CS 151: Mathematical Foundations of Computing
Homework Assignment 03
Spring 2018

Instructions

This assignment is due Sunday, February 18, at 11:59PM (Central Time).

This assignment must be submitted on *Gradescope* (entry code: M3Y24Z). Handwritten submissions are allowed as long as they are legible. Submissions typed in LaTeX or Word are preferred. For instructions on how to submit assignments on *Gradescope* see this guide.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

This assignment is individual. Offering or receiving any kind of unauthorized or unacknowledged assistance is a violation of the University's academic integrity policies, will result in a grade of zero for the assignment and will be subject to disciplinary action.

Part I: Constructing Proofs (100 pt.)

You must write down all proofs in acceptable mathematical language: make sure you mark the beginning and end of the proof, define all variables, use complete, grammatically correct sentences, and give a justification for each assertion (e.g., *by definition of…*). See lecture slides for examples.

***Definitions*:**

- An integer n is **even** if and only if there exists an integer k such that $n = 2k$.
- An integer n is **odd** if and only if there exists an integer k such that $n = 2k + 1$.
- Two integers have the **same parity** when they are both even or when they are both odd.
- Two integers have **opposite parity** when one is even and the other one is odd.
- An integer n is **divisible** by an integer d with $d \neq 0$, denoted $d \mid n$, if and only if there exists an integer k such that $n = dk$.
- A real number r is **rational** if and only if there exist integers a and b with $b \neq 0$ such that $r = a/b$.
- For any real number x, the **absolute value** of x, denoted $|x|$, is defined as follows:

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

1. **Prove each of the following statements using a *direct proof*, a *proof by contrapositive*, a *proof by contradiction*, <u>or</u> a *proof by cases*. In addition to the proof, you must answer the following questions for each statement:**

   - **Which proof method did you use?**
   - **What are the assumptions of the proof (*what you suppose*)?**
   - **What is the conclusion of the proof (*what you need to show*)?**

   a. **(15 pt.) Any two consecutive integers have *opposite parity*.**

   b. **(15 pt.) If a group of 8 kids have won a total of 65 trophies, then at least one of the 8 kids has won at least 9 trophies.**

   c. **(15 pt.) The difference of any *rational* number and any *irrational* number is *irrational*.**

   d. **(15 pt.) For any three integers x, y, and z, if y is *divisible* by x and z is *divisible* by y, then z is *divisible* by x.**

   e. **(20 pt.) For all real numbers x and y, $\max(x, y) = \frac{x+y+|x-y|}{2}$ and $\min(x, y) = \frac{x+y-|x-y|}{2}$.**

   f. **(20 pt.) For any positive integer n, n is even <u>if and only if</u> $7n + 4$ is even.**

   Hint: To prove that a **biconditional statement** of the form $p \leftrightarrow q$ is true, you must show that $p \rightarrow q$ and $q \rightarrow p$ are both true. For example, to prove that for any integer n, n is odd **if and only if** n^2 is odd, you must show that (1) if n is odd, then n^2 is odd, and (2) if n^2 is odd, then n is odd.