MACHINE LEARNING IN PHYSICS
TUTORIAL 06 / PINN

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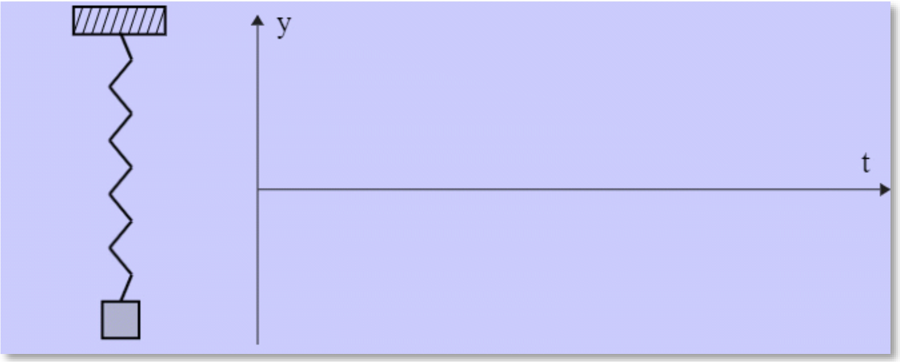
Recap: PINN

Task: use physics-informed neural networks to solve the differential equation describing the damped harmonic oscillator,

$$\frac{d^2x}{dz^2} + 2\zeta\frac{dx}{dz} + x = 0$$

where u is the displacement of an object and z is the dimensionless time.



https://www.mathwarehouse.com/harmonic-motion/interactive-damped-oscillator.php

Recap: Basic Idea

1. Model solution $x(z)$ with a neural network $f(p, \theta)$, where θ are free parameters, $p = (z, x_0, v_0, \zeta)$, and the initial conditions are $x(0) = x_0$, $\frac{dx}{dz} = v_0$.

2. Minimize a *weighted sum* of the components:

   1. $R_{ODE}(\theta)$ imposes an ODE constraint;

   2. $R_C(\theta)$ imposes initial/boundary conditions,

   3. $R_D(\theta)$ imposes constraints provided by data.

PINNs: Basic Idea

The average loss function $R(\theta)$ is weighted sum of

$$R_{ODE}(\theta) = \frac{1}{N_{ODE}} \sum_{i=1}^{N_{ODE}} \left[\mathcal{F}\big(f(p_i,\theta)\big)\right]^2$$

$$R_C(\theta) = \left(f(p_i,\theta)\Big|_{z=0} - x_0\right)^2 + \left(\frac{df(p_i,\theta)}{dz}\Big|_{z=0} - v_0\right)^2$$

$$R_D(\theta) = \frac{1}{N_D} \sum_{i=1}^{N_D} \left[f(p_i,\theta) - x(p_i)\right]^2$$

$\mathcal{F}\big(f(p,\theta)\big) = 0$ is the differential equation.

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PINN: Model

Model Components

$$p = (z, x_0, v_0, \zeta).$$

$$f(p, \theta) = \text{FCN}(\ldots)$$

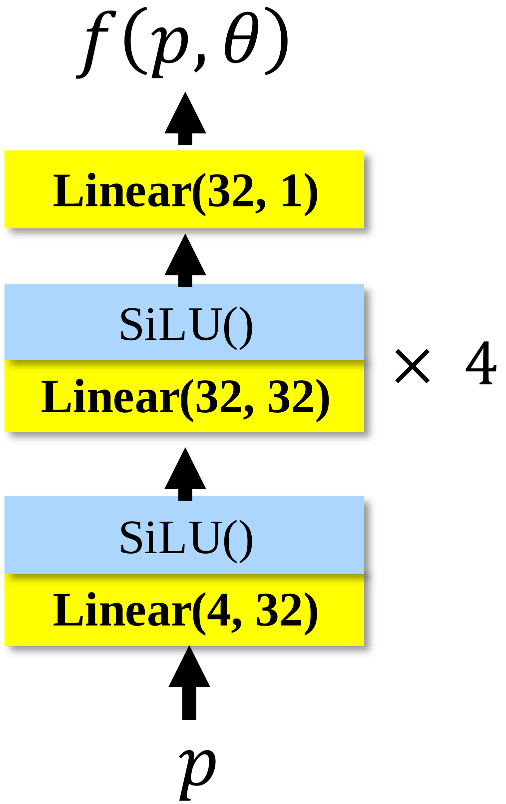
$$x(p) = \text{Solution}(f(p, \theta), \ldots)$$

$$O(\theta) = \text{Objective}(x(p), \ldots)$$

$$R(\theta) = \text{mean}(O^2)$$

Ansatz

$$\boxed{x(z) = \frac{x_0 + v_0\, z + f(p, \theta) z^2}{1 + z^2}}$$



FCN: Fully-Connected Network (aka: Multi-Layer Perceptron)

PINN: Model Training

Domain

$p \in [0, 20] \otimes [-1, 1] \otimes [-2, 2] \otimes [0, 0.5]$

Training

1. Sample size: 250,000
2. Learning rate: 5×10^{-4}
3. Batch size: 200
4. Iterations: 150,000

Training time ~ 18 minutes.