MACHINE LEARNING IN PHYSICS
TUTORIAL 06 / PINN

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Recap: PINN

Task: use physics-informed neural networks to solve the differential equation describing the damped harmonic oscillator,

$$\frac{d^2u}{dz^2} + 2\alpha\frac{du}{dz} + u = 0$$

where u is the displacement of an object and z is the dimensionless time.

https://www.mathwarehouse.com/harmonic-motion/interactive-damped-oscillator.php

Recap: Basic Idea

1. Model solution $u(z)$ with a neural network $f(x,\theta)$, where θ are free parameters, $x = (z, u_0, v_0, \alpha)$, and the initial conditions are $u(0) = u_0, \ \ \frac{du}{dz} = v_0$.

2. Minimize a *weighted sum* of the components:

    1. $R_{ODE}(\theta)$ imposes an ODE constraint;

    2. $R_C(\theta)$ imposes initial/boundary conditions,

    3. $R_D(\theta)$ imposes constraints provided by data.

PINNs: Basic Idea

The average loss function $R(\theta)$ is weighted sum of

$$R_{ODE}(\theta) = \frac{1}{N_{ODE}} \sum_{i=1}^{N_{ODE}} \left[\mathcal{F}\big(f(x_i, \theta)\big)\right]^2$$

$$R_C(\theta) = \left(f(x_i, \theta)\Big|_{z=0} - u_0 \right)^2 + \left(\frac{df(x_i, \theta)}{dz}\bigg|_{z=0} - v_0 \right)^2$$

$$R_D(\theta) = \frac{1}{N_D} \sum_{i=1}^{N_D} [f(x_i, \theta) - u(x_i)]^2$$

$\mathcal{F}\big(f(x, \theta)\big) = 0$ is the differential equation.

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PINN: Model

Model Components

$$x = (z, u_0, v_0, \alpha).$$

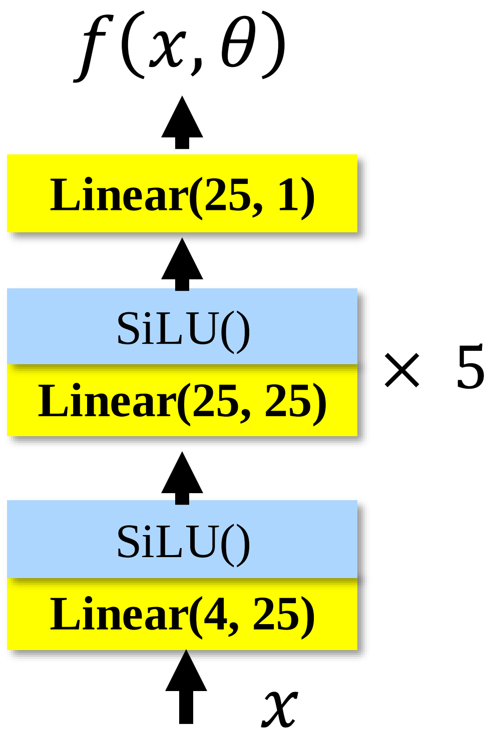
$$f(x, \theta) = \mathrm{FCN}(\dots)$$

$$u(x, \theta) = \mathrm{Solution}(f(x, \theta), \dots)$$

$$O(\theta) = \mathrm{Objective}(u(x, \theta), \dots)$$

$$R(\theta) = \mathrm{mean}(O^2)$$

$$\boxed{u(x, \theta) = \frac{u_0 + v_0\, z + f(x, \theta) z^2}{1 + z^2}}$$



FCN: Fully-Connected Network (aka: Multi-Layer Perceptron)

PINN: Model Training

Domain

$x \in [0, 20] \otimes [-1, 1] \otimes [-2, 2] \otimes [0, 0.5]$

Training

1. Sample size: 60,000
2. Learning rate: 10^{-3}
3. Batch size: 256
4. Iterations: 200,000

Training time ~ 18 minutes.