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Discourse Semantics with Information Structure

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Abstract

The property of projection poses a challenge to formal semantic theories, due to its apparent non-compositional nature. Projected content is therefore typically analyzed as being different from and independent of asserted content. Recent evidence, however, suggests that these types of content in fact closely interact, thereby calling for a more integrated analysis that captures their similarities, while respecting their differences. Here, we propose such a unified, compositional semantic analysis of asserted and projected content. Our analysis captures the similarities and differences between presuppositions, anaphora, conventional implicatures and assertions on the basis of their *information structure*, that is, on basis of *how* their content is contributed to the unfolding discourse context. We formalize our analysis in an extension of the dynamic semantic framework of Discourse Representation Theory (DRT)—called Projective DRT (PDRT)—that employs projection variables to capture the information-structural aspects of semantic content; different constellations of such variables capture the differences between the different types of projected and asserted content within a single dimension of meaning. We formally derive the structural and compositional properties of PDRT, as well as its semantic interpretation. By instantiating PDRT as a mature semantic formalism, we argue that it paves way for a more focused investigation of the information-structural aspects of meaning.

1 Introduction

Any viable semantic theory should capture aspects of meaning beyond that of asserted content alone. One such aspect is the property of *projection*: the indifference of semantic content to the syntactic scope of its embedding operators, such as negation, implication, modal operators and interrogative constructions. In the sentence 'The king of France is bald', for instance, the definite description 'the king of France' triggers a *presupposition* about the existence of a person that is the king of France. If this sentence is embedded under a negation operator ('It is not the case that the king of France is bald'), this presupposition survives, whereas the asserted content of the sentence (i.e. that he is bald) becomes negated. Crucially, projection phenomena—of which presuppositions have traditionally been considered the most paradigmatic case—pose a challenge for semantic theories because of their apparent non-compositional nature. A case in point is the formal treatment of presupposition projection as anaphora resolution proposed by van der Sandt (1992). This analysis involves a two-stage resolution algorithm for presuppositions, formulated in Discourse Representation Theory (DRT; Kamp 1981; Kamp & Reyle 1993; Kamp *et al.* 2011), in which presuppositions are first identified and represented at their introduction site, and accordingly resolved by either binding to an antecedent or accommodation at a suitable discourse context (taking into account pragmatic constraints on resolution). Presuppositions are therefore only resolved after discourse construction is completed, thus interfering with the compositional construction procedure of DRT. In addition to this difficulty in the compositional treatment of presuppositions, another challenge for semantic theories is to account for the full range of projection phenomena; that is, since Potts' redefinition of Conventional Implicatures (CIs; Potts 2003, 2005), the class of projection phenomena has been expanded to include presuppositions and anaphora, as well as CIs (see e.g. Simons *et al.* 2010). Consequently, existing theories of presupposition projection need to be augmented to account for these different types of projected content. This has inspired semantic analyses in which the contributions of CIs, presuppositions and asserted or 'at-issue' content are analyzed independently of each other, in separate dimensions or layers of meaning (see e.g. Geurts & Maier 2003; Potts 2005). However, these accounts have recently been challenged by evidence from various sources suggesting a close interaction between the different types of projected and asserted content (see e.g. Amaral *et al.* 2007; AnderBois *et al.* 2010; Koev 2014; Nouwen 2007; Schlenker 2013). Critically, this calls for a compositional semantic analysis in which the different types of projected and asserted content are analyzed within a single dimension or layer of meaning, thereby capturing their interactions, while respecting their differences.

Here, we propose such a unified semantic analysis of asserted and projected content that captures the similarities and differences between CIs, presuppositions, anaphora and asserted content on the basis of their *information structure*, that is, in terms of *givenness* and *backgroundedness*. This analysis entails that these linguistic phenomena do not differ in terms of *what* type of contribution they make, but rather in *how* their contribution relates to the unfolding discourse context (thus precluding different dimensions or layers of meaning). We formalize our analysis in the dynamic semantic framework of Projective Discourse Representation Theory (Venhuizen 2015; Venhuizen *et al.* 2013, 2014), which adds projection variables to the semantic representations of traditional DRT. It will be shown that different constellations of these projection variables naturally account for the information-structural differences between the different types of projected and asserted content. Hence, PDRT extends the semantic representations of DRT with an explicit notion of information structure. Importantly, by treating projection directly in terms of variable binding, we preclude a two-stage resolution algorithm (cf. van der Sandt 1992) and maintain the compositional machinery of DRT. We discuss how PDRT relates to existing extensions of DRT, which have been proposed to account for similar or related aspects of meaning, and we speculate how it can be integrated with some of the core ideas underlying these different extensions in order to arrive at an even more comprehensive formalism. Crucially, the addition of projection variables to DRT affects its structural and compositional properties in a non-trivial manner. To show the soundness of the PDRT formalism, we therefore derive its formal properties for constructing and combining semantic representations, which we also implemented as part of an open-source Haskell library called PDRT-SANDBOX (Venhuizen & Brouwer 2014). Overall, PDRT thus provides a rich and mature semantic formalism that paves way for a more focused investigation of information-structural aspects of meaning.

2 Information Structure and Dynamic Semantics

Linguistic utterances typically contain different types of information, including references to already established information, backgrounded comments and contributions that are marked as important by the speaker. The way in which these different types of information are organized in a linguistic utterance is called the *information structure* (Halliday 1967) or *information packaging* (Chafe 1974) of the utterance. The information structure is often reflected by the linguistic form of the utterance (Lambrecht 1996), for instance by means of word ordering, choice of referential form, morphological marking and prosody (see Arnold *et al.* 2013, for an overview). For example, when referring to an entity, the use of an indefinite description (e.g. ‘a dog’) signals that the entity has not been introduced before, while the use of a definite description (‘the dog’) or even a pronoun (‘it’) indicates that the entity is specific and familiar to both the hearer and the speaker. Based on these observations, various approaches have classified referential expressions according to their information status, distinguishing them in terms of (various levels of) ‘*givenness*’ (e.g. Ariel 1988; Gundel *et al.* 1993; Prince 1981).

Here, we will argue that asserted and projected content do not differ in terms of *what* type of contribution they make, but rather in *how* their contribution is related to the discourse context; presuppositions, anaphora, CIs as well as asserted content only differ in terms of information structure. These information-structural differences can be described on the basis of the notions of ‘givenness’ and ‘backgroundedness’. We will argue that any viable dynamic semantic formalism should therefore incorporate a means to represent information structure.

2.1 Projection as information structure

The information conveyed by presuppositions has often been described as being similar to the information conveyed by anaphoric expressions (see e.g. Geurts 1999; Kripke 2009; Soames 1979; van der Sandt, 1989, 1992). Like anaphora, presuppositions signal established or *given* information. This correspondence is illustrated by their similar acceptability constraints, exemplified in (1) (adapted from Beaver & Geurts 2011).

(1) a. If a farmer owns a donkey, then he feeds it.
  b. If Fred left, then Mary knows that Fred left.
  c. #If a farmer doesn't own a donkey, he feeds it.
  d. #If Fred didn't leave, then Mary knows that Fred left.

In (1b) and (1d), the factive verb 'knows' triggers the presupposition that its complement is true (i.e. Fred left). In (1b), this presupposition is 'satisfied' as part of the conditional statement, in the same way that the anaphoric relation triggered by the pronoun 'it' in (1a) is satisfied by the earlier introduction of 'a donkey'. If the conditional statement is denied, however, both the presupposition and the anaphoric relation cannot be satisfied, resulting in the infelicity of (1c) and (1d). Interestingly, presuppositions differ from anaphora in that they can occur felicitously in contexts in which their content has not been mentioned before. This is illustrated in (2a–b), which show the bare versions of (1a–b) (i.e. without the conditional statement).

(2) a. ??He feeds it.
  b. Mary knows that Fred left.

Without any context, (2a) is infelicitous because no reference can be established for the pronouns 'he' and 'it'. The sentence in (2b), on the other hand, is felicitous despite the fact that the presupposed content (i.e. 'Fred left') has not been mentioned before. In fact, the failure of the givenness assumption results in a sentence that itself presupposes this content to be true. The process of 'adding' a presupposition to the discourse context in case no antecedent can be found has been called *presupposition accommodation* (Karttunen 1974; Lewis 1979; Stalnaker 1974). Based on these observations about the relation between presuppositions and anaphora, van der Sandt developed a theory of presuppositions that is often referred to as 'Binding and Accommodation Theory' (van der Sandt 1989, 1992; see also Geurts 1999). On this account, presupposition projection is formalized as a type of anaphora resolution. We will describe this analysis in more detail in Section 2.2 below.

Like anaphora and presuppositions, conventional implicatures (CIs; in terms of Potts, 2003, 2005) also have the property of projection. Unlike the former two, however, CIs do not signal given information—in fact, they are infelicitous in contexts where their content has already been established, as illustrated in (3) (from Koev 2014).

(3) ?? If Obama is a socialist, then the President, <u>who is a socialist,</u> will raise taxes.

In (3) the non-restrictive relative clause triggers a conventional implicature (i.e. that the President is a socialist). Just as in the first two sentences of (1) above, the projected content is part of the conditional statement. However, in the case of the CI, this givenness of the projected content renders the entire sentence infelicitous; since CIs signal *novel* information, the CI contribution in (3) seems to be redundant. Formally, Potts (2005) defines CIs as entailments that follow from the conventional meaning of lexical items or constructions, but

Table 1 Information status of projected and asserted content. The information status of different types of semantic content is described in terms of the felicity conditions in different contexts. The contexts can be divided into those in which the content is *given*, *new and backgrounded* or *new and at-issue*. The identifiers '+' and '−' respectively denote the felicity and infelicity of the types of content in the individual contexts.

	Given	New	
		backgrounded	*at-issue*
Anaphora	+	−	−
Presuppositions	+	+	−
CIs	−	+	−
Assertions	−	−	+

that do not affect the truth conditions of the 'at-issue' content (cf. Grice 1975; Horn 2007; Potts 2015). It is the latter property, that is shared between CIs and other projected content. Simons *et al.* (2010) emphasize this point, stating that the property of projection can be explained in terms of 'not at-issueness', which they define using the concept of the 'Question Under Discussion' (QUD; Roberts 1996). The QUD is a set of alternative propositions that represent the topic of a discourse. The goal of the discourse is to resolve this question, and felicitous conversational moves are taken to be those that address the QUD. According to Simons *et al.* (2010), the projection behavior of presuppositions, anaphora and CIs can be explained by the observation that these contributions do not address the at-issue content of the utterance in which they occur (i.e. they are 'not at-issue'). Since operators such as modals and negation typically target at-issue content, not at-issue content remains unaffected by entailment-cancelling operators, thus explaining their projection behavior.

Hence, we can describe the different contributions made by asserted content and the different types of projected content in terms of differences in information structure: asserted content always contributes new information that is at-issue, whereas projected content may refer to given information, or to novel information that is *backgrounded* (i.e. 'not at-issue'; cf. Geurts 2010).[1] In particular, presuppositions and anaphora both signal *given* information. In case the givenness assumption fails, however, presuppositions can be added (i.e. accommodated) to the discourse context as new, backgrounded information. CIs, in turn, can only felicitously occur in contexts in which their information is new, in which case this information is added as backgrounded information. These information-structural properties of the different projection phenomena are summarized in Table 1. This table describes the various contexts in which the different types of information can occur felicitously (indicated by a plus-sign). In what follows, we will formalize this characterization of projected and asserted content in terms of information structure using the framework of Projective Discourse Representation Theory. To this end, we first describe the treatment of projection in traditional DRT and motivate its extension to account for differences in information status.

1 Note that any information that is 'given' will also be 'backgrounded', because it will not aid in resolving the current QUD (cf. Simons *et al.* 2010).

2.2 Projection in Discourse Representation Theory

In order to formally describe the behavior of the different types of projected, as well as asserted content, we need a semantic formalism that allows for capturing these different contributions. Of particular interest for this enterprise is the dynamic semantic formalism of Discourse Representation Theory (DRT; Kamp 1981; Kamp & Reyle 1993). DRT is a mentalist and representational theory of interpretation, originally developed by Hans Kamp in order to account for discourse anaphoric dependencies and the representation of tense in natural language (Kamp 1981). Together with Heim's (1982) introduction of File Change Semantics, DRT established a departure from Montagovian truth-conditional semantics. In contrast to traditional approaches to semantics, dynamic approaches take the discourse context into account in the interpretation of semantic content; anaphoric expressions, for instance, obtain an interpretation by virtue of the context in which they occur, that is, by binding to an accessible antecedent. DRT has proven to be a powerful framework that provides an analysis for a wide range of linguistic phenomena including quantification and plurality (Kamp & Reyle 1993), modal subordination (Roberts 1989), attitude reports (Asher, 1986, 1989; Maier 2009; Zeevat 1996) and discourse structure (Asher and Lascarides 2003). Moreover, DRT formed the basis for one of the most influential theories of presupposition projection: van der Sandt's (1992) 'Binding and Accommodation Theory' (see also Beaver 2002; Bos 2003; Geurts 1999)

In DRT, the meaning of a discourse is represented by means of recursive units called *Discourse Representation Structures* (DRSs). A DRS consists of a set of *discourse referents* and a set of *conditions* on these referents. Conditions may be either *basic*, reflecting a property or a relation between referents, or *complex*, reflecting embedded contexts introduced by semantic operators such as negation, implication and modal expressions. DRSs are often visualized using a box-representation consisting of two parts: the set of referents is represented in the top of the box, and the set of conditions on these referents are shown in the body. Example (4) shows the DRS of a 'donkey sentence' (Geach 1962). Here, the outer DRS contains a single complex condition representing an implication, which in turn consists of two embedded DRSs, both containing two referents, and basic conditions on these referents.

(4) If a farmer owns a donkey, he feeds it.

$$\begin{array}{|c|}\hline \\ \hline \begin{array}{|l|}\hline \text{x y} \\ \hline \text{farmer(x)} \\ \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array} \Rightarrow \begin{array}{|l|}\hline \text{z v} \\ \hline \text{feed(z,v)} \\ \text{z=x} \\ \text{v=y} \\ \hline \end{array} \\ \hline \end{array}$$

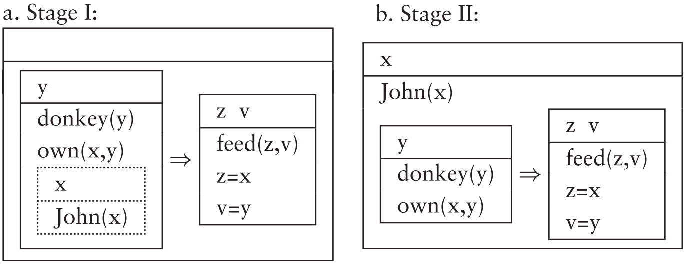
Each (embedded) DRS—each *box*—can be seen as representing a *context*. Taken together, these contexts constitute the logical form of the discourse. Crucially, anaphoric binding of referents is determined on the basis of the accessibility between contexts; in the example above, the antecedent DRS of the implication is accessible from the consequent DRS, and as a result the pronouns 'he' and 'it' are successfully linked to the referents introduced in the antecedent DRS (x and y).

In DRT, *form* determines *interpretation*. That is, the context in which some content appears determines how it is interpreted. Thus, if some content occurs within the (syntactic) scope of, for example, an implication or a negation in the linguistic surface form, it will be interpreted within the logical scope of this operator. This tight correspondence between form and meaning challenges a parsimonious account of projection phenomena. This becomes clear, for instance, if in the above example the indefinite noun phrase ‘a farmer’ is replaced by the proper name ‘John’, resulting in the sentence ‘If John owns a donkey, he feeds it’. Whereas the indefinite noun phrase is interpreted as part of the conditional statement, the proper name triggers a presupposition that is projected to outside the scope of the implication; this results in the reading that there exists a person named ‘John’, for whom it holds that if he owns a donkey, then he feeds it. In the original DRT formulation, this interpretation is obtained by formalizing proper names as always introducing their discourse referents in the global DRS (Kamp 1981; Kamp & Reyle 1993). Similarly, Muskens (1996) proposed to treat proper names as constants, which always refer to some specific entity in the discourse context. However, these solutions cannot be straightforwardly extended to other presupposition triggers—let alone to other projection phenomena, such as conventional implicatures (Potts 2005).

An important advance in the formal analysis of presuppositions was made by van der Sandt (1989, 1992), who proposed a treatment of presupposition projection in terms of anaphora resolution (see also Beaver 2002; Bos 2003; Geurts 1999). On this analysis, presuppositions are resolved in two stages of processing; in the first stage, presuppositions appear marked at their introduction site, and in the second stage the presupposition is resolved either by *binding* to an antecedent or *accommodation* to an accessible context (taking into account semantic as well as pragmatic constraints on resolution). This is illustrated in example (5), which shows the DRT treatment of the proper name version of (4).

(5) If John owns a donkey, he feeds it.

a. Stage I: b. Stage II:



Here, the presupposition triggered by ‘John’ does not have a suitable antecedent, which means that it needs to be accommodated to an accessible discourse context. As shown in (5b), the presupposition is here accommodated to the global discourse context, which corresponds to the desired interpretation on which the presupposition triggered by ‘John’ is projected outside the scope of the implication. Van der Sandt defines several acceptability constraints that determine the interpretation site of presuppositions, including *semantic* constraints, such as the requirement that bound variables cannot become free as a result of projection (‘variable trapping’), and *pragmatic* constraints, such as local and global consistency and informativeness (for more details, see van der Sandt 1992). Moreover, the analysis assumes a preference for global accommodation over local accommodation. On the basis of these properties of projection, Binding and Accommodation Theory can account for the projection behavior of various presupposition triggers, including proper names, definite determiners and possessives.

2.3 Toward a dynamic semantics with information structure

The analysis of presuppositions in DRT makes empirically strong predictions, but at the same time pushes the boundaries of the formalism, as it aims to capture a dimension of meaning that is not inherent to DRT. The main issue with the analysis proposed by van der Sandt is that presupposition projection is treated as a deviation from standard meaning construction. In particular, the two-stage process of deriving the representations is at odds with a compositional treatment for deriving DRSs, since presuppositions are only resolved *after* discourse construction has been completed. Therefore, the intermediate representations with unresolved DRSs are ill-defined with respect to formal properties like accessibility and variable binding, which hinders a compositional formalization in terms of Muskens (1996). Moreover, asserted and presupposed content obtain the same status in the resulting representations. This obliterates the information structure of the discourse and precludes any interpretation in which their contributions are distinguished from each other (cf. Kracht 1994; Krahmer 1998). Critically, however, these limitations are not inherent to Binding and Accommodation Theory, but rather to the formalism in which it is implemented; since DRT cannot capture aspects of meaning that go beyond its logical representation, the contribution made by presuppositions and related phenomena cannot be captured in a satisfactory manner.

In order to resolve this issue, we propose an extension of DRT that allows for an explicit representation of information structure. In particular, the extension provides a uni-dimensional analysis of the contributions made by presuppositions, anaphora, conventional implicatures and asserted content in terms of the notions of ‘givenness’ and ‘backgroundedness’. As shown in section 2.1, differences in information structure affect the way in which semantic content is contributed to the unfolding discourse context—that is, it affects the *context-change potential* of an utterance. Moreover, information structure has been shown to affect various aspects of semantic interpretation, including anaphoric accessibility (e.g. Joshi & Weinstein 1981), and the availability of content for direct denial or correction (e.g. Maier & van der Sandt 2003; van der Sandt 1991; van Leusen 2004). This means that information structure is a crucial part of semantic interpretation and should therefore be accounted for in formal semantic representations—in particular in dynamic approaches to semantics, in which the context-dependent aspects of meaning have a central role. Below, we describe the proposed extension of DRT, called *Projective Discourse Representation Theory*, in more detail, and show how it accounts for the different contributions made by presuppositions, anaphora, conventional implicatures and asserted content.

3 Introducing Projective Discourse Representation Theory

Projective Discourse Representation Theory (PDRT) is a framework that extends DRT with an explicit representation of information structure via the use of projection variables (Venhuizen 2015; Venhuizen *et al.* 2013, 2014). The PDRT analysis of presuppositions basically follows van der Sandt’s treatment of presuppositions, except that projection does not involve movement of semantic content within the representation, but is effectuated by means of variable binding. This eliminates the need for a two-stage resolution algorithm, and thereby provides a more compositional treatment of projection. The dependency between projection variables reflects the information structure of the utterance; locally bound projection variables indicate asserted content, while non-locally bound and free variables indicate projected, that is, backgrounded information. The relation between local and projected contexts is explicated by a set of Minimally Accessible Projection contexts (MAPs), which reflect minimal constraints on projection. PDRT thus provides a parsimonious treatment of projection phenomena by exploiting a central component of traditional DRT: the binding of variables. Below, we first elaborate on the representation of information structure in PDRT. We then describe the treatment of the different projection phenomena and compare the formalism to other variants of Discourse Representation Theory. In section 4, we will work out the formal properties of PDRT in more detail.

3.1 Projective Discourse Representation Structures

The basic structures in PDRT are Projective Discourse Representation Structures (PDRSs). These structures carry the same information as DRSs, with the addition of *projection variables* that explicitly reflect the information status of semantic content: all referents and conditions are associated with a *projection pointer*, and all (embedded) PDRSs obtain a *label* (similar to the context identifiers used in other DRT extensions, e.g. Segmented DRT; Asher and Lascarides 2003, see section 3.3 for more detail).

In a PDRS, asserted content is represented using a pointer bound by the local context, that is, the context in which the information is (syntactically) introduced; this means that the content is interpreted relative to the local PDRS, just like in traditional DRT. Projected content, on the other hand, is represented either with a pointer bound by a non-local context, or by using a free variable as pointer. Hence, all semantic content is represented as part of the context in which it is (syntactically) introduced, and the projection variables reflect the information status relative to the local context. In case the pointer is bound non-locally, the projected content is interpreted at the context indicated by the pointer. In case the pointer occurs freely, the interpretation site remains underspecified, indicating that the content yet needs to be accommodated. The final projection site is restricted by semantic constraints on projection, which are represented in a PDRS as a set of Minimally Accessible Projection contexts (MAPs), indicating accessibility relations between projection variables.

Example (6) shows the PDRS corresponding to example (5). We here use integers as projection variables; labels are represented on top of each PDRS, pointers are indicated using a leftward pointing arrow '←', and the MAPs are shown in the bottom part of each PDRS.

(6) If John owns a donkey, he feeds it.

$$
\begin{array}{c} 1 \\ \begin{array}{|c|} \hline \\ \hline 1 \leftarrow \begin{array}{c} 2 \\ \begin{array}{|l|}\hline 4 \leftarrow x \;\; 2 \leftarrow y \\ \hline 4 \leftarrow \text{John}(x) \\ 2 \leftarrow \text{donkey}(y) \\ 2 \leftarrow \text{own}(x,y) \\ \hline 2 \leq 4 \\ \hline \end{array}\end{array} \Rightarrow \begin{array}{c} 3 \\ \begin{array}{|l|}\hline 4 \leftarrow x \;\; 2 \leftarrow y \\ \hline 3 \leftarrow \text{feed}(x,y) \\ \hline 3 \leq 4 \;\; 3 \leq 2 \\ \hline \end{array}\end{array} \\ \hline \\ \hline \end{array}\end{array}
$$

This sentence contains three projection triggers: the proper name 'John', and the two pronouns 'he' and 'it'. This is represented in the PDRS by means of the projection pointers—for simplicity, we here use a simplified representation for anaphora, in which the equality statements triggered by the pronouns are resolved, but the 'shadow referents' introduced in the PDRS with label 3 remain (see section 3.2.2 for the full account of anaphora in PDRT). The semantic content associated with the projected expressions obtains a pointer that is not bound by the local context; for 'John' and 'he' this is the free pointer 4, indicating co-reference to an unresolved antecedent, and for 'it' this is the label of the antecedent of the implication (pointer 2), indicating binding. All other content is asserted, which is represented using locally bound pointers. The MAPs indicate minimal constraints on projection; here, all projection triggers introduce a *weak subordination* ($\leq$) constraint, which indicates that the projection site is either the same as or accessible from the local discourse context (the MAP constraints will be described in more detail in section 3.2, and formalized in section 4.1.1). Since the MAP constraints are non-deterministic, interpreting the PDRS involves determining the final interpretation site of the projected content based on additional (pragmatic) constraints. Following van der Sandt (1992), presuppositions will be projected to the highest possible context—in the example above, the free projection pointer 4 will therefore be resolved to the global context (i.e. context 1). As a result, the model-theoretic interpretation of (6) is equivalent to the model-theoretic interpretation of the DRS shown in example (5) (see section 4.1.2 for more details on the interpretation of PDRSs).

To see how the use of projection pointers relates to the notions of backgroundedness and at-issueness, it is useful to consider the dynamic interpretation of PDRSs. As in DRT, the basic structures of PDRT simultaneously serve as *content* and *context* (cf. van Eijck & Kamp 1997). Pragmatically, each PDRS can be viewed as providing a (partial) answer to the current Question Under Discussion, and at the same time defining (or restricting) the QUD for any novel information. This becomes especially clear if we consider the use of implication in (P)DRT; the antecedent of the implication functions as a QUD that is addressed by the consequent of the implication. In example (6), for instance, the antecedent of the implication can be paraphrased as the QUD 'What is the case if John owns a donkey?'. The consequent, in turn, provides an answer to this question, namely: 'he feeds it'. Thus, the semantic content of any (P)DRS addresses the local QUD. Given the description of projected content as not at-issue (or backgrounded), that is, not addressing the current QUD (Simons *et al.* 2010), projected material should thus not be part of the semantic content of the (P)DRS in which it is introduced (cf. van der Sandt 1992; see also Geurts 2010 for a DRT account in terms of backgrounding). Rather, projected content should be interpreted in the context in which it does address the local QUD—in PDRT, this context is indicated by the projection pointers. Crucially, the context relative to which the projected content is at-issue may or may not be available in the current discourse structure, as the global DRS itself addresses an (implicit) QUD, relative to which the projected content can be not at-issue; in PDRT, this is reflected by the use of a free pointer. Thus, pointers that are not locally bound indicate content that is not at-issue relative to the local QUD, and hence projects. In case the projected content is at-issue relative to an accessible context in the current discourse structure (i.e. the content addresses the local QUD of that context), its pointer will be bound. If, on the other hand, no such context can be found, the projected content is associated with a free pointer, indicating that its final projection site is unresolved with respect to the current discourse context, and hence does not address any (local) QUD. When more context is added during discourse construction, the projected content may become at-issue relative to an accessible context, which means that its pointer becomes bound.

The analysis of projection in PDRT thus explicitly connects the property of projection to the notions of backgrounding and at-issueness; the projection pointers indicate the context relative to which content is at-issue, which may or may not be the local discourse context—and in fact this context may not even be available in the current discourse context. Note, however, that PDRT is not intended as a full-fledged framework for formalizing QUD structure, as such an analysis needs to take into account additional aspects of discourse structure, for example, rhetorical structure (for recent formalizations of QUD structure in discourse semantics, see Hunter & Abrusán 2017; Reyle & Riester 2016; Riester 2016). Rather, the PDRT representations reflect the information structural aspects of the logico-semantic discourse structure. As we will discuss below, this framework allows for capturing the different contributions of presuppositions, anaphora and conventional implicatures in a uniform manner. Moreover, when compared to other DRT extensions, PDRT is shown to integrate ideas from various other frameworks, thus paving way to an integrated theory of discourse and information structure (for a discussion of possible extensions of the framework, including a treatment of QUD structure, see section 5.2).

3.2 Projection as information structure in PDRT

In PDRT, resolving the projection site of any projected content boils down to determining the right constellation of projection variables, while taking into account the constraints on projection introduced by the MAPs. Crucially, this can be done as soon as the trigger is introduced into the discourse, thereby precluding a two-stage resolution procedure (cf. van der Sandt 1992). The projection site of projected content is thus determined on the lexical level, similar to the resolution of anaphoric expressions in compositional versions of DRT (Muskens 1996). This means that the projection variables, as well as the MAPs, are part of the lexical semantics of projecting expressions. During discourse construction, this semantics is combined with its discourse context, which either results in binding of the projection pointers, or the introduction of a free variable pointer (through the use of different merge operations for asserted and projected content; see section 4.3 and Appendix B for details). Critically, the specific *pragmatic* constraints that determine the projection site of projected content in a given context are not part of the PDRT framework. Similar to the DRT treatment of anaphora, PDRT assumes a separate resolution mechanism for projected content, which takes into account context-dependent pragmatic constraints for determining binding or accommodation of projected content. The framework does, however, incorporate *semantic* constraints on projection, namely by means of the MAPs. Presuppositions, anaphora and conventional implicatures place different constraints on the context in which they are introduced; as described above, conventional implicatures, for instance, differ from presuppositions in that they cannot be felicitously used in a context in which their contribution is given. In PDRT, the MAPs are used, in combination with the regular semantic content of expressions, to formalize these projection constraints.

Table 2 provides a coarse overview of the different properties of anaphora, presuppositions, CIs and asserted content in PDRT. *Projection potential*, which indicates the possibility of semantic content to project to an accessible context, directly reflects the property of *backgroundedness* (or not at-issueness) described in Table 1. This property is represented in a PDRS using subordination constraints in the MAPs, indicating that the content is interpreted in an accessible discourse context. *Content-anaphoricity* constrains expressions

Table 2 Projection properties of projected and asserted content in PDRT. The projection properties, which are represented by means of the projection variables and the PDRS conditions, describe the differences in projection behavior between the different types of semantic content. *Projection potential* indicates that the semantic content can be interpreted non-locally, *content-anaphoricity* states that the content itself needs to be bound to previously introduced information and *context-anaphoricity* means that the interpretation context needs to be bound to an existing context. The identifiers '+' and '−' respectively indicate that the denoted property is or is not a requirement for the individual types of content.

Requirement:	Projection potential	Content-anaphoricity	Context-anaphoricity
Anaphora	+	+	+
Presuppositions	+	−	−
CIs	+	−	+
Assertions	−	−	+

to be *given* in the discourse context; this is indicated by means of an identity constraint on discourse referents, as part of the PDRS conditions. Whereas both presuppositions and anaphora can refer to given information (cf. Table 1), it is only a requirement for anaphora. *Context-anaphoricity*, represented by means of an identity constraint in the MAPs, indicates that the interpretation context of the semantic content is bound to a previously introduced discourse context. This means that the semantic content is added to the existing context. In case the content anaphoricity constraint does not hold, this additional content will be *novel*. Following Table 1, CIs and assertions both introduce novel information, which means that they are context- (but not content-) anaphoric; assertions are added to the local discourse context, and CIs are added to a non-local (projected) context. Finally, presuppositions only have the property of projection potential; they are context- nor content-anaphoric, which means that they can create their own interpretation context in case no suitable interpretation site can be found. This reflects the potential of presuppositions to either bind to existing information or accommodate. Below, the formalization of presuppositions, anaphora and conventional implicatures in PDRT will be described in more detail. This analysis instantiates the projection properties described in Table 2, thereby giving rise to the felicity constraints described in Table 1.

3.2.1 *Presuppositions* Following the analysis by van der Sandt (1992), presuppositions are backgrounded contributions that have an anaphoric character; they may either bind to previously introduced content or accommodate to a suitable level of discourse. In PDRT, this behavior is formalized by identifying presuppositions with a projecting pointer that weakly subordinates the local context. As a result, the pointer may either bind to an accessible context label or it may occur free. In case the presupposed content is anaphorically linked to previously introduced information, the pointer will be bound to the interpretation site of this information. The pointer may also bind to a previously introduced context label, which happens, for example, in the case of 'variable trapping' (cf. van der Sandt 1992) and results in the intermediate accommodation of the presupposed content. Finally, in case the presupposition is not bound or 'trapped', its pointer remains free; this indicates that no suitable interpretation site has been found. Example (7) shows three PDRSs with different information-structural configurations: a PDRS without any projected content in (7a), one with an unresolved presupposition in (7b) and one with a bound presupposition in (7c).

(7) a. A cat purrs b. The cat purrs c. If a cat is happy, the cat purrs

$$\overset{1}{\begin{array}{|l|}\hline 1 \leftarrow x \\ \hline 1 \leftarrow \mathrm{cat}(x) \\ 1 \leftarrow \mathrm{purr}(x) \\ \hline \\ \hline \end{array}} \quad \overset{1}{\begin{array}{|l|}\hline 2 \leftarrow x \\ \hline 2 \leftarrow \mathrm{cat}(x) \\ 1 \leftarrow \mathrm{purr}(x) \\ \hline 1 \leq 2 \\ \hline \end{array}} \quad \overset{1}{\begin{array}{|c|}\hline \\ \hline \overset{2}{\begin{array}{|l|}\hline 2 \leftarrow x \\ \hline 2 \leftarrow \mathrm{cat}(x) \\ 2 \leftarrow \mathrm{happy}(x) \\ \hline \\ \hline \end{array}} \Rightarrow \overset{3}{\begin{array}{|l|}\hline 2 \leftarrow x \\ \hline 2 \leftarrow \mathrm{cat}(x) \\ 3 \leftarrow \mathrm{purr}(x) \\ \hline 3 \leq 2 \\ \hline \end{array}} \\ \hline \\ \hline \end{array}}$$

In (7a), all pointers coincide with the label of the PDRS in which they are introduced. This indicates that the content is interpreted locally, that is, it is asserted. In (7b), on the other hand, the referent and condition introduced by the definite description 'the cat' are associated with a free variable as pointer. This indicates that the presupposition triggered by the definite description still needs to be accommodated. The MAPs of the PDRS constrain this interpretation site; '$1 \leq 2$' indicates that context 2 should be either the same to or accessible from context 1. In the representation in (7b), there is a single accommodation site available to the presupposition, namely the local context 1. Critically, however, each PDRS can be interpreted as being part of a larger discourse representation; the use of a free variable as pointer indicates that the presupposition may still be resolved later on during discourse construction. The underspecified representations thus explicitly represent the *projection potential* of presuppositions, which reflects their backgrounded nature. In case the presupposition is resolved in the discourse context, as illustrated in (7c), the pointer of the projected content is bound by the label of the interpretation site. As a result, the presupposition is only backgrounded with respect to its local context (i.e. context 3, the consequent of the conditional statement), but not with respect to the entire discourse context.

Because of the underspecified nature of presuppositions in PDRT, they affect the context in which they occur differently from asserted content. Instead of simply adding information to an unfolding discourse context, presuppositions place an assumption on their context—the assumption that a suitable projection site can be found. On the interpretation level, however, presuppositions have the same status as asserted content; they are interpreted relative to the interpretation site indicated by the projection variables. When interpreting PDRSs containing free pointers, pragmatic heuristics may be employed in combination with the MAPs in order to determine the final interpretation site of the unresolved presupposition, for instance, by assuming a preference for global accommodation over local accommodation (following Geurts 1999; van der Sandt 1992), or by employing constraints based on Optimality Theory, as proposed by Zeevat (2002). This uni-dimensional interpretation of presupposed and asserted content makes sure that logically equivalent sentences that differ only in terms of information structure, such as (7a) and (7b) above, will obtain the same model-theoretic interpretation.

3.2.2 *Anaphora* In PDRT, the treatment of anaphora follows the treatment of presuppositions, but anaphoric expressions place a stronger constraint on their contexts than regular presuppositions; their content must be given. Just like in regular DRT, anaphoric expressions introduce a referential dependency to an accessible antecedent. Critically, this dependency does not only involve the binding of discourse variables, but also the binding of projection variables; pronouns introduce backgrounded information, like presuppositions, which means that they are interpreted in the same context as their antecedent. An example is shown in (8).

(8) If John beats a donkey, he will be punished.

$$
\begin{array}{|c|}
\multicolumn{1}{c}{1} \\ \hline
\\ \hline
1 \leftarrow \begin{array}{c} 2 \\ \begin{array}{|l|} \hline 4 \leftarrow x \;\; 2 \leftarrow y \\ \hline 4 \leftarrow \text{John}(x) \\ 2 \leftarrow \text{donkey}(y) \\ 2 \leftarrow \text{beats}(x,y) \\ \hline 2 \leq 4 \\ \hline \end{array} \end{array} \Rightarrow \begin{array}{c} 3 \\ \begin{array}{|l|} \hline 5 \leftarrow z \\ \hline 5 \leftarrow z = x \\ 3 \leftarrow \text{punished}(z) \\ \hline 3 \leq 5 \;\; 5 = 4 \\ \hline \end{array} \end{array} \\ \hline
\\ \hline
\end{array}
$$

The antecedent of the pronoun ‘he’ in (8) is the entity introduced by ‘John’, which is itself a presuppositional expression. This anaphoric dependency is represented by means of two equality statements: the PDRS condition ‘$5 \leftarrow z = x$’ indicates that the referent introduced by the pronoun is equated to the referent introduced by ‘John’ (*content-anaphoricity*), and the MAP constraint ‘$5 = 4$’ indicates that the interpretation site of the pronoun equals the interpretation site of the antecedent (*context-anaphoricity*). Thus, the pronoun effectively picks out the exact same referent as the antecedent. This becomes even clearer when we eliminate the equality statements from the MAPs and PDRS conditions in (8), resulting in the simplified representation (8)’:

(8)’ If John beats a donkey, he will be punished.

$$
\begin{array}{|c|}
\multicolumn{1}{c}{1} \\ \hline
\\ \hline
1 \leftarrow \begin{array}{c} 2 \\ \begin{array}{|l|} \hline 4 \leftarrow x \;\; 2 \leftarrow y \\ \hline 4 \leftarrow \text{John}(x) \\ 2 \leftarrow \text{donkey}(y) \\ 2 \leftarrow \text{beats}(x,y) \\ \hline 2 \leq 4 \\ \hline \end{array} \end{array} \Rightarrow \begin{array}{c} 3 \\ \begin{array}{|l|} \hline 4 \leftarrow x \\ \hline 3 \leftarrow \text{punished}(x) \\ \hline 3 \leq 4 \\ \hline \end{array} \end{array} \\ \hline
\\ \hline
\end{array}
$$

Here, the equality statement ‘$z = x$’ is resolved by replacing all (bound) occurrences of ‘z’ with ‘x’, and the MAP equality ‘$5 = 4$’ is eliminated by replacing all (bound) occurrences of ‘5’ by ‘4’. In contrast to the DRT treatment of anaphora, this procedure does not involve moving or eliminating any semantic material; the projected referent introduced by the pronoun ('$4 \leftarrow x$') remains part of the universe of the local PDRS, despite the fact that it coincides with a previously introduced referent. We call this a *shadow referent*. Due to the distinction made in PDRT between the introduction and interpretation site of semantic content, identical projected referents (matching in referent and pointer) are allowed to be introduced in multiple universes. However, just like in DRT, discourse referents cannot be *interpreted* in multiple accessible universes, because of ambiguous bindings (formally, this means that the PDRS is not 'pure'; see Appendix A.4, Definition 16). Beyond the direct correspondence between the representations in (8) and (8)', another advantage of employing shadow referents is the explicit representation of the information-structural contribution of anaphora; picking out a previously introduced entity and re-introducing it within the current discourse context. This not only aids the direct alignment between (larger) texts and semantic representations, but also emphasizes the correspondence between anaphora and presuppositions.

3.2.3 *Conventional Implicatures* As described above, conventional implicatures signal backgrounded—not at-issue—information that is not yet available in the discourse context. Based on this observation, Venhuizen *et al.* (2014) propose a uni-dimensional analysis of CIs and at-issue content, formalized in terms of PDRT. On this account, CIs attach their novel contribution to the unfolding discourse via an *anchor* (following syntactic treatments of CIs, e.g. Del Gobbo 2003; Heringa 2012; Nouwen 2007). Critically, this anchor must be (locally) *specific*, that is, it must identify a specific discourse referent in the local discourse context. This assumption follows from the backgrounded nature of CIs; if the anchor is non-specific, any novel information that is added to the description of the referent referred to by the anchor will contribute to its identification, and will thus be at-issue. That is, since referents that are newly introduced into the discourse context are considered at-issue content (i.e. addressing the—local—QUD), any content that is contributed to these referents will be at-issue as well. As described above, CIs are by definition not at-issue and can therefore only attach to a specific anchor (for more details, see Venhuizen *et al.* 2014).

In sum, CIs are considered elaborations on the description of the referent referred to by their specific anchor. Example (9) illustrates how this analysis of CIs is implemented in PDRT.[2]

(9) Jake, a famous boxer, lives in Utrecht.

$$
\begin{array}{c}
1 \\
\begin{array}{|l|}
\hline
2 \leftarrow x \quad 3 \leftarrow y \\
\hline
2 \leftarrow \text{Jake}(x) \\
3 \leftarrow \text{famous\_boxer}(y) \\
3 \leftarrow y = x \\
1 \leftarrow \text{lives\_in\_Utrecht}(x) \\
\hline
1 \leq 2 \quad 1 < 3 \quad 3 = 2 \\
\hline
\end{array}
\end{array}
$$

Here, the presupposition (triggered by the proper name *Jake*) is associated with a free pointer, which is related to its introduction context via weak subordination ($1 \leq 2$). The CI triggered by the nominal appositive (*a famous boxer*) is also assigned a free pointer and

2 For reasons of brevity, the CI content and the at-issue content are represented as a single condition.

in addition introduces two specific accessibility constraints as part of the MAPs. Firstly, the interpretation site of the CI is equated with the interpretation site of its anchor ($3 = 2$; reflecting *context-anaphoricity*), and secondly, this interpretation site strictly subordinates the CI's introduction context ($1 < 3$; reflecting *projection potential*). The CI thus effectively constrains the interpretation site of the anchor; since the interpretation site of the CI is non-local and identical to that of its anchor, the anchor can only be accommodated non-locally. These MAP constraints directly instantiate the CI content as backgrounded, novel information: the strict subordination constraint signals the backgrounded nature, and the identity constraint indicates that the CI content is added to an existing discourse context, in the same way that asserted content is added to the local discourse context.

It should be noted that the MAP constraints in (9) cannot be satisfied within the current discourse representation. This is the case because the CI requires a non-local context to accommodate to ($1 < 3$), but no such context is available in this representation. Nonetheless, the PDRS shown in (9) is felicitous. Like DRSs, PDRSs are considered a *partial* model representing the information conveyed by some piece of discourse; their interpretation is determined via an embedding function into a *total* model (Kamp & Reyle 1993). This means that any PDRS will be implicitly embedded as part of a larger context, hence creating an additional interpretation site that subordinates the 'global' discourse context and allows for satisfying the MAP constraints in (9) (see also the analysis of indexicals proposed by Hunter 2013, where an additional global DRS is introduced to which indexical expressions accommodate). However, since such an additional global context does not have a truth-conditional effect on the interpretation of semantic content, we can obtain the desired truth conditions of (9) by simply ignoring the strong subordination constraint to the global discourse context and interpreting the CI as part of context 1 (we employ this strategy in the interpretation of PDRSs via a translation to DRSs, as described in section 4.1.2).

The MAP constraints introduced by CIs generate straightforward predictions about CI (in)felicity; in particular, they prevent CIs from being cancelled. This is illustrated in (10), which shows the PDRS for example (3) from above.

(10) ??If Obama is a socialist, then the President, *who is a socialist*, will raise taxes.

$$
\begin{array}{c}
1 \\
\begin{array}{|c|}
\hline
\\
\hline
1 \leftarrow \begin{array}{c} 2 \\ \begin{array}{|l|} \hline 5 \leftarrow x \\ \hline 5 \leftarrow \text{Obama}(x) \\ \mathbf{2} \leftarrow \textbf{socialist}(x) \\ \hline 2 \leq 5 \\ \hline \end{array} \end{array}
\Rightarrow
\begin{array}{c} 3 \\ \begin{array}{|l|} \hline 5 \leftarrow x \;\; 6 \leftarrow y \\ \hline 5 \leftarrow \text{President}(x) \\ \mathbf{6} \leftarrow \textbf{socialist}(y) \\ 6 \leftarrow y = x \\ 3 \leftarrow \text{raise\_taxes}(x) \\ \hline 3 \leq 5 \;\; 3 < 6 \;\; 6 = 5 \\ \hline \end{array} \end{array}
\\
\hline
\\
\hline
\end{array}
\end{array}
$$

In this example, the presuppositions triggered by 'Obama' and 'the President', introduced in respectively the antecedent and the consequent of an implication, refer to the same entity. This is reflected by the fact that these conditions affect the same referent and are assigned the same pointer. As such, the contribution made by the CI (that the president is a socialist) is the same as the contribution in the antecedent of the implication (that Obama is a socialist). Critically, this equivalence renders the antecedent of the implication void. This happens because the identity constraint introduced by the CI ($6 = 5$) forces the CI content to project out of the consequent to the interpretation site of its anchor. Since the MAP constraint in the antecedent ($2 \leq 5$) indicates that the interpretation site of the CI content is accessible from the antecedent's local context, the content contributed by the antecedent is already established within the context in which it is introduced. Hence, the conditional statement is not locally informative (cf. van der Sandt 1992), and therefore the sentence is infelicitous.

3.3 Position within the DRT family

PDRT extends DRT with a representation of information structure, via the introduction of projection pointers. However, various extensions of DRT already exist, which aim at dealing with different aspects of linguistic meaning, including presuppositions and other not at-issue content (e.g. Geurts & Maier 2003; Hunter 2013; Krahmer 1998), underspecification (e.g. Reyle 1993, 1995; van Leusen & Muskens 2003) and discourse relations (e.g. Asher 1993; Asher and Lascarides 2003). Below, we will describe how PDRT differs from these other extensions, and where it can be integrated with existing extensions in order to create an even more comprehensive formalism.

3.3.1 *DRT extensions for projected content* PDRT incorporates an explicit notion of information structure, which can be employed to represent the different contributions made by presuppositions, anaphora and CIs without introducing separate levels or dimensions of meaning. As a result, PDRT provides a more comprehensive and parsimonious analysis of projected and asserted content than existing formalisms aimed at dealing with these types of contributions. In particular, Krahmer's (1998) Presuppositional DRT does not directly extend to projection phenomena beyond presuppositions. Whereas Layered DRT (Geurts & Maier 2003, 2013) incorporates different types of meaning, including presuppositions, conversational implicatures and indexicals, it assumes a strict interpretational distinction between the different types of content. As a result, this formalism cannot capture interpretational interactions between these different types of content, such as anaphoric dependencies and VP-ellipsis (see e.g. AnderBois *et al.* 2010). It remains an open issue to decide whether an additional semantic layer is required to account for conversational implicatures. With respect to the treatment of indexicals, the PDRT analysis seems highly compatible with the DRT treatment of indexical expressions in terms of presupposition resolution, as proposed by Hunter (2010, 2013). Hunter implements an additional global DRS to capture the discourse situation and uses specific interpretation constraints for indexicals in order to obtain their rigid interpretation; in a PDRT version of this analysis, these constraints could be represented using the projection variables and the MAPs. We will describe this extension of PDRT in more detail in the discussion (section 5).

Schlenker (2011) develops an inherently different account of presuppositions in DRT, which aims at incorporating Heim's (1983) satisfaction-based view on presuppositions into the DRT formalism. Similar to the PDRT analysis, Schlenker employs variables to indicate the context in which a presupposition must be *satisfied*; satisfaction here means that the context indicated by the presupposition should entail it. In contrast, presuppositions in PDRT are added to their interpretation context, which means that the presupposition inherits all referential and interpretational properties from its projection site. As a result, the accounts make different predictions about semantic interpretation; for instance, they differ on the interpretation of referring expressions that are more descriptive than their antecedent, as illustrated in example (11).

(11) A woman was arrested in Berlin yesterday. <u>The mother of three</u> is suspected of murder.

Here, the underlined definite description '*the mother of three*' simultaneously co-refers with the indefinite description '*a woman*' and provides more information about its referent. On Schlenker's account, the presupposition triggered by the definite description requires the context in which the indefinite '*a woman*' is introduced to entail that this woman is a mother of three. However, this seems to constrain the context too strongly; these sentences can naturally be used in a context in which '*a woman*' introduces a novel entity, without any entailments of the number of children she has. In the PDRT analysis, by contrast, the projected content is attached to the description of '*a woman*', and therefore becomes part of the novel contribution of the discourse.

3.3.2 *Underspecification approaches* In the PDRT analysis, projection is indicated by means of variable binding, which means that no semantic content needs to be moved within the representation. This lack of movement is also inherent to the treatment of presuppositions in underspecification formalisms (e.g. Keller 1997; van Leusen & Muskens 2003), where different possible interpretation sites can be constrained within the semantic representations. In contrast to the underspecification approaches, however, the resolution of projected content in PDRT does not require a two-stage procedure; instead, projected content is directly linked to its (possibly underspecified) interpretation site during discourse construction. This direct interpretation of presuppositions is in line with recent experimental findings that provide evidence against a two-step serial processing model of presupposition (e.g. Chemla & Bott 2013).

Another difference between DRT and underspecification approaches concerns the level of semantic interpretation. As described in section 4.1.2 above, the information structure represented in PDRSs is considered to be a discourse-level phenomenon, which does not directly affect the model-theoretic interpretation of discourse structures, but rather their compositional properties (but see also the discussion in section 5.2). Underspecification approaches do, however, assume underspecification to affect the model-theoretic interpretation. With respect to the theory of presupposition, this means that sentences containing unresolved presuppositions have a meaning that is inherently different from sentences without unresolved presuppositions. In the account proposed by Keller (1997), for instance, the following sentence obtains an underspecified interpretation due to the underspecification of the scope of the negation:

(12) Theo doesn't love his rabbit.

The underspecification may be resolved by either anaphorically referring back to the presupposed rabbit, thereby confirming non-local accommodation of the presupposition, or by directly denying the presupposition (as in: 'In fact, he doesn't have a rabbit'). Without either of these resolutions, however, the meaning of the sentence remains underspecified. In the PDRT analysis, by contrast, 'his rabbit' will be associated with a free variable as pointer. Depending on the context in which the sentence occurs, different pragmatic constraints are employed to resolve this presupposition (cf. van der Sandt 1992); without any context, it will be accommodated to the global discourse context (based on the constraint that favors

Table 3 Comparison of identifiers in various DRT extensions. Comparison of the structural properties of the identifiers used in UDRSs (Reyle 1993), LDRSs (Geurts & Maier 2003), SDRSs (Asher and Lascarides 2003) and PDRSs. See the text for a description of each of the individual properties. The '+' and '−' indicate whether the identifiers of the DRT extension have the denoted property or not.

	UDRS	LDRS	SDRS	PDRS
Labels as variables	−	−	+	+
Content labels (referents & conditions)	+	+	−	+
Discourse context labels (DRSs)	−	−	+	+
Other context labels	−	+	−	+

global accommodation over local accommodation), but in the context of a direct denial, it will be accommodated locally (based on the local coherence constraint). Thus, PDRT captures the underspecified nature of presuppositions, without requiring an underspecified semantics.

3.3.3 *DRT extensions with context identifiers* The projection variables in PDRT are used to represent differences in information structure. Other DRT variants have also employed different types of identifiers to indicate discourse contexts and the relations between them. Most notably, the identifiers used in Underspecified DRSs (UDRSs; Reyle 1993, 1995) and the compositional structures from Logical Description Grammar (LDG; van Leusen 2007; van Leusen & Muskens 2003) are subject to similar context subordination constraints as the ones introduced in the MAPs of a PDRS. Critically, however, these identifiers are constants, and thus do not adhere to the same principles as the projection variables of PDRT. Moreover, the identifiers in existing underspecified representations cannot be used to refer to contexts that are not part of the current discourse context. As described above, PDRT does allow for such underspecification; free pointers indicate the underspecified interpretation site of presuppositions. This is similar to the presupposition layer indicated by the labels used in LDRSs (Geurts & Maier 2003, 2013), but does not imply a distinct level of interpretation for presuppositions and asserted content. Another formalism that employs discourse variables is Segmented DRT (Asher 1993; Asher and Lascarides 2003). The identifiers associated with SDRSs are variables, like in PDRT, but in addition they obtain a model-theoretic interpretation. This is not the case for the projection variables from PDRT; they operate on the level of the discourse structure, rather than on the level of interpretation (see section 4.1.2). On the structural level, SDRSs and PDRSs differ in the labeling of semantic content (the referents and conditions in a PDRS all obtain a pointer), and the use of identifiers for contexts that are not part of the current discourse structure. Table 3 summarizes the basic structural differences between the DRT variants that employ context identifiers. Critically, the expressive nature of PDRT stems from the employment of variable identifiers that label semantic content, as well as discourse-internal and -external contexts; this allows for a direct alignment between the semantic representations and the linguistic surface structure (e.g. via the use of 'shadow referents' for anaphoric expressions), and for capturing the information-structural differences between presuppositions, anaphora, CIs and asserted content.

The differences between PDRT and the formalisms described above are largely due to the fact that they were developed in order to incorporate different aspects of semantic meaning. It is therefore interesting to investigate the compatibility of PDRT with these other extensions, as to develop an even more comprehensive formalism; we will come back to this point in the discussion (section 5.2).

4 Formalizing Projective Discourse Representation Theory

In what follows, we will work out the formal definitions underlying the PDRT framework and show how it extends the formalization of traditional DRT, while remaining faithful to the basic DRT notions, such as variable binding. We first describe the syntax and semantics of Projective Discourse Representation Structures, and how binding and accessibility is formalized in PDRT. We then formally describe the compositional properties of PDRSs (see also Venhuizen 2015). Based on the definitions described here, the PDRT framework has been formally implemented—alongside traditional DRT—as part of a Haskell library called ‘PDRT-SANDBOX’[3] (Venhuizen & Brouwer 2014); this implementation allows for empirical validation and experimentation with the proposed formalism.

4.1 Syntax and semantics

4.1.1 *Syntax* Formally, a PDRS is a quadruple consisting of a *label*, a set of *projected referents* (i.e. discourse referents associated with a projection pointer; see Definition 9, Appendix A.1), a set of *projected conditions* (i.e. PDRS conditions associated with a projection pointer; see Definition 2) and a set of *MAPs* (representing accessibility relations between projection variables). The syntax of PDRSs is defined as follows:

Definition 1 (PDRS). A PDRS P is defined as a quadruple:
$\langle l, \mathrm{U}, \mathrm{C}, \mathrm{M}\rangle$, where:

(i) l is a projection variable;

(ii) $\mathrm{U} = \{\delta_1 ... \delta_n\}$ is a finite set of projected referents (also referred to as the *universe*), with $\delta_i = v_i \leftarrow x_i$, such that v_i is a projection variable, and x_i is a discourse referent;

(iii) $\mathrm{C} = \{\chi_1 ... \chi_m\}$ is a finite set of projected conditions, with $\chi_i = v_i \leftarrow \gamma_i$, such that v_i is a projection variable, and γ_i is a PDRS condition (see Definition 2);

(iv) $\mathrm{M} = \{\mu_1 ... \mu_k\}$ is a finite set of MAPs, with $\mu_i = v_1 \leq v_2$ or $\mu_i = v_1 \nleq v_2$, such that v_1 and v_2 are projection variables.

The definition of PDRS conditions basically follows the definition of DRS conditions proposed by Bos (2003). Besides the standard logical operators for negation ($\neg$), disjunction ($\vee$) and implication ($\Rightarrow$), this definition also includes modal operators for logical necessity ($\Box$), and possibility ($\Diamond$) as well as a hybrid condition (:), which associates a variable ranging over possible worlds with a DRS, and can be used to represent sentential complements (see Bos 2003). The following definition describes these PDRS conditions:

3 PDRT-SANDBOX is available at: http://hbrouwer.github.io/pdrt-sandbox/

Definition 2 (PDRS Conditions). PDRS conditions may be either basic or complex and are defined as follows:

(i) $R(x_1, ..., x_n)$ is a basic PDRS condition, with $x_1...x_n$ are discourse referents and R is a relation symbol for an n-place predicate;

(ii) $\neg P$, $\Box P$ and $\Diamond P$ are complex PDRS conditions, with P is a PDRS;

(iii) $P_1 \vee P_2$ and $P_1 \Rightarrow P_2$ are complex PDRS conditions, with P_1 and P_2 are PDRSs;

(iv) $x : P$ is a complex PDRS condition, with x is a discourse referent and P is a PDRS;

(v) PDRS conditions are only defined on the basis of clauses i–iv above.

Together, Definitions 1 and 2 define the syntax of PDRSs. Note that this extends the PDRS syntax proposed in Venhuizen *et al.* (2013), which did not include the set of Minimally Accessible PDRS-contexts (MAPs). MAPs introduce constraints on context accessibility, similar to the accessibility constraints for Unresolved DRSs introduced by Reyle (1993, 1995). As shown in section 3.2, these constraints can be used to describe the information status of different projection phenomena. The MAPs are defined over the set *PDRS-contexts*, which includes all sub-PDRSs of the global PDRS, as well as the projected contexts indicated by a free pointer. Formally, there are two types of MAPs: $v_1 \leq v_2$ indicates that PDRS-context v_2 is accessible from PDRS-context v_1, and $v_1 \not\leq v_2$ indicates that PDRS-context v_2 is *not* accessible from PDRS-context v_1. The first constraint represents *weak subordination*, that is, v_1 is the same context as v_2 or subordinated by it. The second constraint indicates that v_1 is not the same context as v_2, nor subordinated by it; it may thus either be the case that v_2 subordinates v_1, or that there exists no subordination relation between v_1 and v_2. These two basic MAP types can be combined in order to formulate the stronger accessibility constraints used in section 3.2: *strict subordination* ($v_1 < v_2$) is defined as $\{v_1 \leq v_2, v_2 \not\leq v_1\}$, and *identity* ($v_1 = v_2$) is defined as $\{v_1 \leq v_2, v_2 \leq v_1\}$.

4.1.2 *Semantic interpretation* One of the strengths of the traditional DRT framework is that its basic structures have a model-theoretic interpretation, providing them with a truth-conditional semantics. This model-theoretic interpretation can be derived in two ways: via a translation from DRSs to first-order logic (see e.g. Blackburn *et al.* 2001; Bos 2003; Muskens 1996) or by providing a direct interpretation using an embedding function that verifies a DRS in a given model (Kamp & Reyle 1993). PDRT provides a richer semantic formalism than DRT, since it incorporates explicit information about the information status of semantic content. Yet, PDRSs inherit all interpretational properties from DRSs, including the translation to first-order logic, as demonstrated by the translation from PDRT into DRT formulated in Venhuizen *et al.* (2013) and implemented in PDRT-SANDBOX (Venhuizen & Brouwer 2014). This translation involves moving all projected content to the appropriate interpretation site, while taking into account the constraints posed on projection by the MAPs. In case the projection site is underspecified, pragmatic heuristics are employed to determine the context to which the content will be accommodated. In particular, we employ the basic heuristic that global accommodation is preferred over local accommodation (cf. Geurts 1999; van der Sandt 1992). As shown in section 3.2.3, the MAP constraints may not always be satisfiable within the current discourse structure due to a lack of embedding contexts. Since the model-theoretic interpretation of DRSs is only defined with respect to the current discourse structure, however, accommodating the projected content to the global discourse context will result in the appropriate truth-conditions.

As a result of the translation from PDRT to DRT, the information status of semantic content does not affect its truth-conditional interpretation. This is in line with traditional theories of information structure, on which it is a discourse-level phenomenon that affects the felicity conditions rather than the truth-conditions of a discourse (cf. Chafe 1974). Moreover, it underlines the uni-dimensional treatment of projected and asserted content; presuppositions, anaphora and CIs differ from each other and from asserted content in terms of *how* they contribute to the unfolding discourse context, not in terms of the *type* of contribution. The PDRT analysis thus predicts no difference between presuppositions, anaphora, CIs and asserted content at the truth-conditional level; this prediction is supported by recent empirical evidence on the truth-conditional contributions of presuppositions (e.g. Abrusán & Szendroi 2013) and conventional implicatures (e.g. Syrett & Koev 2014), which shows that the truth value judgement of projected content directly affects the truth value of the sentence in which it occurs. When interpreting PDRSs via a translation to DRSs, all information-structural aspects of the discourse are disregarded. It should be noted, however, that certain properties of projected content, such as the speaker-oriented nature of CIs, may have an effect on the semantic interpretation. The PDRT formalism allows for incorporating these notions via a more elaborate interpretation function (see Venhuizen *et al.* 2014); we will address this point in the discussion (section 5).

4.2 Binding and accessibility

The treatment of anaphora is central to the formulation of traditional DRT. Determining when a discourse variable can be anaphorically linked to an earlier introduced referent involves the identification of free and bound referents. In DRT, a variable can be bound by a referent in case it occurs in the set of accessible universes. Accessibility is here defined based on a subordination relation between DRSs. More specifically, the universes that are accessible from a referent introduced in DRS K are the universe of K itself, and those of any DRS that directly, or indirectly subordinates K. Subordination, in turn, is defined as follows: DRS K_1*directly* subordinates DRS K_2 iff K_2 occurs in a condition in K_1, or K_1 serves as the antecedent of K_2 in an implication, and DRS K_1*indirectly* subordinates DRS K_2 iff K_2 is a sub-DRS of a DRS that directly subordinates K_1.

Crucially, the addition of pointers and labels to PDRSs affects this definition non-trivially, since projected content appears *in situ*, while it inherits the interpretational properties from its interpretation site. This discrepancy is illustrated in example (13).

(13) It is not the case that John is a vegetarian, he is a vegan.

$$
\begin{array}{c}
1\\
\begin{array}{|l|}
\hline
4 \leftarrow y\\
\hline
1 \leftarrow \neg \begin{array}{c} 2\\ \begin{array}{|l|}\hline 3 \leftarrow x\\ \hline 3 \leftarrow \text{John}(x)\\ 2 \leftarrow \text{vegetarian}(x)\\ \hline 2 \leq 3\\ \hline \end{array}\end{array}\\
1 \leftarrow \text{vegan}(y)\\
4 \leftarrow y = x\\
\hline
1 \leq 4 \quad 4 = 3\\
\hline
\end{array}
\end{array}
$$

The sentence in (13) contains an unresolved presupposition, triggered by 'John', and an anaphoric expression 'he'. In the PDRS, the presupposition is represented with an underspecified interpretation site (PDRS-context 3), and the anaphoric expression is represented by using equality statements in both the conditions ('$4 \leftarrow y = x$') and MAPs ('$4 = 3$') of PDRS 1. In order to obtain the desired interpretation of this PDRS, the variable 'x' in the condition '$4 \leftarrow y = x$' must be bound by the discourse referent introduced by the proper name 'John'. However, this discourse referent is introduced in an embedded PDRS (labeled 2), which is not accessible according to DRT's accessibility rules (Kamp & Reyle 1993). In PDRT, however, accessibility is determined based on the *interpretation site* of semantic content, as indicated by the projection pointers and the MAPs. In (13), the MAP constraints '$1 \leq 4$' and '$4 = 3$' indicate that context 3 (i.e. the context in which the referent introduced by the proper name 'John' is interpreted) should be accessible from context 1 (i.e. the context in which the anaphoric expression is interpreted). Hence, the variable 'x' in the condition '$4 \leftarrow y = x$' appears bound.

The example in (13) shows that in PDRT variables may be bound by projected referents whose *introduction* site is not hierarchically accessible; the only requirement is that the *interpretation* site of the projected referent is accessible. In order to account for this non-hierarchical binding, the definition of binding in PDRT requires an adjusted notion of context subordination, which determines the accessibility relations between all interpretation contexts; these include contexts introduced by (sub-)PDRS, as well as projected contexts introduced by the free pointers. The accessibility relations between these contexts should take into account the structural subordination constraints from DRT, as well as the additional constraints introduced by the MAPs. We formalize this using a graph-structure, called the *projection graph*, which contains underspecified accessibility relations for unresolved projected contexts. In what follows, we first describe how to derive the projection graph of a PDRS, and then define binding for both projection variables and projected referents, using the accessibility constraints that can be derived from the projection graph. These definitions can in turn be used to describe structural properties of PDRSs, as shown in the Appendix A.4.

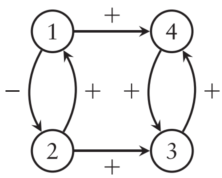
4.2.1 *Projection graph* A projection graph is a partial order over PDRS-contexts, which can be derived from the logical structure of the PDRS and the accessibility constraints in the MAPs. The projection graph is a directed labeled graph (E, V, l), consisting of a set of edges E, a set of vertices V (i.e. PDRS-contexts), and a labeling function l that maps edges to the labels '+' and '−' (signalling accessibility and inaccessibility, respectively).

The projection graph of a PDRS can be derived directly by traversing the PDRS structure, as shown in Definition 10 in Appendix A. Just like in DRT, a PDRS is accessible from itself and from any other PDRS that it subordinates; that is, the antecedent of an implication is accessible from its consequent, and the context indicated by the pointer of a condition is accessible from all PDRSs within that condition. Moreover, the projection graph of a PDRS incorporates the additional accessibility constraints provided by the MAPs, as well as the constraint that pointers can only indicate contexts that are accessible from the PDRS in which the pointer is introduced. Importantly, the way in which the projection graph is derived assumes that all projection variables in a PDRS indicate unique discourse contexts; that is, there cannot be any duplicate uses of projection variables in the PDRS from which the graph is derived, as these cannot be distinguished in the resulting projection graph (in other words, the PDRS must be *pure*; see Definition 16 in the appendix).

Example (14) shows the projection graph for example (13), as derived using the derivation procedure shown in Definition 10, Appendix A.2.[4] The notation used in (14a) defines a projection graph as a set of labeled edges, where an edge between vertices a and b with label l is indicated as $\{\langle a, b\rangle \mapsto l\}$. The graphical representation is shown in (14b); here the spatial ordering loosely reflects the hierarchical structure of PDRS-contexts.

(14) a. $\{\langle 1, 2\rangle \mapsto -, \langle 2, 1\rangle \mapsto +, \langle 2, 3\rangle \mapsto +, \langle 1, 4\rangle \mapsto +, \langle 4, 3\rangle \mapsto +, \langle 3, 4\rangle \mapsto +\}$

b.



The graph-theoretic properties of the projection graph reflect the information theoretic properties of the PDRS. The fact that the graph is not fully connected (i.e. weakly connected) reflects the underspecified nature of the PDRS: there is no accessibility relation from 4 to 1, indicating that context 4 may still be resolved to be the same as context 1, or a context dominating it. The accessibility relations between all other contexts can be derived using basic graph-theoretic inferences. In particular, based on the transitive nature of the accessibility relation, we can infer that context 2 is not accessible from context 3 (i.e. $\langle 3, 2\rangle \mapsto -$): since context 3 is accessible from 1 (through context 4), and context 2 is not accessible from 1, it must follow that context 2 is not accessible from context 3.

We can now define the accessibility between PDRS-contexts in a PDRS as finding a path p between two vertices, such that all edges in the path indicate a positive accessibility relation:

Definition 3 (PDRS accessible universes). The universe of PDRS-context π_j is accessible from PDRS-context π_i in PDRS P with projection graph $\mathcal{G}$, that is, $\pi_i \leq \pi_j$ (in $\mathcal{G}$), iff:

(i) There is a path p from π_i to π_j in $\mathcal{G}$, that is, $p = \mathit{path}(\pi_i, \pi_j, \mathcal{G}) \neq \emptyset$;

(ii) p consists only of positive edges, that is, $\mathit{pathlab}(p) = \{+\}$.

Here, $\mathit{pathlab}(p)$ is the path-label of a path p, defined as the (unordered) set of labels of the edges that make up a path (cf. Zou *et al.* 2014):

$$\mathit{pathlab}(p) = \bigcup_{e \in p} \mathit{lab}(e), \text{ where } \mathit{lab}(e) \text{ is } e\text{'s edge label}$$

Note that the first constraint in Definition 3 rules out the accessibility of underspecified contexts, unless it is explicitly specified in the MAPs. This therefore introduces an important design principle on PDRSs, which states that the MAPs should explicitly reflect *all*

4 For the purpose of readability, we do not show the loop edges that describe the accessibility of a context to itself.

accessibility relations required to obtain the desired interpretation (that is, both MAP constraints $1 \leq 4$ and $4 = 3$ in example (13) are required for securing variable binding).

Based on the projection graph, we can define variable binding in PDRT. As described above, in order to determine whether some referent is bound, the interpretation site of the referent, as well as the interpretation site of its possible antecedents, must be determined. This is done on the basis of the projection variables, which in turn may also occur *free* or *bound* in a PDRS. Therefore, we first define variable binding for projection variables, and accordingly describe binding of projected referents in PDRT.

4.2.2 *Binding of projection variables* The definition for free and bound projection variables in PDRT parallels the DRT definition of free and bound referents. DRS referents can be bound by a referent introduced in the universe of an accessible DRS. Similarly, projection variables can be bound by the label of some accessible PDRS. This is formalized below.

Definition 4 (Projection Variable Binding). A projection variable v, introduced in PDRS P_i with label π_i is bound in global PDRS P (represented as: $boundpvar(v, \pi_i, P)$) iff there exists a sub-PDRS P_j in P, such that:

(i) P_j is accessible from P_i in the projection graph $\mathcal{G}$ of P, that is, $P_i \leq P_j$;

(ii) The label of P_j is v, that is, $lab(P_j) = v$.

Informally, a projection variable v is bound in case it occurs as the label of a PDRS P_j that is accessible from the introduction site P_i of v (which has label π_i). Thus, the binding of projection variables in PDRT crucially depends on the introduction site of the projection variable, just like in the DRT definition for bound referents. Critically, however, this introduction site may be a projected context itself (e.g. as part of a projected condition). Therefore, the binding of projection variables is not simply determined based on context subordination (as is the case for the binding of referents in DRT), but rather using the PDRT notion of accessibility in terms of the projection graph defined above. The set of all free projection variables in a PDRS P, indicated by $\mathcal{F}_\pi(P)$, can now be derived by traversing the PDRS and checking for all projected referents in the universes, conditions and MAPs of P whether they occur free (i.e. not bound). For each (embedded) projection variable, binding is determined relative to the *global* PDRS, in order to make sure that all possible projected antecedents are taken into account. The full definition is shown in Appendix A, Definition 11.

4.2.3 *Binding of projected referents* Based on the definition of free and bound projection variables, we can define the binding of *projected referents* in PDRT, that is, discourse referents combined with a pointer (formalized in Definition 9 in the appendix). As described above, the binding of a projected referent is defined relative to its *interpretation site* and that of its potential antecedent. More formally, a projected referent $p \leftarrow r$ is bound by projected referent $p' \leftarrow r$ in case $p' \leftarrow r$ is introduced in a universe in the global PDRS and p' is accessible from p in the projection graph. This notion of binding of projected referents is formalized as follows (here, $\Pi(P)$ represents the set of all ptojection variables in P; see Definition 8, Appendix A.1):

Definition 5 (Projected Referent Binding). A projected referent $p \leftarrow r$ is bound in global PDRS P ($boundpref(p \leftarrow r, P)$) iff there exists a PDRS-context $\pi_j \in \Pi(P)$, such that:

(i) π_j is accessible from the interpretation site of the projected referent ($p \leq \pi_j$);

(ii) $\pi_j \leftarrow r$ is introduced in some universe of P, that is, there exists some PDRS $P_j \leq P$, such that $\pi_j \leftarrow r \in U(P_j)$.

The set of all free discourse referents of a PDRS P, represented as $\mathcal{F}_{\mathcal{R}}(P)$, can now be defined straightforwardly by traversing the conditions of P and determining for each referent x_i occurring in a relation-condition ($p \leftarrow R(x_1, \ldots, x_n)$) or a propositional condition ($p \leftarrow x : K$) whether $p \leftarrow x_i$ is free in P relative to the label of its introduction context l. This is formalized in Definition 12 in Appendix A.

Following this definition of binding, we can now formally determine whether the variable 'x' in the condition '$4 \leftarrow y = x$' in example (13) is bound by the discourse referent introduced by the proper name 'John'. That is, we need to determine whether the projected referent '$4 \leftarrow x$' is bound by the projected referent '$3 \leftarrow x$'. Following definition 5, this is the case if (i) '$3 \leftarrow x$' appears in a universe in the global PDRS, and (ii) there is a positive edge between context 4 and context 3 in the projection graph of the PDRS. Now it is straightforward to derive that (i) and (ii) are both satisfied: '$3 \leftarrow x$' appears in the universe of PDRS 2, and it follows from the projection graph shown in 4.2.1 that context 4 is accessible from 3. Thus, it follows that 'x' appears bound in this PDRS.

4.3 Composition

As the aim of a semantic formalism is to construct complex meaning representations, we need a way to combine structures in order to create larger meaning representations. The combination of structures in PDRT basically extends the traditional DRT notion of 'merge'. Combining two structures in DRT means creating a novel DRS that contains all referents and conditions of both conjuncts, and resolves any anaphoric dependencies. In PDRT, the merging of discourse structures should take into account the information status of the merged information, as represented by the projection variables. To this end, we define two different types of merge: one for *asserted* content, and one for *projected* content (cf. Venhuizen *et al.* 2013, 2014). These two merges reflect the different ways in which content can be contributed to the unfolding discourse context; asserted (*new*) content is simply added to the discourse context, while projected (*old*) content is either linked to information available in the discourse context, or added as established information. In what follows, we describe the two merge operations of PDRT and investigate the interactions between them, and then formalize a compositional version of PDRT in line with Muskens (1996).

4.3.1 *Assertive Merge* Assertive Merge is an operation between two PDRSs P_1 and P_2 that results in a PDRS in which the asserted content of both P_1 and P_2 remains asserted (i.e. occurring with a pointer that is bound by the label of the local PDRS). Here, PDRS P_1 represents the novel information that is added to the unfolding discourse context, represented by P_2. This operation is similar to traditional DRS Merge (Kamp & Reyle 1993): it takes the union of the sets of referents, conditions and MAPs from both merge arguments, and renames any overlapping, bound discourse referents using α-*conversion* (the *rename* function; see Appendix A, Definition 17). In addition, all asserted content from P_1 must remain asserted in the resulting PDRS; this is achieved by renaming all pointers bound by the label of P_1 to the label of (the α-converted version of) P_2. The formal definition for Assertive Merge (indicated by the + operator) is provided below (here, $\Pi(P)$ describes the set of projection variables of PDRS P and $\Delta(P)$ describes the discourse referents in P).

Definition 6 (Assertive Merge). Given two PDRSs P_i and P_j, such that $P_i = \langle l_i, U_i, C_i, M_i \rangle$ and $P_j = \langle l_j, U_j, C_j, M_j \rangle$, the assertive merge $P_i + P_j$ is defined as follows:

$$P_i + P_j = \langle l_{j'}, (U_i \cup U_{j'}), (C_i \cup C_{j'}), (M_i \cup M_{j'}) \rangle [l_i \backslash l_{j'}]$$

where: $P_{j'} = \langle l_{j'}, U_{j'}, C_{j'}, M_{j'} \rangle = \mathit{rename}(P_j, \Pi(P_i), \Delta(P_i))$

The result of this merge procedure is a PDRS in which all asserted content from P_1 and P_2 remains asserted. Since the pointers of the projected content are not affected in this merge procedure, this content either remains projected, or becomes asserted due to binding of projection variables. This procedure therefore effectively implements van der Sandt's (1992) formalization of presupposition projection as anaphora resolution; in PDRT, presuppositions are resolved in the same way as anaphora, namely by binding to an antecedent during discourse construction. Note that this behavior is a direct result of the way in which projected content is represented in PDRT, and therefore does not require any additional resolution procedures, as in van der Sandt's (1992) original formulation.

An example of Assertive Merge is shown in (15). Here, the two PDRSs that are being merged only contain asserted content; in the resulting PDRS, this content remains asserted. There are no overlapping variables between the two PDRSs, so no additional variables need to be renamed.

(15) A dog barks. A cat meows.

$$\begin{array}{c} 1 \\ \begin{array}{|l|} \hline 1 \leftarrow x \\ \hline 1 \leftarrow \mathrm{dog}(x) \\ 1 \leftarrow \mathrm{bark}(x) \\ \hline \\ \hline \end{array} \end{array} + \begin{array}{c} 2 \\ \begin{array}{|l|} \hline 2 \leftarrow y \\ \hline 2 \leftarrow \mathrm{cat}(y) \\ 2 \leftarrow \mathrm{meow}(y) \\ \hline \\ \hline \end{array} \end{array} = \begin{array}{c} 2 \\ \begin{array}{|l|} \hline 2 \leftarrow x \;\; 2 \leftarrow y \\ \hline 2 \leftarrow \mathrm{dog}(x) \\ 2 \leftarrow \mathrm{bark}(x) \\ 2 \leftarrow \mathrm{cat}(y) \\ 2 \leftarrow \mathrm{meow}(y) \\ \hline \\ \hline \end{array} \end{array}$$

4.3.2 *Projective Merge* Not all content is added to the discourse context as *novel* information. Some content might be backgrounded, or refer to already established information. This way of adding information to the discourse context is formalized as Projective Merge (cf. Bos's 2003, α-operator). When two PDRSs are combined using Projective Merge, the result is a PDRS in which the asserted content of the first argument of the merge is projected. This is achieved by merging the contents of both PDRSs, without renaming the pointers of the projected PDRS; this way, the content becomes projected since its pointers are not bound anymore by the local PDRS. In order to ensure the accessibility of projected contexts (see section 4.2), a MAP constraint reflecting this accessibility relation is added to the merged PDRS. The formal definition for Projective Merge (indicated using the $*$ operator) is provided below (again, $\Pi(P)$ and $\Delta(P)$ describe the sets of projection variables and discourse referents in P, respectively).

Definition 7 (Projective Merge). Given two PDRSs P_i and P_j, such that $P_i = \langle l_i, U_i, C_i, M_i\rangle$ and $P_j = \langle l_j, U_j, C_j, M_j\rangle$, the projective merge $P_i * P_j$ is defined as follows:

$$P_i * P_j = \langle l_{j'}, U_i \cup U_{j'}, C_i \cup C_{j'}, M_i \cup M_{j'} \cup \{l_{j'} \leq l_i\}\rangle$$

where: $P_{j'} = \langle l_{j'}, U_{j'}, C_{j'}, M_{j'}\rangle = \mathit{rename}(P_j, \Pi(P_i), \Delta(P_i))$

In both merge operations, the order of the arguments can crucially affect the interpretation of the resulting PDRS; in Projective Merge, only the content of the first argument becomes projected, and both Assertive and Projective Merge allow for free variables from the second argument to become bound by content in the first argument (this includes discourse referents as well as projection variables). Moreover, due to the asymmetry between Assertive and Projective Merge, not only the order of the arguments, but also the order of application of different merge operations may affect the resulting PDRS. This is demonstrated in example (16), which shows the different ways in which PDRSs P_1, P_2 and P_3 can be combined using Assertive and Projective Merge.

(16)

$$P_1 = \overset{1}{\begin{array}{|l|}\hline 1 \leftarrow x \\ \hline 1 \leftarrow \text{man}(x) \\ \hline \\ \hline \end{array}} \quad P_2 = \overset{2}{\begin{array}{|l|}\hline 2 \leftarrow y \\ \hline 2 \leftarrow \text{sister}(y) \\ 2 \leftarrow \text{of}(y,x) \\ \hline 2 \leq 1 \\ \hline \end{array}} \quad P_3 = \overset{3}{\begin{array}{|l|}\hline \\ \hline 3 \leftarrow \text{love}(x,y) \\ \hline 3 \leq 2 \\ \hline \end{array}}$$

a. $(P_1 + P_2) * P_3 =$

$$\overset{3}{\begin{array}{|l|}\hline 2 \leftarrow x \;\; 2 \leftarrow y \\ \hline 2 \leftarrow \text{man}(x) \\ 2 \leftarrow \text{sister}(y) \\ 2 \leftarrow \text{of}(y,x) \\ 3 \leftarrow \text{love}(x,y) \\ \hline 3 \leq 2 \;\; 2 \leq 2 \\ \hline \end{array}}$$

b. $P_1 + (P_2 * P_3) =$

$$\overset{3}{\begin{array}{|l|}\hline 3 \leftarrow x \;\; 2 \leftarrow y \\ \hline 3 \leftarrow \text{man}(x) \\ 2 \leftarrow \text{sister}(y) \\ 2 \leftarrow \text{of}(y,x) \\ 3 \leftarrow \text{love}(x,y) \\ \hline 3 \leq 2 \;\; 2 \leq 3 \\ \hline \end{array}}$$

c. $P_1 * (P_2 + P_3) = (P_1 * P_2) + P_3 =$

$$\overset{3}{\begin{array}{|l|}\hline 1 \leftarrow x \;\; 3 \leftarrow y \\ \hline 1 \leftarrow \text{man}(x) \\ 3 \leftarrow \text{sister}(y) \\ 3 \leftarrow \text{of}(y,x) \\ 3 \leftarrow \text{love}(x,y) \\ \hline 3 \leq 1 \;\; 3 \leq 3 \\ \hline \end{array}}$$

The result of the operations in (16a) is a PDRS with a single presupposition containing the (asserted) content of P_1 and P_2. However, if we change the order of merge applications, as shown in (16b), the status of the information in resulting PDRS changes. Since the MAPs constrain the interpretation site of the content of PDRSs P_2 and P_3, the presupposition triggered by the Projective Merge is 'cancelled', and no content is projected. Now, the order of merge applications does not matter if the Projective Merge operation appears sequentially before the Assertive Merge operation, as this prevents presupposition cancellation. This is shown in (16c). Interestingly, all three merged PDRSs correspond to unique natural language examples; (16a) can be paraphrased as '*The man who has a sister loves her*', (16b) as '*A man loves his sister*' and (16c) as '*The man has a sister and loves her*'. This suggests that these interactions between asserted and projected content are in fact reflected in the way linguistic meaning is constructed.

4.3.3 *Compositional PDRT* The merge operations described so far allow for the combination of *complete* structures, representing for example sentences or propositions. Ideally, however, we could build up structures from even smaller building blocks, such as words.

We thus need a way to express the meaning of unresolved structures that still need to be combined with some additional content in order to form a complete PDRS. Following Muskens's (1996) definition of Compositional DRT, we can define a compositional version of PDRT, in which the unresolved lambda-structures contain PDRSs that are combined using Assertive and Projective Merge. With this machinery, we can deal with projection in a straightforward and intuitive way on the lexical level, so that it becomes an inherent part of meaning composition: presupposition triggers introduce a projective merge to insert their content in their local context, instead of introducing a constant (cf. Kamp & Reyle 1993; Muskens 1996) or requiring a *post hoc* projection mechanism (such as the algorithm proposed by van der Sandt 1992).

Example (17) shows an example of the lexical semantics of the proper name 'John'; a lambda-PDRS of type '$(e \rightarrow t) \rightarrow t$' that uses a Projective Merge to combine the PDRS that introduces 'John' with an unresolved predicate (cf. the lexical semantics introduced by Bos 2003).

(17)

$$\text{`}\textit{John}\text{'}: \quad \lambda p.(\ \begin{array}{|l|} \multicolumn{1}{c}{1} \\ \hline 1 \leftarrow x \\ \hline 1 \leftarrow \text{John}(x) \\ \hline \\ \hline \end{array} \ * \ p(x))$$

When combined with an unresolved structure of type '$e \rightarrow t$', the pointer associated with 'John' ('1' in the example above) will become free due to the Projective Merge, and therefore projects. Note, however, that the presupposition may still become bound later on during discourse construction, when it is combined with an antecedent PDRS that matches its label. This treatment of presupposition projection as part of discourse construction is much like the treatment of anaphoric expressions in dynamic semantic formalisms, such as Dynamic Predicate Logic (Groenendijk & Stokhof 1991). Just like in the case of anaphoric expressions, determining the projection site of a presupposition in PDRT is part of constructing its lexical semantics. This emphasizes a context-dependent view of meaning construction.

A set of lexical items together with their semantics is shown in Appendix B. One of the main advantages of the PDRT representation over the traditional DRT representations is that the use of different types of merge nicely captures the correspondence between projecting and non-projecting sibling items. For instance, the definite determiner 'the' and the indefinite determiner 'a' obtain the same lexical semantic representation, except for the fact that the argument of 'the' is contributed to its direct context using Projective Merge, whereas the argument of 'a' uses Assertive Merge. A similar minimal pair is constituted by the lexical semantics of the factive verb 'know' and that of its non-factive equivalent 'believe' (see Appendix B, Table 5).

5 Discussion

Projective Discourse Representation Theory is a novel semantic formalism in which information structure is explicitly part of the semantic representations. PDRT extends traditional DRT with a notion of information structure through the addition of projection variables. Crucially, different constellations of these variables capture the information-structural differences between the contributions made by different types of projected content, as well as asserted content. Moreover, we have shown how these differences derive from the compositional construction procedure of PDRSs. As such, PDRT is the first formalism that provides a uni-dimensional, compositional analysis of presuppositions, anaphora, CIs and asserted content. Below, we describe several implications of the PDRT analysis, speculate about a more extensive notion of interpretation that takes into account information-structural aspects of meaning, and formulate different applications of PDRT.

5.1 Implications of the PDRT analysis

The PDRT analysis of presuppositions follows van der Sandt (1992) in treating presuppositions as a species of anaphoric expressions that can be bound to previously introduced antecedents. In contrast to van der Sandt (1992), however, the anaphoric nature of presuppositions is not stipulated, but rather follows from the way in which they are encoded in the information structure of the discourse. Presuppositions are not at-issue with respect to the local Question Under Discussion and hence associated with a projecting pointer, which may become bound as part of the larger discourse context (when the presupposed content becomes at-issue). Crucially, this analysis allows for a uniform treatment of presuppositions and other projection phenomena that are not necessarily anaphoric in nature, such as conventional implicatures. In fact, several expressions and constructions that are traditionally categorized as (anaphoric) presuppositions can be better explained in terms of not at-issueness with respect to the local QUD. For instance, sortally restricted predicates such as ‘bachelor’, which is taken to presuppose that the predicated individual is an adult male, are not obviously anaphoric in nature, as they do not introduce a referent that can be bound. These predicates are better viewed as providing backgrounded information that is not relevant to the local QUD (cf. Geurts 2010). In PDRT, this means that ‘bachelor’ introduces projected PDRS conditions for ‘adult’ and ‘male’, which elaborate on the description of the predicated individual, similar to the contribution made by conventional implicatures (see Appendix B for the lexical semantics of ‘bachelor’).

On the PDRT analysis, differences between asserted content and different types of projected content are explained in terms of differences in information structure, which are represented through different constellations of projection variables. As such, presuppositions, anaphora, CIs and asserted content are not treated as being different types of content; they are all interpreted within a single dimension of meaning. The PDRT analysis thus naturally predicts that these different contributions may interact on the level of semantic interpretation, for instance in terms of mutual anaphoric dependencies. This prediction is consistent with recent evidence emphasizing such a close interaction between different types of projected and non-projected content (see e.g. Amaral *et al.* 2007; AnderBois *et al.* 2010; Koev 2014; Nouwen 2007; Schlenker 2013). AnderBois *et al.* (2010), for instance, point out that a strong separation between at-issue content and CIs is challenged by the observation that various semantic phenomena ‘cross the meaning boundary’ between these different types of content. In particular, anaphoric dependencies, presuppositions as well as ellipsis from the at-issue part of a sentence can be resolved as part of a conventional implicature, as well as the other way around. This is illustrated in (18) (from AnderBois *et al.* 2010):

(18) John$_x$, <u>who nearly killed a$_y$ woman with his$_x$ car</u>, visited her$_y$ in the hospital.

In this example, the underlined non-restrictive relative clause triggers a conventional implicature. Interestingly, the anaphoric expression ‘his’ that is introduced as part of the

CI content, is bound by the presuppositional antecedent 'John', which is introduced as part of the asserted content. Moreover, the indefinite noun phrase 'a woman', introduced as part of the CI, serves as the antecedent for the pronoun 'her', which occurs in the asserted part of the sentence. This bidirectional dependency calls for a uni-dimensional analysis of CIs, presuppositions, anaphora and at-issue content. Indeed, this challenges theories of projection that as assume different 'layers' or 'dimensions' of meaning (see e.g. Geurts & Maier 2003; Potts 2005), whereas it follows naturally from the PDRT analysis.

Hence, by extending DRT with projection variables to represent information status, we increase the expressiveness of its semantic representations to naturally capture the fine-grained differences, similarities and interactions between asserted and different types of projected content. Indeed, this extension comes at a cost of increased complexity of the semantic formalism. Yet, we believe that the trade-off between increased complexity and increased expressiveness pays out. PDRT employs projection variables to indicate the interpretation site of linguistic content, thereby increasing the expressiveness of semantic representations, while not requiring any machinery beyond that already present in traditional DRT. That is, as was demonstrated in the definitions above, projection variables behave just like traditional DRS referents in terms of their binding properties, and the way in which they can be resolved (note, however, that unlike DRS referents, projection variables do not obtain a model-theoretic interpretation). Moreover, the introduction of projection variables results in a simpler construction procedure for representations containing projected content than the one proposed by van der Sandt (1992), since presuppositions do not need to be resolved *post hoc*, as the projection variables can become bound already during discourse construction. The result is a purely *dynamic* analysis of projection, in which projection triggers provide content with 'projection potential' on the lexical level—using Projective Merge—and the context determines where the content will be accommodated by binding the projection variables during discourse construction.

5.2 Information structure in interpretation

As discussed in section 4.1.2 above, PDRSs have a model-theoretic interpretation that can be derived via the translation of PDRSs to DRSs. However, this translation comes at the cost of losing information about how the information is structured within the discourse (e.g. the status of presupposed material), since during translation all content is moved to the interpretation site indicated by the pointers. There are various ways in which the formal models can be extended in order to incorporate the different aspects of meaning that can be represented in PDRT.

A straightforward extension would be the incorporation of indexical expressions in PDRT. In order to account for the interpretational properties of indexical expressions, such as *I* and *here*, Hunter (2013) proposes a DRT analysis in which indexicals, and other presupposition triggers, are accommodated to a special global DRS context, called K_0, which is evaluated with respect to the extra-linguistic context, that is, the actual world (see also, Hunter & Asher 2005). Such an analysis is highly compatible with the PDRT approach, since the use of free pointers for presuppositions already suggests their readiness to be bound by the label of some higher (possibly extra-linguistic) context. In fact, the availability of such a context is already assumed by the PDRT analysis of conventional implicatures proposed in Venhuizen *et al.* (2014), as CIs require their anchor to project out of the local context via strict subordination, even in case the local context is itself the global context of the discourse, as shown in example (9) above.

Another way to extend the interpretation of PDRSs concerns the incorporation of different 'belief states'. For instance, Venhuizen *et al.* (2014) propose that the projection pointers from PDRT may be exploited to incorporate information about the speaker-oriented nature of CIs into the model. CIs are generally interpreted as contributing subjective content that is attributed to the speaker of the utterance. This information may be reflected in PDRT by the use of special pointers that indicate a context in a *subjective model*, which is the state of affairs in the world according to some specific discourse agent, for example, the speaker, the hearer or some agent introduced in the discourse (see Venhuizen *et al.* 2014, for more details).

Finally, on a more conceptual level, the PDRT framework is highly compatible with the semantic structures from SDRT, which extend DRT with a formal treatment of rhetorical structure (Asher 1993; Asher and Lascarides 2003). As described in section 3.3 above, SDRT employs variables to indicate discourse contexts, which accordingly obtain a model-theoretic interpretation that reflects how the discourse is rhetorically structured. Since PDRSs are already associated with a context label, it seems straightforward to extend their interpretation to serve as SDRT's discourse labels; they can simultaneously serve as binders for projection variables, and as the variables that constitute the rhetorical structure of a discourse. In addition to incorporating a notion of rhetorical structure into the PDRS representations, such a unification of PDRT and SDRT would allow for presupposition resolution constraints driven by rhetorical structure (cf. Asher & Lascarides 1998). Of particular interest for the resolution of projection phenomena is SDRT's concept of the Right Frontier Constraint (RFC). In SDRT, the RFC constrains how novel information can be attached to the discourse, and how anaphoric dependencies are resolved (Asher 2008; Asher and Lascarides 2003). Since projection in PDRT is treated on a par with anaphora resolution, the RFC could be employed to constrain the possible projection sites for projected content by constraining the projection graph (see section 4.2). Moreover, a combination of PDRT and SDRT would allow for a more detailed investigation of the relation between discourse structure and QUD structure, as already alluded to in section 3.1 (following Hunter & Abrusán 2017; Riester 2016).

5.3 Application of PDRT

With respect to the applicability of the representations from PDRT, and its position within the broader enterprise of semantic theory, it is important to consider the theoretical as well as the practical perspective. From a theoretical point of view, PDRT opens up the way to model and investigate the semantic properties of information-structural aspects of meaning. As was already shown by the analysis of conventional implicatures presented in Venhuizen *et al.* (2014), formalizing the behavior of specific linguistic phenomena in a semantic framework contributes significantly to the understanding of the semantic properties underlying this behavior. PDRT could be employed to gain a more fine-grained insight into the projection behavior of different kinds of projection triggers; for instance, PDRT could be used to investigate the different contextual constraints underlying the behavior of *weak* and *strong* presupposition triggers (Karttunen 1971). Conversely, as PDRT aims to treat projection as a property that is inherent to the way in which discourse representations are constructed, the PDRT analysis might contribute to the development of a unified analysis of projected content, in line with analyses that aim to explain projection in terms of *at-issueness* (Simons *et al.* 2010; Tonhauser *et al.* 2013). Moreover, the representations from PDRT may be used to investigate aspects of meaning beyond projection phenomena. For instance, the projection variables may also be used to represent differences in the scopal properties of linguistic quantifiers. Moreover, the additional level of information available in the representations of PDRT allows for the formalization of different syntactic constructions, and their interaction with linguistic meaning. For example, the MAPs may be employed to explicitly represent the notion of *givenness* as an ordering of projection sites. Givenness has been shown to critically affect the choice of syntactic structure in, for example, dative alternation (see Bresnan *et al.* 2007) and genitive alternation (see Rosenbach 2014).

From a practical point of view, the direct correspondence between the representations of PDRT and the linguistic surface structure make it an attractive semantic formalism for the purpose of natural language generation (see, e.g., Basile and Bos 2013). Moreover, the formalization of the construction and interpretation procedure makes PDRT a suitable formalism for computational applications. As described above, the formal definitions of PDRT have been implemented as part of PDRT-SANDBOX, a widely applicable NLP library (Venhuizen & Brouwer 2014). PDRT also provides the formal backbone underlying the semantic representations in the Groningen Meaning Bank (GMB; Basile *et al.* 2012; Bos *et al.* 2017). These existing implementations make PDRT a practically useful semantic framework for investigating linguistic phenomena using large-scale computational methods, which have become the standard in computational linguistics. This type of data-driven analysis is exemplified by the study presented in Venhuizen (2015), which investigates the information-structural properties of referential expressions based linguistic features derived from the PDRT analysis. Hence, the use of PDRT as a semantic formalism in large-scale data-driven analyses may aid in arriving at a more fine-grained understanding of information-structural aspects of meaning.

6 Conclusion

In this paper, we have presented Projective Discourse Representation Theory, an extension of traditional DRT that incorporates an explicit representation of the information status of semantic content via the use of 'projection variables'. PDRT effectively generalizes the DRT treatment of anaphora to account for the behavior of projection phenomena in general. We have shown how this allows for the derivation of a uni-dimensional analysis of asserted ('at-issue') content and projected ('not at-issue') content. The framework offers explicit—*semantic*—constraints on the resolution of presuppositions, anaphora and conventional implicatures, while it follows DRT in assuming an extraneous mechanism for their context-dependent— *pragmatic*—resolution. Semantic constraints are formalized in terms of Minimally Accessible Projection contexts (MAPs), which are defined as part of the lexical semantics of projecting expressions. As such, the formalism makes qualitative predictions about the behavior of different projection phenomena during discourse construction.

The PDRT formalism provides a rich representational scheme for formalizing and investigating a variety of linguistic phenomena. The addition of projection variables to discourse representation structures has been shown to provide a maximal information gain into the semantic representations, using minimal additional machinery. Yet, the incorporation of projection variables affects the formal properties of DRT non-trivially, because the hierarchical structure of the PDRT representations does not reflect their logical structure. In order to account for this, we have provided formal definitions of the variable binding and accessibility properties of PDRSs, as well as their compositional behavior (see also the 'PDRT-SANDBOX' implementation; Venhuizen & Brouwer 2014).

In sum, by extending DRT with projection variables, PDRT effectively employs traditional semantic notions such as variable binding to account for aspects of meaning that were not traditionally considered part of the semantic contribution of an utterance. As such, PDRT opens up new directions in the representation and investigation of linguistic meaning, in which non-truth-conditional aspects of meaning, such as information status, may contribute to and interact with model-theoretic semantic interpretations.

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Appendices

A PDRT: Additional Definitions

A.1 Variables

$\Pi(P)$ represents the set of projection variables of PDRS P:

Definition 8 (Projection variables in a PDRS).

(i) $\Pi(\langle l, U, C, M\rangle) = \{l\} \cup \bigcup_{u\in U} \Pi(u) \cup \bigcup_{c\in C} \Pi(c) \cup \bigcup_{m\in M} \Pi(m)$;

(ii) $\Pi(p \leftarrow x) = \Pi(p \leftarrow R(x_1 \ldots x_n)) = \{p\}$;

(iii) $\Pi(p \leftarrow \neg K) = \Pi(p \leftarrow \Diamond K) = \Pi(p \leftarrow \Box K) = \Pi(p \leftarrow x : K) = \{p\} \cup \Pi(K)$;

(iv) $\Pi(p \leftarrow K_1 \Rightarrow K_2) = \Pi(p \leftarrow K_1 \vee K_2) = \{p\} \cup \Pi(K_1) \cup \Pi(K_2)$;

(v) $\Pi(p_1 \leq p_2) = \Pi(p_1 \not\leq p_2) = \{p_1, p_2\}$.

$\mathcal{R}(P)$ represents the set of projected referents from PDRS P:

Definition 9 (Projected Referents).

(i) $\mathcal{R}(\langle l, U, C, M\rangle) = U \cup \bigcup_{c\in C} \mathcal{R}(c)$

(ii) $\mathcal{R}(p \leftarrow R(x_1, \ldots, x_n)) = \bigcup_{x\in\{x_1,\ldots,x_n\}} \{p \leftarrow x\}$

(iii) $\mathcal{R}(p \leftarrow \neg K) = \mathcal{R}(p \leftarrow \Diamond K) = \mathcal{R}(p \leftarrow \Box K) = \mathcal{R}(K)$

(iv) $\mathcal{R}(p \leftarrow K_1 \Rightarrow K_2) = \mathcal{R}(p \leftarrow K_1 \vee K_2) = \mathcal{R}(K_1) \cup \mathcal{R}(K_2)$

(v) $\mathcal{R}(p \leftarrow x : K) = \{p \leftarrow x\} \cup \mathcal{R}(K)$

A.2 Projection Graph

The projection graph of PDRS P, $pg(P)$, can be derived using the following procedure:

Definition 10 (PDRS to Projection Graph).

(i) $pg(\langle l, U, C, M\rangle) = \{\langle l, l\rangle \mapsto +\} \cup \bigcup_{v\leftarrow x \in U}\{\langle l, v\rangle \mapsto +\} \cup \bigcup_{c\in C} pg(l, c) \cup \bigcup_{m\in M} pg(l, m)$

(ii) $pg(l,\ v \leftarrow R(x_1, \ldots, x_n)) = \{\langle l,\ v\rangle \mapsto +\}$

(iii) $pg(l, v \leftarrow \neg P) = pg(l, v \leftarrow \Diamond P) = pg(l, v \leftarrow \Box P) = pg(l, v \leftarrow x : P)$
$= \{\langle l, v\rangle \mapsto +, \langle lab(P), l\rangle \mapsto +, \langle l, lab(P)\rangle \mapsto -\} \ \cup\ pg(P)$

(iv) $pg(l,\ v \leftarrow P_1 \vee P_2) = \{\langle l,\ v\rangle \mapsto +,\ \langle lab(P_1),\ l\rangle \mapsto +,\ \langle l,\ lab(P_1)\rangle \mapsto -,\ \langle lab(P_2),\ l\rangle \mapsto +,$
$\langle l,\ lab(P_2)\rangle \mapsto -\} \cup pg(P_1) \cup pg(P_2) \cup \{\langle lab(P_1),\ lab(P_2)\rangle \mapsto -,\ \langle lab(P_2),\ lab(P_1)\rangle \mapsto -\}$

(v) $pg(l,\ v \leftarrow P_1 \Rightarrow P_2) = \{\langle l,\ v\rangle \mapsto +,\ \langle lab(P_1),\ l\rangle \mapsto +,\ \langle l,\ lab(P_1)\rangle \mapsto -,\ \langle lab(P_2),\ l\rangle \mapsto +,$
$\langle l,\ lab(P_2)\rangle \mapsto -\} \cup pg(P_1) \cup pg(P_2) \cup \{\langle lab(P_1),\ lab(P_2)\rangle \mapsto -,\ \langle lab(P_2),\ lab(P_1)\rangle \mapsto +\}$

(vi) $pg(l,\ v_1 \leq v_2) = \{\langle l,\ v_1\rangle \mapsto +,\ \langle l,\ v_2\rangle \mapsto +,\ \langle v_1,\ v_2\rangle \mapsto +\}$

(vii) $pg(l, v_1 \nleq v_2) = \{\langle l, v_1\rangle \mapsto +, \langle l, v_2\rangle \mapsto +, \langle v_1, v_2\rangle \mapsto -\}$

A.3 Variable Binding

In the definitions below, *freepvar*(p, l, P) means that projection variable p, introduced in a PDRS labeled l, is *not* bound in PDRS P (i.e. *freepvar*(p, l, P) = ¬*boundpvar*(p, l, P)). Analogously, *freepref*($p \leftarrow x$, l, P)} means that projected referent $p \leftarrow x$, introduced in a PDRS labeled l, is *not* bound in PDRS P (that is, *freepref*($p \leftarrow x$, l, P)} = ¬*boundpref*($p \leftarrow x$, l, P)}).

$\mathcal{F}_\pi(P)$ represents the set of free projection variables in PDRS P:

Definition 11 (Free Projection Variables). Given a global PDRS P, we can define the set of free projection variables of PDRS $P' = \langle l,\ U,\ C,\ M\rangle$ as follows:

(i) $\mathcal{F}_\pi(\langle l, U, C, M\rangle) = \{p \mid p \leftarrow u \in U \wedge \mathit{freepvar}(p, l, P)\} \cup \bigcup_{c\in C} \mathcal{F}_\pi(c, l) \cup \bigcup_{m\in M} \mathcal{F}_\pi(m, l)$

(ii) $\mathcal{F}_\pi(p \leftarrow R(...), l) = \{p \mid \mathit{freepvar}(p, l, P)\}$

(iii) $\mathcal{F}_\pi(p \leftarrow \neg P_1, l) = \mathcal{F}_\pi(p \leftarrow \Diamond P_1, l) = \mathcal{F}_\pi(p \leftarrow \Box P_1, l) = \mathcal{F}_\pi(p \leftarrow x : P_1, l) =$
$\{p \mid \mathit{freepvar}(p, l, P)\} \cup \mathcal{F}_\pi(P_1)$

(iv) $\mathcal{F}_\pi(p \leftarrow P_1 \Rightarrow P_2, l) = \mathcal{F}_\pi(p \leftarrow P_1 \vee P_2, l) = \{p \mid \mathit{freepvar}(p, l, P)\} \cup \mathcal{F}_\pi(P_1) \cup \mathcal{F}_\pi(P_2)$

(v) $\mathcal{F}_\pi(p_1 \leq p_2, l) = \{\mathit{freepvar}(p_1, l, P)\} \cup \{\mathit{freepvar}(p_2, l, P)\}$

$\mathcal{F}_R(P)$ represents the set of free projected referents in PDRS P:

Definition 12 (Free Projected Referents). Given a global PDRS P, we can define the set of free projected referents of PDRS $P' = \langle l,\ U,\ C,\ M\rangle$ as follows:

(i) $\mathcal{F}_{\mathcal{R}}(\langle l, U, C, M\rangle) = \bigcup_{c\in C} \mathcal{F}_{\mathcal{R}}(c, l)$

(ii) $\mathcal{F}_{\mathcal{R}}(p \leftarrow R(x_1, \ldots, x_n), l) = \{p \leftarrow x_i \mid x_i \in \{x_1, \ldots, x_n\} \wedge \mathit{freepref}(p \leftarrow x_i, l, P)\}$

(iii) $\mathcal{F}_{\mathcal{R}}(p \leftarrow \neg P_1, l) = \mathcal{F}_{\mathcal{R}}(p \leftarrow \Diamond P_1, l) = \mathcal{F}_{\mathcal{R}}(p \leftarrow \Box P_1, l) = \mathcal{F}_{\mathcal{R}}(P_1)$

(iv) $\mathcal{F}_{\mathcal{R}}(p \leftarrow x : P_1, l) = \{p \leftarrow x \mid \mathit{freepref}(p \leftarrow x, l, P)\} \cup \mathcal{F}_{\mathcal{R}}(P_1)$

(v) $\mathcal{F}_{\mathcal{R}}(p \leftarrow P_1 \Rightarrow P_2, l) = \mathcal{F}_{\mathcal{R}}(p \leftarrow P_1 \vee P_2, l) = \mathcal{F}_{\mathcal{R}}(P_1) \cup \mathcal{F}_{\mathcal{R}}(P_2)$

A.4 Structural properties

Based on the definitions of free and bound variables in PDRT, we can define several properties of PDRSs. Firstly, a PDRS without any free projected referents is called a *proper* PDRS (cf. the definitition of proper DRSs defined in Kamp & Reyle 1993; Kamp *et al.* 2011):

Definition 13 (Properness). A PDRS P is proper iff P does not contain any free projected referents: $\mathcal{F}_{\mathcal{R}}(P) = \emptyset$.

If a PDRS contains free projection variables, this means that there are still unresolved presuppositions; a PDRS without any free projection variables is called a *non-presuppositional* PDRS:

Definition 14 (Non-presuppositionality). A PDRS P is non-presuppositional iff P does not contain any free pointers: $\mathcal{F}_{\pi}(P) = \emptyset$.

Not all content in a non-presuppositional PDRS needs to be asserted; some pointers may be bound by labels of accessible contexts (this is also referred to as 'intermediate accommodation'). A PDRSs without any projected content is called *projectionless*, or *plain*:

Definition 15 (Plainness). A PDRS P is plain iff all projection variables in P are locally accommodated: For all P', such that $lab(P') \leq lab(P)$, it holds that $\mathcal{F}_{\pi}(P') = \emptyset$.

The property of *purity* refers to the occurrence of duplicate uses of variables. The definition of PDRS purity consists of two parts: one describing impurity with respect to discourse referents, and the other impurity with respect to projection variables. This is formally defined as follows (here, $\mathcal{U}(P)$ indicates the union of all universes in P):

Definition 16 (Purity). PDRS P is pure iff:

(i) P does not contain any otiose uses of discourse referents (i.e. P does not contain any unbound, duplicate uses of discourse referents): For all P_1, P_2, such that $P_1 < P_2 \leq P$, and $lab(P_1) = p_1$ and $lab(P_2) = p_2$, it holds that: $\{r_1 \mid p_1 \leftarrow r_1 \in \mathcal{U}(P)\} \cap \{r_2 \mid p_2 \leftarrow r_2 \in \mathcal{U}(P)\} = \emptyset$;

(ii) P does not contain any otiose uses of projection variables (i.e. P does not contain any unbound, duplicate uses of projection variables): For all P_1, P_2, such that $P_1 < P_2 \leq P$, and $lab(P_1) = p_1$ and $lab(P_2) = p_2$, it holds that: $\{p_1\} \cap (\{p_2\} \cup \mathcal{F}_{\pi}(P)) = \emptyset$.

A.5 Composition

The renaming function $\mathit{rename}(P, \Pi(P'), \Delta(P'))$ performs alpha conversion on a PDRS P based on the projection variables ($\Pi(P')$) and discourse referents ($\Delta(P')$) from PDRS P':

Definition 17 (PDRS Renaming). Given the function: *rename*(P, Π, Δ), let:

$$\Pi_T = \bigcup_{i=1}^{|\Pi(P)|} \{(p_i, p_i')\} \text{ where } p_i \in \Pi(P) \text{ and } p_i' \in \mathit{newpvars}(\Pi(P))$$

$$\Delta_T = \bigcup_{i=1}^{|\Delta(P)|} \{(x_i, x_i')\} \text{ where } x_i \in \Delta(P) \text{ and } x_i' \in \mathit{newdrefs}(\Delta(P))$$

where *newpvars* is a function that takes a set of projection variables and returns a set of the same length containing novel projection variables, and *newdrefs* does the same for discourse referents.

Now, define *rename*$'(P)$ as follows:

(i) $\mathit{rename}'(\langle l, U, C, M\rangle) = \langle l', U', C', M'\rangle$
where: $l' = \mathit{trpvar}(l, l)$

$$U' = \bigcup\nolimits_{p \leftarrow r \in U} (\mathit{trpvar}(p, l) \leftarrow \mathit{trdref}(r, p, l))$$
$$C' = \bigcup\nolimits_{c \in C} \mathit{rename}'(c, l)$$
$$M' = \bigcup\nolimits_{p_1 \leq p_2 \in M} \mathit{trpvar}(p_1, l) \leq \mathit{trpvar}(p_2, l) \cup \bigcup\nolimits_{p_1 \not\leq p_2 \in M} \mathit{trpvar}(p_1, l) \not\leq \mathit{trpvar}(p_2, l)$$

(ii) $\mathit{rename}'(p \leftarrow R(x_1, \ldots, x_n), l) = p' \leftarrow R(x_1', \ldots, x_n')$

(iii) $\mathit{rename}'(p \leftarrow \neg P_1, l) = p' \leftarrow \neg(\mathit{rename}'(P_1))$

(iv) $\mathit{rename}'(p \leftarrow \Box P_1, l) = p' \leftarrow \Box(\mathit{rename}'(P_1))$

(v) $\mathit{rename}'(p \leftarrow \Diamond P_1, l) = p' \leftarrow \Diamond(\mathit{rename}'(P_1))$

(vi) $\mathit{rename}'(p \leftarrow P_1 \vee P_2, l) = p' \leftarrow (\mathit{rename}'(P_1) \vee \mathit{rename}'(P_2))$

(vii) $\mathit{rename}'(p \leftarrow P_1 \Rightarrow P_2, l) = p' \leftarrow (\mathit{rename}'(P_1) \Rightarrow \mathit{rename}'(P_2))$

(viii) $\mathit{rename}'(p \leftarrow x : P_1, l) = p' \leftarrow (x' : \mathit{rename}'(P_1))$

where: $p' = \mathit{trpvar}(p, l)$

$x_i' = \mathit{trdref}(x_i, p, l)$

The translate function *trpvar*(p, l) determines for a projection variable p whether it is free relative to its local context l, given global PDRS P. If this is not the case, p is translated based on the set of tuples of projection variables Π_T.

$$\mathit{trpvar}(p, l) = \begin{cases} p & \text{if } \mathit{freepvar}(p, l, P) \\ p' & \text{otherwise, where: } (p, p') \in \Pi_T \end{cases}$$

Similarly, *trdref*(x, p, l) determines whether a discourse referent x is free relative to pointer p and local context l, given global PDRS P. If this is not the case, x is translated based on the set of tuples of discourse referents Δ_T.

$$\mathit{trdref}(x, p, l) = \begin{cases} x & \text{if } \mathit{freepref}(p \leftarrow x, l, P) \\ x' & \text{otherwise, where: } (x, x') \in \Delta_T \end{cases}$$

B PDRT: Lexical semantics

Tables 4 and 5 show the lexical semantics for a set of English lexical items (cf. the lexical DRT semantics proposed by Bos 2003).

Table 4 PDRT representations for a set of lexical items

Item	PDRT Semantics
$\text{a}_{\langle et,\langle et,t\rangle\rangle}$	$\lambda p.\lambda q.((\overset{1}{\boxed{\begin{array}{l} 1 \leftarrow \text{x} \\ \hline ~ \\ \hline ~ \end{array}}} + p(\text{x})) + q(\text{x}))$
$\text{the}_{\langle et,\langle et,t\rangle\rangle}$	$\lambda p.\lambda q.((\overset{1}{\boxed{\begin{array}{l} 1 \leftarrow \text{x} \\ \hline ~ \\ \hline ~ \end{array}}} + p(\text{x})) * q(\text{x}))$
$\text{dog}_{\langle e,t\rangle}$	$\lambda x.(\overset{1}{\boxed{\begin{array}{l} ~ \\ \hline 1 \leftarrow \text{dog}(x) \\ \hline ~ \end{array}}})$
$\text{John}_{\langle et,t\rangle}$	$\lambda p.(\overset{1}{\boxed{\begin{array}{l} 1 \leftarrow \text{x} \\ \hline 1 \leftarrow \text{John(x)} \\ \hline ~ \end{array}}} * p(\text{x}))$
$\text{someone}_{\langle et,t\rangle}$	$\lambda p.(\overset{1}{\boxed{\begin{array}{l} 1 \leftarrow \text{x} \\ \hline 1 \leftarrow \text{person(x)} \\ \hline ~ \end{array}}} + p(\text{x}))$
$\text{happy}_{\langle et,et\rangle}$	$\lambda p.\lambda x.(\overset{1}{\boxed{\begin{array}{l} ~ \\ \hline 1 \leftarrow \text{happy(x)} \\ \hline ~ \end{array}}} + p(x))$
$\text{his}_{\langle et,\langle et,t\rangle\rangle}$	$\lambda p.\lambda q.(\overset{2}{\boxed{\begin{array}{l} 2 \leftarrow \text{y} \\ \hline 2 \leftarrow \text{male(y)} \\ \hline ~ \end{array}}} * ((\overset{1}{\boxed{\begin{array}{l} 1 \leftarrow \text{x} \\ \hline 1 \leftarrow \text{of(x,y)} \\ \hline 1 \leq 2 \end{array}}} + p(x)) * q(x)))$
$\text{walk}_{\langle\langle et,t\rangle,\langle et,t\rangle\rangle}$	$\lambda p.\lambda q.(p(\lambda x.(\overset{1}{\boxed{\begin{array}{l} ~ \\ \hline 1 \leftarrow \text{walk}(x) \\ \hline ~ \end{array}}} + q(x))))$
love $_{\langle\langle et,t\rangle,\langle\langle et,t\rangle,\langle et,t\rangle\rangle\rangle}$	$\lambda p.\lambda q.\lambda r.(q(\lambda x.(p(\lambda y.(\overset{1}{\boxed{\begin{array}{l} ~ \\ \hline 1 \leftarrow \text{love}(x,y) \\ \hline ~ \end{array}}} + r(x))))))$

Table 5 PDRT representations for a set of lexical items (cont.)

<table>
<tr><th>Item</th><th>PDRT Semantics</th></tr>
<tr><td>be_bachelor
$\langle\langle et,t\rangle,\langle et,t\rangle\rangle$</td><td>$$\lambda p.\lambda q.(p(\lambda x.((\overset{3}{\begin{array}{|l|}\hline \\ \hline 3 \leftarrow \text{man}(x) \\ 3 \leftarrow \text{adult}(x) \\ \hline 3 \leq lab(p) \\ \hline\end{array}} * \overset{1}{\begin{array}{|l|}\hline \\ \hline 1 \leftarrow \neg \overset{2}{\begin{array}{|l|}\hline \\ \hline 2 \leftarrow \text{married}(x) \\ \hline \\ \hline\end{array}} \\ \hline \\ \hline\end{array}}) + q(x))))$$
where: $lab(p)$ returns the first label of the unresolved PDRS p</td></tr>
<tr><td>believe
$\langle\langle et,t\rangle,\langle\langle et,t\rangle,\langle et,t\rangle\rangle\rangle$</td><td>$$\lambda p.\lambda q.\lambda r.(q(\lambda x.(\overset{1}{\begin{array}{|l|}\hline 1 \leftarrow \text{k} \\ \hline 1 \leftarrow \text{believe}(x,\text{k}) \\ 1 \leftarrow \text{k}: p(\lambda y.(\overset{2}{\begin{array}{|c|}\hline \quad \\ \hline \quad \\ \hline \quad \\ \hline\end{array}})) \\ \hline \\ \hline\end{array}} + r(x))))$$</td></tr>
<tr><td>know
$\langle\langle et,t\rangle,\langle\langle et,t\rangle,\langle et,t\rangle\rangle\rangle$</td><td>$$\lambda p.\lambda q.\lambda r.(q(\lambda x.(\overset{1}{\begin{array}{|l|}\hline 1 \leftarrow \text{k} \\ \hline 1 \leftarrow \text{know}(x,\text{k}) \\ 1 \leftarrow \text{k}: p(\lambda y.(\overset{2}{\begin{array}{|c|}\hline \quad \\ \hline \quad \\ \hline \quad \\ \hline\end{array}})) * p(\lambda z.(\overset{3}{\begin{array}{|c|}\hline \quad \\ \hline \quad \\ \hline \quad \\ \hline\end{array}})) \\ \hline \\ \hline\end{array}} + r(x))))$$</td></tr>
</table>

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