Quantum Computing

Annexure – I

Complex **Conjugate** (A^*): Negate the imaginary part

Matrix **Transpose** (T): Interchange rows and columns

$[A^*]^T = [A^T]^* = A^\dagger$ (dagger)

Unitary Matrix: $U\,U^\dagger = I$

Hermitian Matrix: $H = H^\dagger$

Eigen Value & Vector $A\vec{v} = \lambda\vec{v}$

$|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

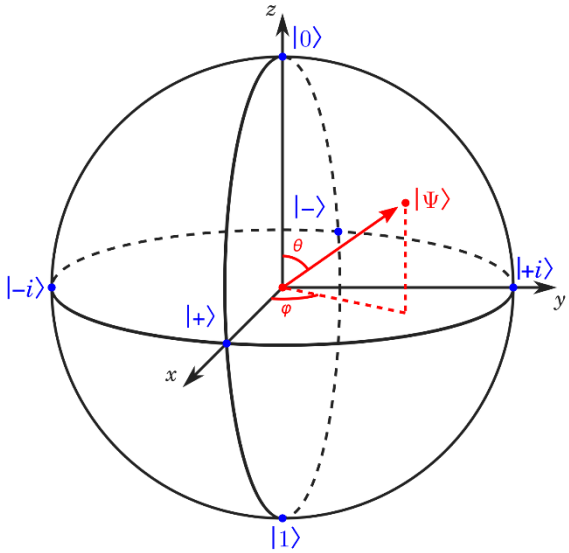
α: How much the qubit is in $|0\rangle$ state.

β: How much the qubit is in $|1\rangle$ state.

$|0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}, |1\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$

Probability: $|\alpha|^2 + |\beta|^2 = 1$

Bloch Sphere:



$$|+\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|i\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|-i\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

Gates

$$\boldsymbol{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \boldsymbol{Y} = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \qquad \boldsymbol{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Hadamard\ \boldsymbol{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathrm{H}|0\rangle \rightarrow |+\rangle$$
$$\mathrm{H}|+\rangle \rightarrow |0\rangle$$
$$\mathrm{H}|1\rangle \rightarrow |-\rangle$$
$$\mathrm{H}|-\rangle \rightarrow |1\rangle$$

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\mathrm{i}\frac{\pi}{2}} \end{bmatrix}$$

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\mathrm{i}\frac{\pi}{4}} \end{bmatrix}$$

CNOT: Controlled NOT Gate: $If\ the\ Control\ Qubit\ is$ 1, $the\ Target\ bit\ is\ flipped.$

Toffoli Gate: Controlled NOT Gate with 2 control qubits: $If\ all\ the\ Control\ Qubit\ are$ 1, $the\ Target\ bit\ is\ flipped.$