roofit_example

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1 RooFit Tutorial: Fit Cosmological Models to Type 1a Supernova Data

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1.1 Introduction

In this tutorial, I show how to use the probability modeling package **RooFit**, released with the CERN data analysis package **ROOT**, to perform simple fits of cosmological models to Type 1a supernova data. See the **minuit_example.ipynb** to get some background to this problem.

prerequisite: some familiarity with **Python** and **ROOT** is helpful.

1.2 Other Dependencies

This tutorial uses the github package **histutil**, which contains some simple ROOT-based utilities. To install this package do

git clone https://github.com/hbprosper/histutil.git
and source the setup.sh script.

1.3 PART 1: Create Probability Model

Note: if you want to make your plots interactive, do **%jsroot on**. But, it is off by default so that the LaTex annotation works correctly.

```
In [1]: import os, sys
        import ROOT
        from histutil import setStyle, mkgraph, mkgraphErrors, mkhist1, Scribe
        %jsroot off
```

```
Welcome to JupyROOT 6.10/02
```

1.3.1 Model parameters

- ID: model identifier
- free: specifies whether parameter is free
- name: name of parameter
- guess: starting (or fixed) value of parameter
- step: step size during minimization

- min, max: parameter range

```
In [2]: #                      ID    free, name,  guess,    step,   min,  max
        PARAMS = {'LCDM' :     [0, [(True,  'OM',     1,    1.e-3,     0,   10),
                                    (True,  'OL',     0,    1.e-3,     -10, 10),
                                    (True,  'H0',    70,    1.e-2,     0,  200)]],

                  'phantom':   [1, [(False, 'OM',     1,    1.e-3,     0,   10),
                                    (False, 'OL',     0,    1.e-3,     -10, 10),
                                    (True,  'H0',    70,    1.e-2,     0,  200),
                                    (True,  'n',      2,    1.e-3,     0,   10)]]
                 }
        # define ranges for redshifts and distance moduli
        ZMIN  = 0.0
        ZMAX  = 1.5
        MUMIN = 32.0
        MUMAX = 48.0
```

1.3.2 Choose model

- MODEL = 'LCDM' or 'phantom'

```
In [3]: MODEL = 'phantom'
```

1.3.3 Compile C++ classes Model and CosmicCode

- **Model** defines $\Omega(a)$ for the cosmological models
- **CosmicCode** computes the distance modulus
- The model identifier, ID, is used later in some inline C++ code. Therefore, ID needs to be added to the ROOT namespace; that is, ROOT needs to know about it.

```
In [4]: def compileCode(modelname, modelparams):
            ROOT.gROOT.ProcessLine(open('../CosmicCode.cc').read())

            # make sure model name is valid
            if not modelparams.has_key(modelname):
                sys.exit("** unknown model %s" % modelname)

            # return model id and model parameters
            ID, params = modelparams[modelname]

            # add ID to ROOT namespace
            ROOT.gROOT.ProcessLine('int ID = %d;' % ID)
            return params

In [5]: params = compileCode(MODEL, PARAMS)
```

1.3.4 A useful bit of ROOT magic: mixing in a bit of C++

Here we create a bit of C++ code, which will be visible to both ROOT and Python. The object **code** and the function **distanceModulus** are used below in the definition of the probability model.

```
In [6]: %%cpp -d
        Model model(ID);
        CosmicCode code(model);
        double distanceModulus(double z, double OM, double OL, double H0, double n=2)
        {
            double p[4] = {OM, OL, H0, n};
            return code.distanceModulus(z, p);
        }


        phantom model
```

check that **ROOT.distanceModulus** is visible to Python

```
In [7]: print ROOT.distanceModulus(1, 0.3, 0.7, 70, 2)

43.9845970443
```

1.3.5 Create workspace

Create an empty workspace called **Type1a**. This is useful for at least two reasons. First, you may wish to save the probability model you have created to a ROOT file. Second, it is much more convenient to use the workspace methods to build a model than to use the **RooFit** C++ classes directly.

```
In [8]: # suppress all messages except fatal ones
        ROOT.RooMsgService.instance().setGlobalKillBelow(ROOT.RooFit.WARNING)

        wspace = ROOT.RooWorkspace('Type1a')


RooFit v3.60 -- Developed by Wouter Verkerke and David Kirkby
                Copyright (C) 2000-2013 NIKHEF, University of California & Stanford University
                All rights reserved, please read http://roofit.sourceforge.net/license.txt
```

1.3.6 Create parameters

Create RooFit parameters using the workspace factory method.

```
In [9]: def createParameters(wspace, params,
                             zmin=ZMIN, zmax=ZMAX, mumin=MUMIN, mumax=MUMAX):
            # measured redshift
            wspace.factory('z[%f, %f]' % (zmin, zmax))

            # measured distance modulus
            wspace.factory('x[%f, %f]' % (mumin, mumax))

            # distance modulus uncertainty
            wspace.factory('dx[0, 2]')

            wspace.defineSet('set_data',  'z,x,dx')

            # cosmological parameters
            args = 'z'
            for (free, name, guess, step, xmin, xmax) in params:
                cmd = '%s[%f, %f, %f]' % (name, guess, xmin, xmax)
                wspace.factory(cmd)
                args += ',%s' % name
                if not free:
                    wspace.var(name).setConstant()

            wspace.defineSet('set_arguments', args)

            wspace.Print()
            return args

In [10]: arguments = createParameters(wspace, params)
         print 'arguments for distanceModulus: (%s)' % arguments

arguments for distanceModulus: (z,OM,OL,H0,n)

RooWorkspace(Type1a) Type1a contents

variables
---------
(H0,OL,OM,dx,n,x,z)

named sets
----------
set_arguments:(z,OM,OL,H0,n)
set_data:(z,x,dx)
```

1.3.7 Load Type 1a supernova data

```
In [11]: # ------------------------------------------------------------
         # read Type Ia data
```

```
# format: name, z, x, dx
# name:     name of supernova
# z         measured redshift of supernova
# x +/- dx: measured distance modulus
# ------------------------------------------------------------
def readData(wspace, filename):
    import os
    from array import array
    from string import split, atof
    if not os.path.exists(filename):
        sys.exit("** can't open file %s" % filename)

    # skip first 5 lines and convert 2nd through 4th
    # columns to floats
    data = map(lambda x: map(atof, x[1:-1]),
                   map(split,
                           open(filename).readlines()[5:]))

    # create a RooFit data set
    set_data = wspace.set('set_data')
    dataset = ROOT.RooDataSet('data', 'Type 1a data', set_data)

    print "number of observations: %d" % len(data)
    print "%5s\t%10s\t%10s +/- %-10s" % ('', 'z', 'x', 'dx')

    # and fill it with the supernova data
    # also copy data to arrays
    z  = array('d')
    x  = array('d')
    dz = array('d')
    dx = array('d')
    for ii, d in enumerate(data):
        z.append(d[0])
        x.append(d[1])
        dz.append(0)
        dx.append(d[2])

        set_data['z'].setVal(z[-1])
        set_data['x'].setVal(x[-1])
        set_data['dx'].setVal(dx[-1])
        dataset.add(set_data)

        if ii % 100 == 0:
            print "%5d\t%10.3f\t%10.4f +/- %-10.4f"%\
              (ii, z[-1], x[-1], dx[-1])

    # import data set into workspace
    getattr(wspace, 'import')(dataset)
```

```
            wspace.Print()
            return (z, x, dz, dx)
```

```
In [12]: data = readData(wspace, '../SCPUnion2.1_mu_vs_z.txt')
```

```
number of observations: 580
                     z                x +/- dx
    0            0.028          35.3466 +/- 0.2239
  100            0.065          37.3067 +/- 0.1628
  200            0.194          39.9615 +/- 0.1264
  300            0.620          43.2280 +/- 0.3903
  400            0.710          43.0220 +/- 0.1843
  500            0.564          42.3729 +/- 0.2920

RooWorkspace(Type1a) Type1a contents

variables
---------
(H0,OL,OM,dx,n,x,z)

datasets
--------
RooDataSet::data(z,x,dx)

named sets
----------
set_arguments:(z,OM,OL,H0,n)
set_data:(z,x,dx)
```

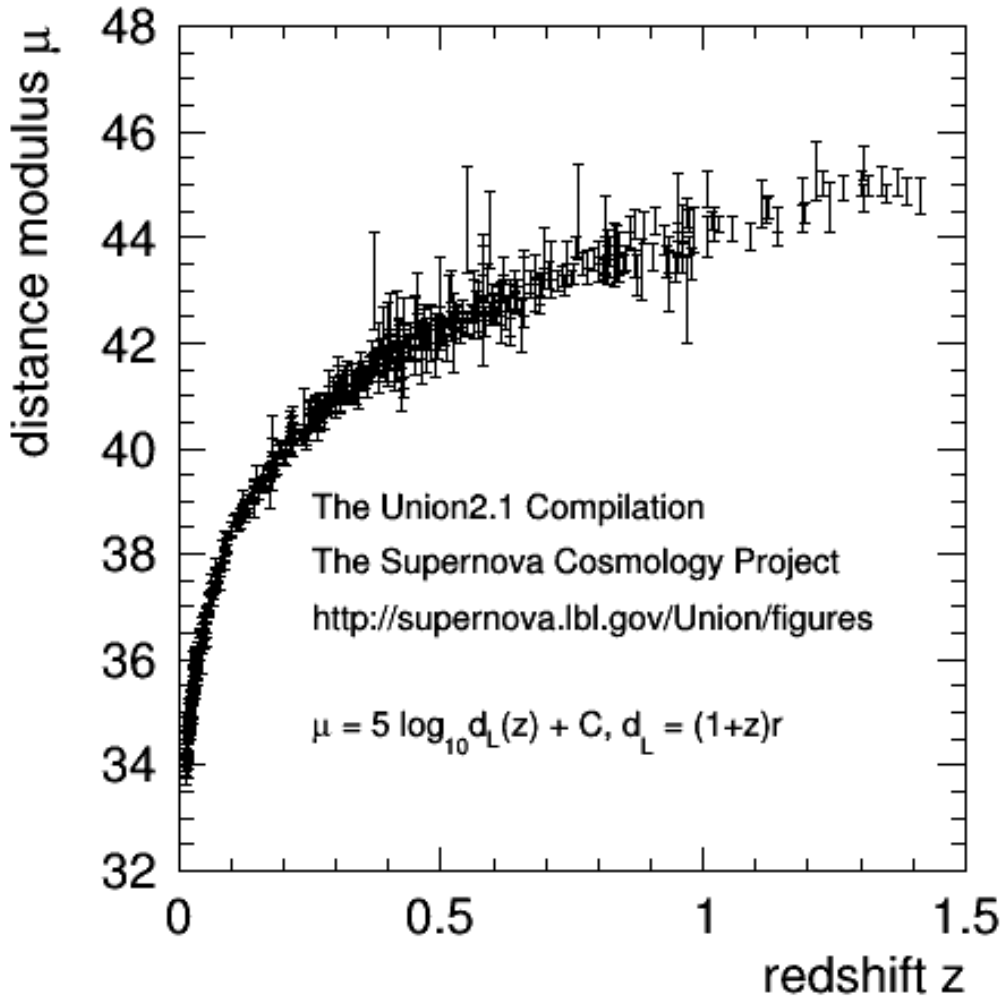
```
In [13]: setStyle()
```

```
         def plotData(data, code, zmin=ZMIN, zmax=ZMAX, mumin=MUMIN, mumax=MUMAX):
             from array import array

             print "plot data"

             z, x, dz, dx = data
             ndata   = len(z)
             print "number of observations: %d" % ndata

             g = mkgraphErrors(z, x, dz, dx,
                               "redshift z",
                               "distance modulus #mu",
                               zmin, zmax,
                               ymin=mumin,
```

```
                                ymax=mumax,
                                color=ROOT.kBlack)
            ROOT.SetOwnership(g, 0)
            g.SetName('dataplot')
            g.SetTitle('')
            g.SetMarkerSize(0.2)

            c = ROOT.TCanvas("fig_data", "SN1a data", 500, 500)
            ROOT.SetOwnership(c, 0)

            c.cd()
            g.Draw("ap")

            xpos = 0.32
            ypos = 0.50
            textsize = 0.035

            scribe = Scribe(xpos, ypos, textsize)
            ROOT.SetOwnership(scribe, 0)
            scribe.write("The Union2.1 Compilation")
            scribe.write("The Supernova Cosmology Project")
            scribe.write("http://supernova.lbl.gov/Union/figures")
            scribe.write("")
            scribe.write("#mu = 5 log_{10}d_{L}(z) + C, d_{L} = (1+z)r")
            c.SaveAs('.pdf')
            c.Draw()

            return g
```

```
In [14]: gd = plotData(data, ROOT.code)
```

```
plot data
number of observations: 580


Info in <TCanvas::Print>: pdf file ./fig_data.pdf has been created
```



1.3.8 Create probability model

We assume a Gaussian probability density function (pdf),

$$f(x \mid z, \sigma, \theta) = \text{Gauss}(x, \mu(z, \theta), \sigma),$$

where θ are the parameters of the cosmological model. Neglecting correlations amongst the data, the likelihood of the data is

$$p(x \mid z, \sigma, \theta) = \prod_{i=1}^{N} f(x_i \mid z_i, \sigma_i, \theta),$$

with $N = 580$. For a Bayesian calculation, we would, in addition, need to specify a prior density $\pi(\theta)$ for the cosmological parameters. We assume that z_i and σ_i are known with negligible uncertainty.

```
In [15]: def createModel(wspace, arguments):
             #-------------------------------------------------------------
             # create distance modulus expression
             # "mu".
             # note use of compiled C++ program
             # for expressions that are too
             # complicated to be written inline
             #-------------------------------------------------------------
             print "create model"

             set_arguments = wspace.set('set_arguments')
             cmd = 'distanceModulus(%s)' % arguments
             print cmd

             mu  = ROOT.RooFormulaVar('mu', '#mu', cmd, ROOT.RooArgList(set_arguments))

             # import the "mu" expression into workspace
             getattr(wspace,'import')(mu)

             # finally, create probability model
             wspace.factory('Gaussian::model(x, mu, dx)')
             wspace.Print()

In [16]: createModel(wspace, arguments)

create model
distanceModulus(z,OM,OL,H0,n)

RooWorkspace(Type1a) Type1a contents

variables
---------
(H0,OL,OM,dx,n,x,z)

p.d.f.s
-------
RooGaussian::model[ x=x mean=mu sigma=dx ] = 0.601823

functions
--------
RooFormulaVar::mu[ actualVars=(z,OM,OL,H0,n) formula="distanceModulus(z,OM,OL,H0,n)" ] = 42.7578

datasets
--------
RooDataSet::data(z,x,dx)

named sets
----------
```

```
set_arguments:(z,OM,OL,H0,n)
set_data:(z,x,dx)
```

1.3.9 Save workspace to a ROOT file

```
In [17]: filename = '%s_workspace.root' % MODEL
         if wspace.writeToFile(filename) != 0:
             print "** problem saveing workspace to ROOT file %s" % filename
```

1.4 PART 2: Perform Fit

Ordinarily, this part would be performed in a separate session, which would require loading the workspace back into memory as follows:

```
filename = '%s_workspace.root' % MODEL
rfile = TFile(filename)
if not rfile.IsOpen():
    exit("** can't open file %s" % filename)

name = 'Type1a'
wspace = rfile.Get(name)
if wspace == None:
    exit("** can't get workspace %s from file %s" % (name, filename))
```

However, we shall just continue with the workspace already in memory.

```
In [18]: wspace.Print()
```

```
RooWorkspace(Type1a) Type1a contents

variables
---------
(H0,OL,OM,dx,n,x,z)

p.d.f.s
-------
RooGaussian::model[ x=x mean=mu sigma=dx ] = 0.601823

functions
--------
RooFormulaVar::mu[ actualVars=(z,OM,OL,H0,n) formula="distanceModulus(z,OM,OL,H0,n)" ] = 42.7578

datasets
--------
RooDataSet::data(z,x,dx)
```

```
named sets
----------
set_arguments:(z,OM,OL,H0,n)
set_data:(z,x,dx)
```

```
In [19]: def performFit(wspace, params, printLevel=-1):
             print "="*80
             print "performing fit..."
             swatch = ROOT.TStopwatch()
             swatch.Start()

             # save results of fit to results
             ROOT.RooMsgService.instance().setGlobalKillBelow(ROOT.RooFit.FATAL)
             r = wspace.pdf('model').fitTo(wspace.data('data'),
                                           ROOT.RooFit.Save(),
                                           ROOT.RooFit.PrintLevel(printLevel))
             print "real time: %10.3f s" % swatch.RealTime()
             print "="*80
             r.Print()
             print "="*80

             results = []
             npar = 0
             for p in params:
                 free, name = p[:2]
                 results.append((wspace.var(name).getVal(), wspace.var(name).getError()))
                 if free:
                     npar += 1
             return (results, npar)

In [20]: results, npar = performFit(wspace, params)

================================================================================
performing fit...
real time:      3.049 s
================================================================================
================================================================================

  RooFitResult: minimized FCN value: 1447.61, estimated distance to minimum: 8.69769e-10
                covariance matrix quality: Full, accurate covariance matrix
                Status : MINIMIZE=0 HESSE=0

    Floating Parameter    FinalValue +/-  Error
  --------------------  --------------------------
                    H0    7.0425e+01 +/-  4.53e-01
                     n    2.8213e+00 +/-  1.87e-01
```

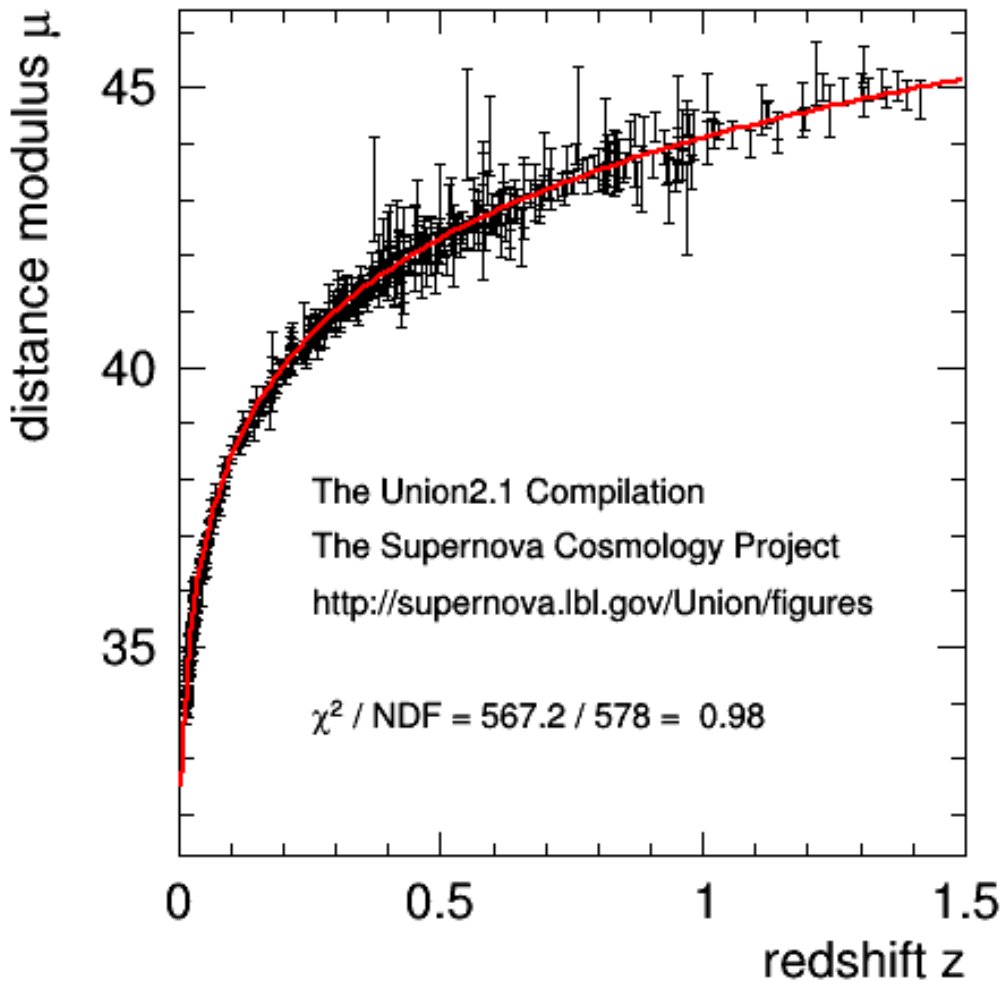
1.5 Plot Results

```
In [21]: def annotate(modelname, scribe, results, offset=0.01):
             if modelname == 'LCDM':
                 OM = tuple(results[0])
                 OL = tuple(results[1])
                 scribe.write("#LambdaCDM model")
                 scribe.write("")
                 scribe.write("#Omega(a) = #frac{#Omega_{M}}{a^{3}} + "
                       "#frac{(1 - #Omega_{M} - #Omega_{#Lambda})}{a^{2}}"
                               " + #Omega_{#Lambda}",
                               offset)
                 scribe.write("")
                 scribe.write("#Omega_{M} = %5.2f #pm %-5.2f" % OM,
                               offset)
                 scribe.write("#Omega_{#Lambda} = %5.2f #pm %-5.2f " % OL,
                               offset)
             else:
                 n, dn = results[3]
                 x = 3.0/(2*n)
                 G = ROOT.TMath.Gamma(x)
                 T = G*ROOT.TMath.Sqrt(2.718)*2**x/n
                 scribe.write("phantom model")
                 scribe.write("")
                 scribe.write("#Omega(a) = #frac{#Omega_{M}}{a^{3}} + "
                           "#frac{e^{a^{n}-1} - #Omega_{M}}{a^{3}}", offset)
                 scribe.write("")
                 scribe.write("H_{0}t = #sqrt{e} 2^{3/(2n)} #Gamma(3/(2n), a^{n}/2)/n", offset)
                 scribe.write("")
                 scribe.write("H_{0}t_{rip} = #sqrt{e} 2^{3/(2n)} #Gamma(3/(2n))/n = %4.2f" % T,
                               offset)
                 scribe.write("")
                 scribe.write("where #Gamma(s, x) = #int_{0}^{x} t^{s-1} e^{-t} dt", offset)
                 scribe.write("and n = %4.2f" % n, offset)

In [22]: def plotModel(modelname, code, results, npar,
                       gd, data,
                       zmin=ZMIN, zmax=ZMAX, mumin=MUMIN, mumax=MUMAX):
             from array import array
             p = array('d')
             for value, error in results:
                 p.append(value)

             z, x, dz, dx = data
             ndata = len(z)
```

```
# compute chisq
chi2 = 0.0
for i in xrange(ndata):
    mu = code.distanceModulus(z[i], p)
    c = (x[i] - mu)/dx[i]
    chi2 += c*c
NDF = ndata - npar # number of degrees of freedom

# compute curve
nz = 100
zstep = (zmax - zmin) / nz
zz = array('d')
mu = array('d')
for ii in xrange(nz):
    zz.append( (ii+0.5)*zstep )
    mu.append( code.distanceModulus(zz[-1], p) )

g = mkgraph(zz, mu,
            "redshift z",
            "distance modulus #mu",
            zmin, zmax, color=ROOT.kRed, lwidth=2)
ROOT.SetOwnership(g, 0)
g.SetName('model')
g.SetTitle('')

c = ROOT.TCanvas("fig_%s_fit" % modelname, "SN1a model fit", 500, 500)
ROOT.SetOwnership(c, 0)

c.cd()
g.Draw('ac')
gd.Draw('psame')
g.Draw('csame')

xpos = 0.32
ypos = 0.50
textsize = 0.035
scribe = Scribe(xpos, ypos, textsize)
ROOT.SetOwnership(scribe, 0)
scribe.write("The Union2.1 Compilation")
scribe.write("The Supernova Cosmology Project")
scribe.write("http://supernova.lbl.gov/Union/figures")
scribe.write("")
scribe.write("#chi^{2} / NDF = %5.1f / %d = %5.2f" % (chi2, NDF, chi2/NDF))

c.Draw()
c.SaveAs(".pdf")
```

```
In [23]: plotModel(MODEL, ROOT.code, results, npar, gd, data)
```

```
Info in <TCanvas::Print>: pdf file ./fig_phantom_fit.pdf has been created
```

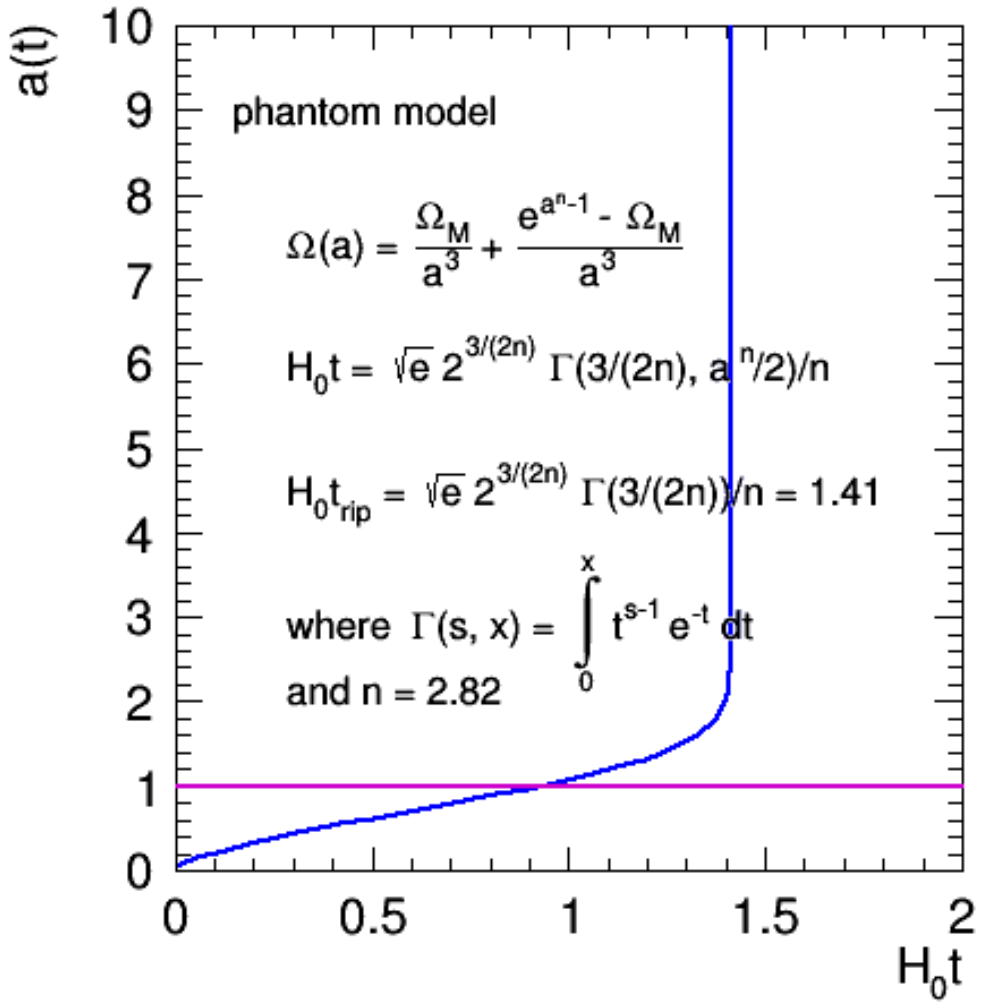



```
In [24]: def plotScaleFactor(modelname, code, results, amax=10, tmax=2):
             from array import array

             p = array('d')
             for value, error in results:
                 p.append(value)

             # create a vs H0 t plot
             a = array('d'); a.fromlist(code.N*[0])
             t = array('d'); t.fromlist(code.N*[0])
```

```
    code.scaleFactor(amax, p, t, a)
    g = mkgraph(t, a,
                "H_{0}t}", "a(t)",
                0, tmax, color=ROOT.kBlue, lwidth=2)
    ROOT.SetOwnership(g, 0)
    g.SetName('scaleFactor')

    # create horizontal line at a = 1
    x = array('d'); x.append(0); x.append(tmax)
    y = array('d'); y.append(1); y.append(1)
    glineH = mkgraph(x, y, '', '', 0, tmax,
                     color=ROOT.kMagenta+1, lwidth=2)
    ROOT.SetOwnership(glineH, 0)
    glineH.SetName('line')

    c = ROOT.TCanvas("fig_%s_scaleFactor" % modelname,
                     "SN1a scalefactor",
                     500, 500)
    ROOT.SetOwnership(c, 0)
    c.cd()
    htmp = mkhist1('htmp', 'H_{0}t', 'a(t)',
                       50, 0, tmax, ymin=0, ymax=10)
    ROOT.SetOwnership(htmp, 0)
    htmp.Draw()
    g.Draw('csame')
    glineH.Draw('csame')

    offset = 0.05
    xpos = 0.25
    ypos = 0.87
    scribe = Scribe(xpos, ypos)
    ROOT.SetOwnership(scribe, 0)
    annotate(modelname, scribe, results, offset)
    c.Draw()
    c.SaveAs(".pdf")
```

```
In [25]: plotScaleFactor(MODEL, ROOT.code, results)
```

```
Info in <TCanvas::Print>: pdf file ./fig_phantom_scaleFactor.pdf has been created
```


a(t)
10
9
8
7
6
5
4
3
2
1
0
phantom model
$\Omega(a) = \frac{\Omega_M}{a^3} + \frac{e^{a^n-1} - \Omega_M}{a^3}$
$H_0 t = \sqrt{e}\, 2^{3/(2n)}\, \Gamma(3/(2n), a^n/2)/n$
$H_0 t_{rip} = \sqrt{e}\, 2^{3/(2n)}\, \Gamma(3/(2n))/n = 1.41$
where $\Gamma(s, x) = \int_0^x t^{s-1} e^{-t}\, dt$
and n = 2.82
0
0.5
1
1.5
2
$H_0 t$

In [ ]: