (below 45) – 31 (below 40) = 17 (between 40 to 45)

Lagrange’s Interpolation Method/Formula

Applicable even if the values of x are not at equal distance (unlike to Newton’s Forward/Backward interpolation formula)