

# Harnessing Projection

## A Formal Implementation of

### Projective Discourse Representation Theory

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# In this talk

- ☞ What is Projective Discourse Representation Theory?
- ☞ We present PDRT-SANDBOX: an NLP Haskell library that implements the PDRT formalism (as well as DRT).

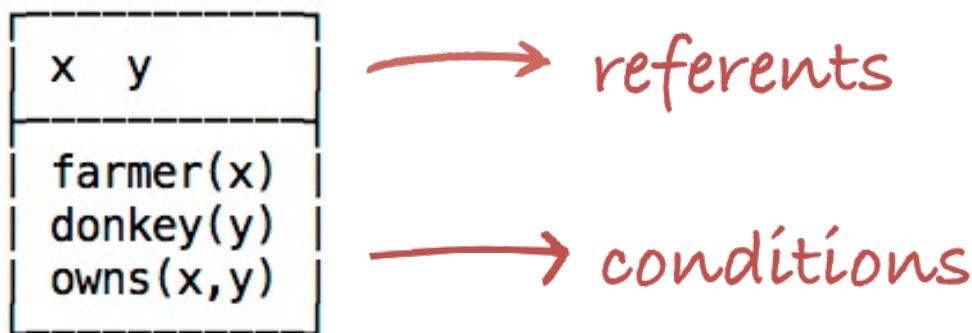


# Discourse Representation Theory

A representational, dynamic formalism for representing meaning.

Kamp (1981)

*A farmer owns a donkey.*

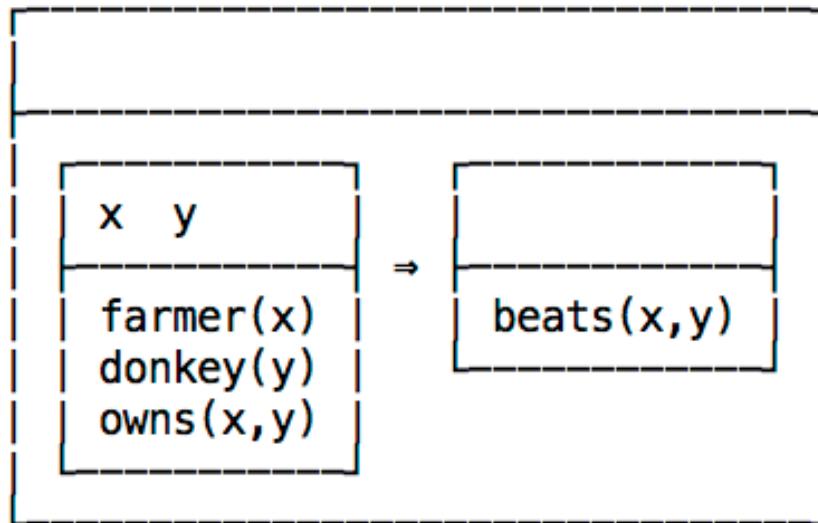


# Discourse Representation Theory

A representational, dynamic formalism for representing meaning.

Kamp (1981)

*If a farmer owns a donkey, he beats it.*



# DRT's pros



Wide coverage: anaphora, tense, presupposition

(van der Sandt, 1992; Kamp & Reyle, 1993)



Model-theoretic interpretation: FOL translation

(Kamp & Reyle, 1993; Bos, 2004)



“Compositional”: Montague Semantics

(cf. Muskens, 1996)

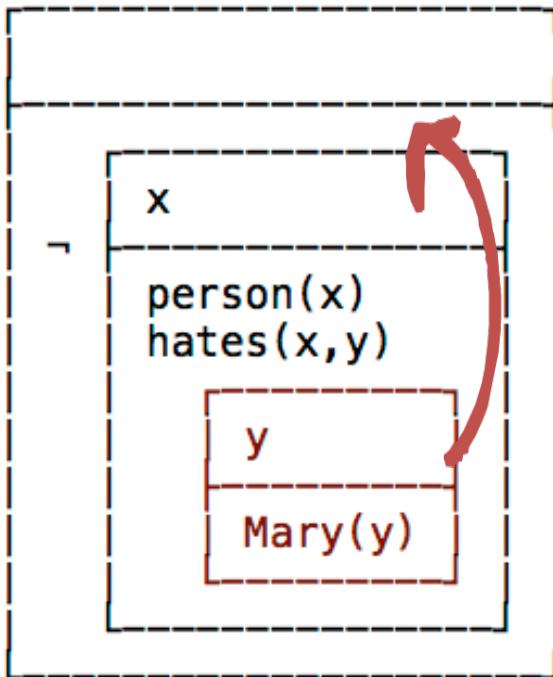
Problem: these properties do not always combine very well

# Projection vs. Compositionality

A **presupposition** is a piece of backgrounded information.

Projection refers to the property of being indifferent to semantic operators.

*Nobody hates Mary.* → *Mary exists*



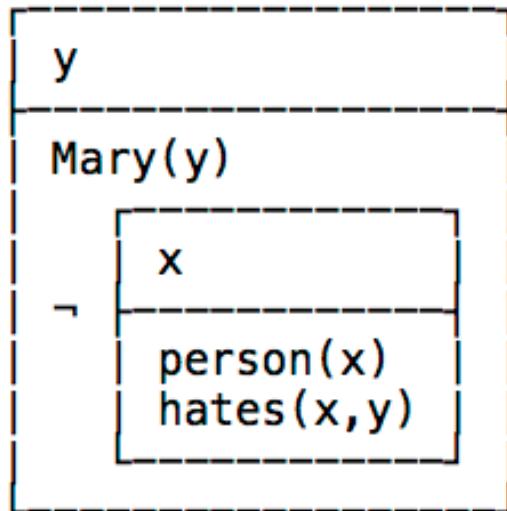
van der Sandt (1992): In DRT, presuppositions are moved in a second step of processing

# Projection vs. Compositionality

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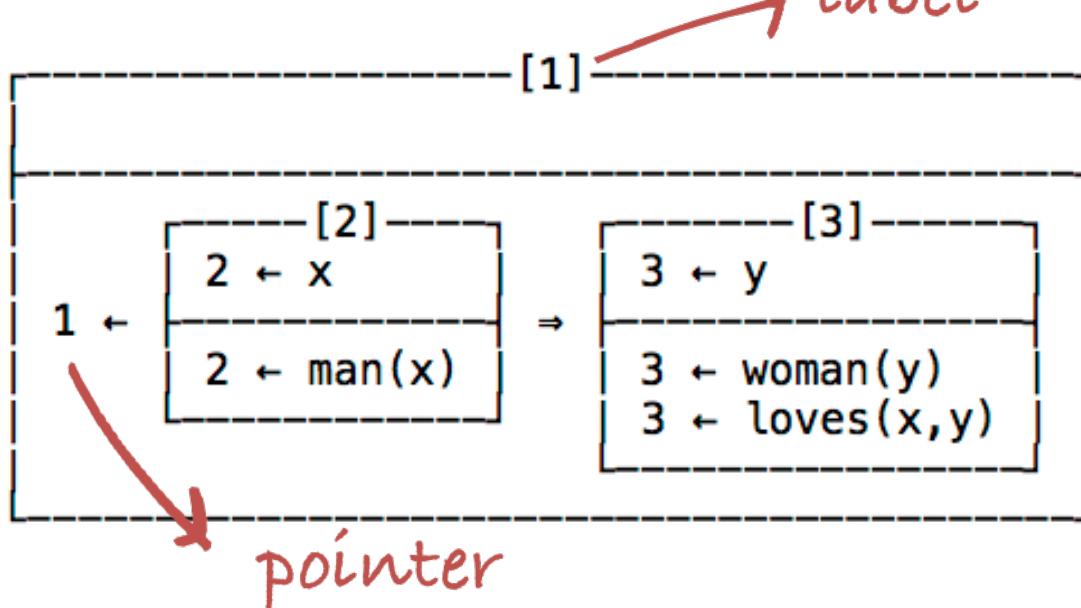
Sandt (1992): In DRT,  
How to reconcile this  
with a compositional,  
Montague-style  
construction procedure?

# A theoretical solution

## Projective Discourse Representation Theory: Adding Projection Pointers to DRSs

(Venhuizen, Bos, & Brouwer, 2013)

*Every man loves some woman.*

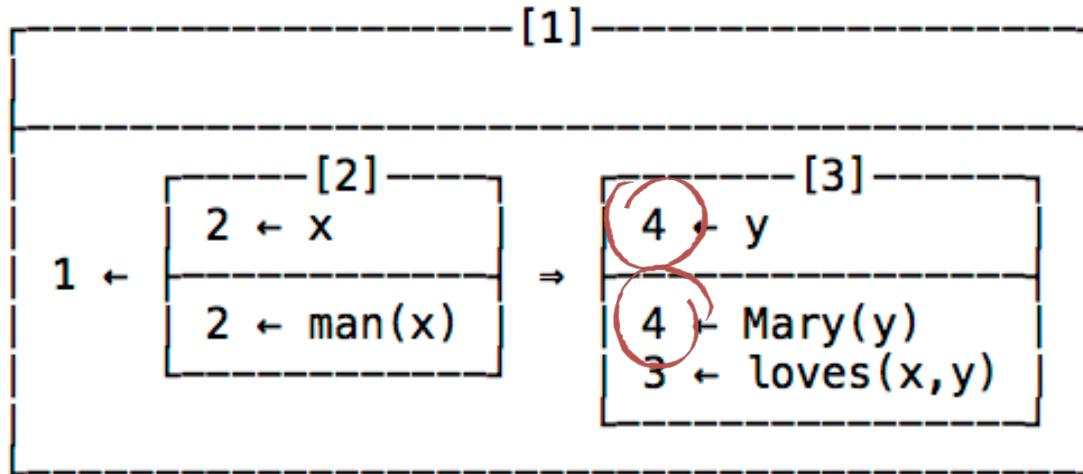


# A theoretical solution

## Projective Discourse Representation Theory: Adding Projection Pointers to DRSs

(Venhuizen, Bos, & Brouwer, 2013)

*Every man loves Mary.*



# But then, what about:

- Merging discourse structures?
- Anaphoric binding?
- Accessible contexts?
- Unresolved (P)DRSs?
- Semantic interpretation?

Let's implement it!



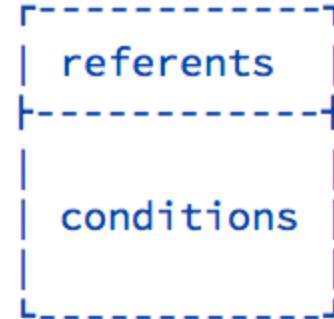
# Background: DRS definitions

Basic DRS:

$\langle \{\text{DRS referents}\}, \{\text{DRS conditions}\} \rangle$

$\hookrightarrow_{x,y}$

$\downarrow$   
 $\text{John}(x), \text{lives}(x,y),$   
 $\neg \text{DRS}, \text{DRS}_1 \Rightarrow \text{DRS}_2$



Complex DRS:

- Merge

$$\begin{array}{c|c} \hline & \text{Di} \\ \hline \end{array} + \begin{array}{c|c} \hline & \text{Dj} \\ \hline \end{array} = \begin{array}{c|c} \hline & \text{Di} \cup \text{Dj} \\ \hline \end{array}$$
$$\begin{array}{c|c} \hline & \text{Ci} \\ \hline \end{array} + \begin{array}{c|c} \hline & \text{Cj} \\ \hline \end{array} = \begin{array}{c|c} \hline & \text{Ci} \cup \text{Cj} \\ \hline \end{array}$$

- Lambda DRS

$$\lambda x. \begin{array}{c|c} \hline & x \\ \hline \end{array}, \quad \lambda k_1. \lambda k_2. \begin{array}{c|c} \hline & \neg (k_1 + k_2) \\ \hline \end{array}, \quad \lambda k. k(\lambda x. \begin{array}{c|c} \hline & x \\ \hline \end{array}), \dots, \text{etc.}$$

# Basic PDRSs

Basic PDRS:

$\langle \text{label}, \{\text{p-referents}\}, \{\text{p-conditions}\} \rangle$

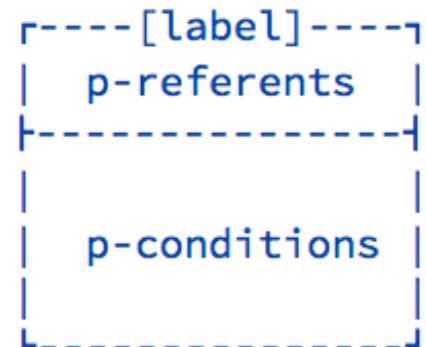
where:

p-referent:  $\langle \text{pointer}, \text{DRS referent} \rangle$

p-condition:  $\langle \text{pointer}, \text{PDRS condition} \rangle$

pointer, label: projection variables

1, 2



*asserted content:* label = pointer

*projected content:* label  $\leq$  pointer  $\rightarrow$  accessible context

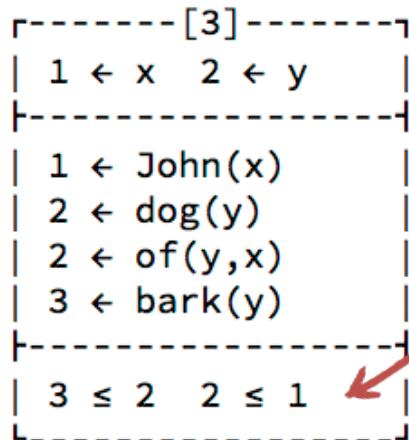
*presupposed content:* pointer occurs free in the global PDRS

# Accessibility of contexts

Context C1 subordinates context C2 ( $C2 \leq C1$ ) iff

- (i) C2 is embedded in C1
- (ii) C2 is the antecedent of C2
- (iii) C2 is a MAP with respect to C1

*John's dog barks*



Revised definition for basic PDRSs:  
 $\langle \text{label}, \{\text{MAPs}\}, \{\text{p-referents}\}, \{\text{p-conditions}\} \rangle$

Minimally Accessible  
Projection context (MAPs)

# Complex PDRSs: Merge

## Assertive Merge

keep all asserted  
content from  
 $\text{PDRS}_1$  asserted

$$\begin{array}{c} \boxed{[1]} \\ | \\ \boxed{D1} \\ | \\ \boxed{C1} \\ | \\ \boxed{M1} \end{array} + \begin{array}{c} \boxed{[2]} \\ | \\ \boxed{D2} \\ | \\ \boxed{C2} \\ | \\ \boxed{M2} \end{array} = \begin{array}{c} \boxed{[2]} \\ | \\ \boxed{D1' \cup D2} \\ | \\ \boxed{C1' \cup C2} \\ | \\ \boxed{M1' \cup M2} \end{array}$$

where:  $\text{PDRS}_1' = \alpha\text{-convert } (\ell_1, \ell_2) \text{ PDRS}_1$

## Projective Merge

project all  
asserted content  
from  $\text{PDRS}_1$

$$\begin{array}{c} \boxed{[1]} \\ | \\ \boxed{D1} \\ | \\ \boxed{C1} \\ | \\ \boxed{M1} \end{array} * \begin{array}{c} \boxed{[2]} \\ | \\ \boxed{D2} \\ | \\ \boxed{C2} \\ | \\ \boxed{M2} \end{array} = \begin{array}{c} \boxed{[2]} \\ | \\ \boxed{D1 \cup D2} \\ | \\ \boxed{C1 \cup C2} \\ | \\ \boxed{M1 \cup M2 \cup 2 \leq 1} \end{array}$$

Caution: prevent  
accidental  
variable binding!

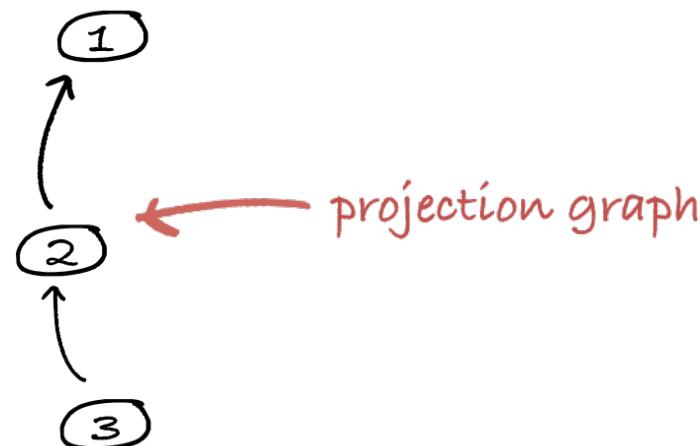
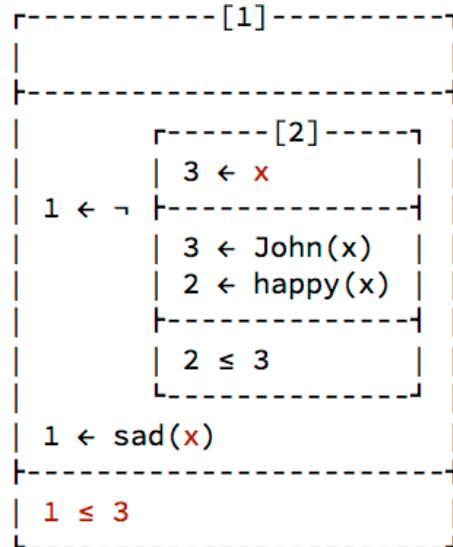
$\alpha$ -convert all  
bound variables  
in context PDRS

define free and  
bound variables  
in PDRT

# Free and bound variables in PDRT

Binding of **pointers** in PDRT defined just like **referents** in DRT: determined by accessible PDRSs.

*It is not the case that John is happy. He is sad.*



What about **referents** in PDRT? Non-hierarchical binding through projection!

# Complex PDRSs: Unresolved structures

Following Muskens (1996) we can define unresolved PDRSs as lambda terms.

a	$\lambda p. \lambda q. (($ <table border="1"><tr><td>1</td></tr><tr><td><math>1 \leftarrow x</math></td></tr><tr><td></td></tr></table> $+ p(x)) + q(x))$	1	$1 \leftarrow x$		
1					
$1 \leftarrow x$					
the	$\lambda p. \lambda q. (($ <table border="1"><tr><td>1</td></tr><tr><td><math>1 \leftarrow x</math></td></tr><tr><td></td></tr></table> $+ p(x)) * q(x))$	1	$1 \leftarrow x$		
1					
$1 \leftarrow x$					
John	$\lambda p. (($ <table border="1"><tr><td>1</td></tr><tr><td><math>1 \leftarrow x</math></td></tr><tr><td><math>1 \leftarrow \text{John}(x)</math></td></tr></table> $* p(x))$	1	$1 \leftarrow x$	$1 \leftarrow \text{John}(x)$	
1					
$1 \leftarrow x$					
$1 \leftarrow \text{John}(x)$					
man	$\lambda x. (($ <table border="1"><tr><td>1</td></tr><tr><td><math>1 \leftarrow \text{man}(x)</math></td></tr><tr><td></td></tr></table> $+ q(e))))$	1	$1 \leftarrow \text{man}(x)$		
1					
$1 \leftarrow \text{man}(x)$					
walks	$\lambda p. \lambda q. (p(\lambda x. (($ <table border="1"><tr><td>1</td></tr><tr><td><math>1 \leftarrow e</math></td></tr><tr><td><math>1 \leftarrow \text{walk}(e)</math></td></tr><tr><td><math>1 \leftarrow \text{Theme}(e, x)</math></td></tr></table> $+ q(e))))$	1	$1 \leftarrow e$	$1 \leftarrow \text{walk}(e)$	$1 \leftarrow \text{Theme}(e, x)$
1					
$1 \leftarrow e$					
$1 \leftarrow \text{walk}(e)$					
$1 \leftarrow \text{Theme}(e, x)$					

```
man :: PDRSRef -> PDRS
man x = PDRS 1
[]
[]
[PCon 1 (Rel (PDRSRel man) [x])]
```

Functions in Haskell  
are already defined  
as lambda terms!

# PDRT-SANDBOX in action!

\*Main> let johnx x = (PDRS (1) [] [PRef 1 (PDRSRef "x")] [PCon (1) (Rel (PDRSRel "John") [PDRSRef "x"])])) <<\*>> (x (PDRSRef "x"))  
\*Main> johnx

$$\lambda k\theta. ( \begin{array}{|l} \hline 1 \leftarrow x \\ \hline \end{array} * \begin{array}{|l} \hline k\theta(x) \\ \hline \end{array} )$$

\*Main> let happyx x = PDRS (1) [] [] [PCon (1) (Rel (PDRSRel "happy") [x])]  
\*Main> happyx

$$\lambda r\theta. ( \begin{array}{|l} \hline 1 \leftarrow happy(r\theta) \\ \hline \end{array} )$$

\*Main> printAMerge (johnx happyx) (stringToPDRS "<1,{},{{<1,not<2,{<3,x>},{<3,Mary(x)>},{<2,sad(x)>},{(2,3)}>>},{}>"")

$$\begin{array}{c} \begin{array}{|l} \hline 1 \leftarrow x \\ \hline \end{array} + \begin{array}{|l} \hline 1 \leftarrow \neg \begin{array}{|l} \hline 3 \leftarrow x \\ \hline \end{array} \\ \hline 2 \leftarrow \neg \begin{array}{|l} \hline 3 \leftarrow Mary(x) \\ \hline 2 \leftarrow sad(x) \\ \hline 2 \leq 3 \\ \hline \end{array} \\ \hline \end{array} = \begin{array}{|l} \hline 1 \leftarrow x \\ \hline 1 \leftarrow John(x) \\ \hline 4 \leftarrow happy(x) \\ \hline 1 \leftarrow \neg \begin{array}{|l} \hline 3 \leftarrow x_1 \\ \hline \end{array} \\ \hline 4 \leftarrow \neg \begin{array}{|l} \hline 3 \leftarrow Mary(x_1) \\ \hline 6 \leftarrow sad(x_1) \\ \hline 6 \leq 3 \\ \hline \end{array} \\ \hline 4 \leq 1 \\ \hline \end{array} \end{array}$$

# PDRT-SANDBOX

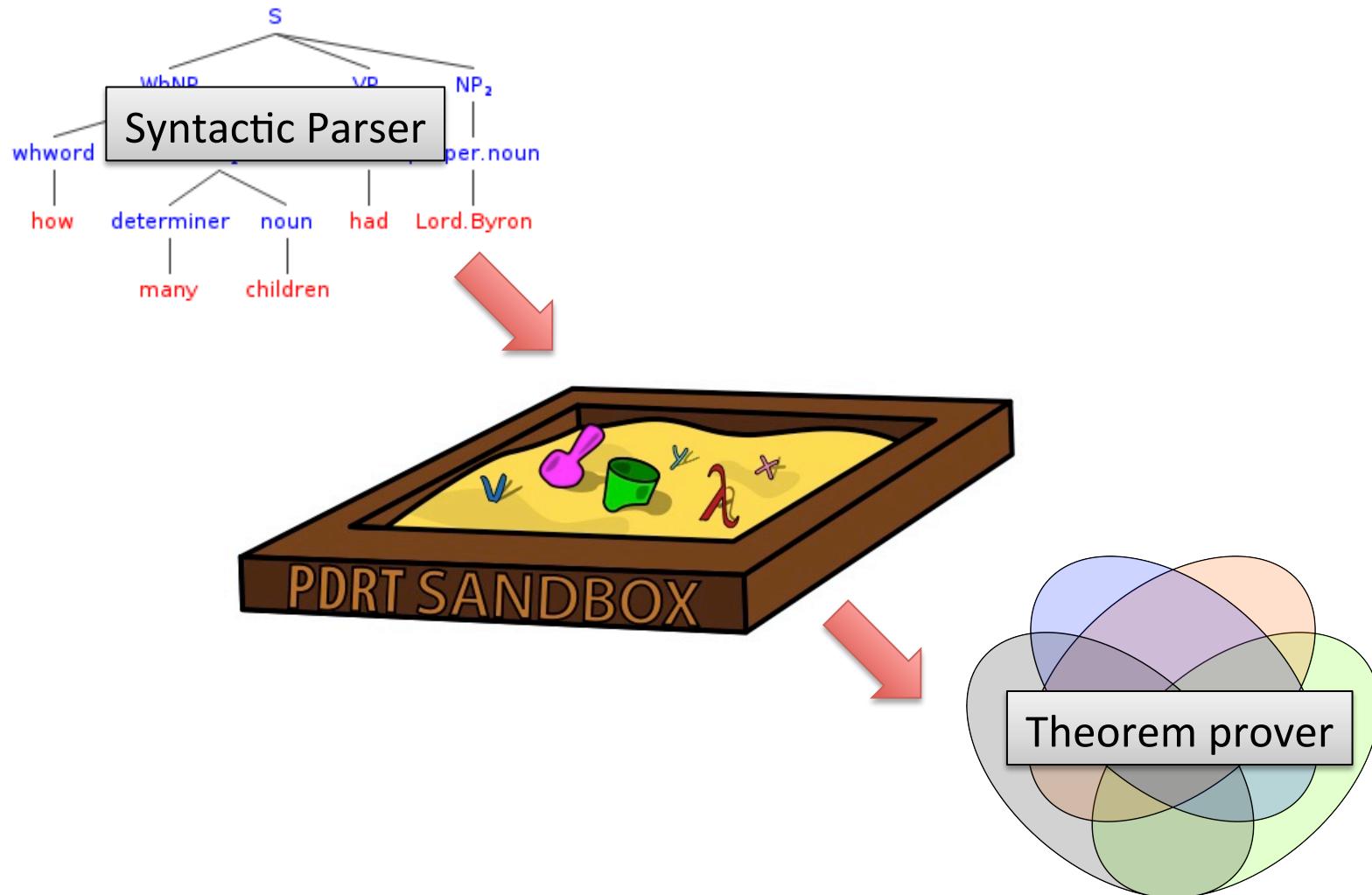
A full-fledged NLP Haskell library that allows for:

- ☛ defining your own PDRSs and DRSs
- ☛ showing and manipulating structures
- ☛ combining unresolved structures
- ☛ translating PDRSs into DRSs and FOL formulas
- ☛ various input/output formats
- ☛ and much more...

Ask for a demo!

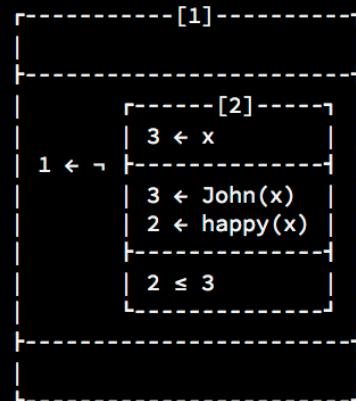


# Combining PDRT-SANDBOX with other tools

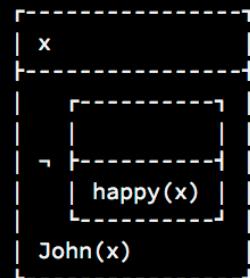


# PDRT to DRT and FOL Translations

```
Prelude Data.PDRS> let notjohnhappy = PDRS (1) [] [] [PCon (1) (Neg (PDRS (2) [(2,3)] [PRef 3 (PDRSRef "x"))] [PCon (3) (Rel (PDRSRel "John") [PDRSRef "x"])] ,PCon (2) (Rel (PDRSRel "happy") [PDRSRef "x"])))]  
Prelude Data.PDRS> notjohnhappy
```



```
Prelude Data.PDRS> pdrsToDRS notjohnhappy
```



```
Prelude Data.PDRS> pdrsToFOL notjohnhappy
```

```
∃x(¬happy(w,x) ∧ John(w,x))
```

# Output formats

```
Prelude Data.PDRS> let notjohnhappy = PDRS (1) [] [] [PCon (1) (Neg (PDRS (2) [(2,3)] [PRef 3 (PDRSRef "x")]) [PCon (3) (Rel (PDRSRel "John") [PDRSRef "x"]),PCon (2) (Rel (PDRSRel "happy") [PDRSRef "x"])])]
Prelude Data.PDRS> notjohnhappy
[1]
  [2]
    3 ← x
    1 ← ¬
      3 ← John(x)
      2 ← happy(x)
      2 ≤ 3
Prelude Data.PDRS> Linear notjohnhappy
1:[|(1,¬2:[(3,x)|(3,John(x)),(2,happy(x))|(2,3)])|]
Prelude Data.PDRS> Set notjohnhappy
<1,{},{} ,{(1,¬2,{(2,3)}, {(3,x)}, {(3,John(x)),(2,happy(x))})}>>
Prelude Data.PDRS> Debug notjohnhappy
PDRS (1) [] [] [PCon (1) (Neg (PDRS (2) [(2,3)] [PRef 3 (PDRSRef "x")]) [PCon (3) (Rel (PDRSRel "John") [PDRSRef "x"]),PCon (2) (Rel (PDRSRel "happy") [PDRSRef "x"])])]
Prelude Data.PDRS> show $ Debug notjohnhappy
"\nPDRS (1) [] [] [PCon (1) (Neg (PDRS (2) [(2,3)] [PRef 3 (PDRSRef \"x\")] [PCon (3) (Rel (PDRSRel \"John\") [PDRSRef \"x\"]),PCon (2) (Rel (PDRSRel \"happy\") [PDRSRef \"x\"])]))]\n"
Prelude Data.PDRS> writeFile "example1.hs" (show $ Debug notjohnhappy)
```

# Lexical semantics

```
Prelude Data.PDRS> let a p q = AMerge (AMerge (PDRS 1 [] [PRef 1 (PDRSRef "x")] []) (p (PDRSRef "x1")))) (q (PDRSRef "x"))
Prelude Data.PDRS> a
```

```
λk0.λk1. ( ( [1] + k0(x) ) + k1(x) )
|-----|
| 1 ← x |
|-----|
|-----|
|-----|
|-----|
```

```
Prelude Data.PDRS> let man x = PDRS 1 [] [] [PCon 1 (Rel (PDRSRel "man") [x])]
Prelude Data.PDRS> man
```

```
λr0. [1]
|-----|
|-----|
| 1 ← man(r0) |
|-----|
|-----|
```

```
Prelude Data.PDRS> let walks p q = (p (\x -> AMerge (PDRS 1 [] [PRef 1 (PDRSRef "e")] [PCon 1 (Rel (PDRSRel "walks") [PDRSRef "e"])]), PCon 1 (Rel (PDRSRel "Theme") [PDRSRef "e",x]))) (q (PDRSRef "e"))))
Prelude Data.PDRS> (walks (a man))
```

```
λk0. ( ( [1] + [1] ) + ( [1] + k0(e) ) )
|-----| |-----| |-----|
| 1 ← x | | 1 ← man(x) | | 1 ← e
|-----| |-----| |-----|
|-----| |-----| |-----|
|-----| |-----| |-----|
```

# Projection Table/Graph

```
Prelude Data.PDRS> example
```

```
[1]
| 8 ← x
|
| 8 ← P(x)
|   [2]
|   | 9 ← y
|   |
|   1 ← ◇
|     [3]
|     | 2 ← Q(y)
|     | 9 ← R(x,y)
|     |
|     2 ≤ 9
|
|   [4]      [5]
|   | 4 ← y   | 5 ← N(x)
|   1 ← |-----| → |-----|
|   | 4 ← M(x) | 5 ← N(x)
|   |-----|     | 4 ← N(y)
|   |
|   [6]      [7]
|   | 1 ≤ 8
|   |
|   [8]
```

```
Prelude Data.PDRS> projectionTable example
```

[Type]	[Content]	[Intro st]	[Proj. st]
Ref	x	1	8
Con	P(x)	1	8
Con	◇ 2	1	1
Ref	y	2	9
Con	Q(y)	2	2
Con	R(x,y)	2	9
Con	4 ⇒ 5	1	1
Ref	y	4	4
Con	M(x)	4	4
Con	N(x)	5	5
Con	N(y)	5	4

```
Prelude Data.PDRS> projectionGraph example
```

```
array (1,9) [(1,[8,1]),(2,[9,2,1]),(3,[],(4,[4,1]),(5,[5,4]),(6,[]),(7,[]),(8,[]),(9,[])]
```

# Boxer and PDRT-SANDBOX: A happy marriage?

```
p264344@zardoz:~$ curl -d'The sloth moved very slowly.' http://localhost:7777/boxer?semantics=pdrs
%% This output was generated by the following command:
%% /home/p262301/dev/candc/bin/boxer --stdin --semantics pdrs
%% The sloth moved very slowly .
id(1,1).
sem(1,[1001:[tok:'The',pos:'DT',lemma:the,namex:'0f'],1002:[tok:sloth,pos:'NN',lemma:sloth,namex:'0'],1003:[tok:moved,pos:'VBD',lemma:move,namex:'0'],1004:[tok:very,pos:'RB',lemma:very,namex:'0'],1005:[tok:slowly,pos:'RB',lemma:slowly,namex:'0'],1006:[tok:'.',pos:'.',lemma:'.',namex:'0']],_G1867:drs([_G1483:[1001]:_G1508,_G1867:[1002]:pred(_G1508,sloth,n,0),_G1867:[1003]:pred(_G1892,move,v,0),_G1867:[1004]:role(_G1892,_G1508,agent,1),_G1867:[1005]:pred(_G1892,slowly,a,0),_G1867:[1006]:pred(_G1892,very,a,0))])."
..

Prelude Data.PDRS> let sloth = boxerToPDRS "sem(1,[1001:[tok:'The',pos:'DT',lemma:the,namex:'0'],1002:[tok:sloth,pos:'NN',lemma:sloth,namex:'0'],1003:[tok:moved,pos:'VBD',lemma:move,namex:'0'],1004:[tok:very,pos:'RB',lemma:very,namex:'0'],1005:[tok:slowly,pos:'RB',lemma:slowly,namex:'0'],1006:[tok:'.',pos:'.',lemma:'.',namex:'0']],_G1867:drs([_G1483:[1001]:_G1508,_G1867:[1002]:pred(_G1508,sloth,n,0),_G1867:[1003]:pred(_G1892,move,v,0),_G1867:[1004]:role(_G1892,_G1508,agent,1),_G1867:[1005]:pred(_G1892,slowly,a,0),_G1867:[1006]:pred(_G1892,very,a,0))))."
Prelude Data.PDRS> sloth
r-----[1]-----
| 2 ← x1 1 ← x2 |
+-----+
| 2 ← sloth(x1)
| 1 ← move(x2)
| 1 ← Agent(x2,x1)
| 1 ← slowly(x2)
| 1 ← very(x2)
+-----+
|-----|
Prelude Data.PDRS> :t sloth
sloth :: PDRS
```

# Why Haskell?

