

# STAT 640: Homework 7

Due **Wednesday, March 23, 11:59pm MT** on the course Canvas webpage. Please follow the homework guidelines on the syllabus.

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## Problem 1

For this question, use the same data and model as Problem 4 on Homework 6. Consider the null hypothesis that there is no difference in chick weight at age 6 between chicks on Diets 1, 2, and 4. (Note: this hypothesis does not involve Diet 3.)

**a.** Provide the form of the linear model for weight as a function of diet for the entire dataset of age 6 chicks. In other words, copy your answer to Problem 4a from Homework 6.

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**Answer:**

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**b.** Find  $\mathbf{A}$  such that  $\mathbf{A}\boldsymbol{\beta} = \mathbf{0}$  corresponds to this hypothesis.

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**Answer:**

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**c.** Compute  $RSS_H - RSS$  using only  $\hat{\boldsymbol{\beta}}$ ,  $\mathbf{A}$ , and  $\mathbf{X}$ .

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**Answer:**

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**d.** Conduct an F-test to test this hypothesis. Provide the test statistic, p-value, and a conclusion statement.

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**Answer:**

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**e.** Check your answer to (c) by fitting a model that corresponds to the null hypothesis and calculating  $RSS_H$ .

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**Answer:**

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## Problem 2

Prove Proposition 5.11. That is, under the conditions of that proposition, show that

$$\frac{RSS_H - RSS}{RSS} = \frac{R^2 - R_H^2}{1 - R^2}$$

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**Answer:**

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## Problem 3

Consider the two regression lines

$$Y_{ki} = \beta_k x_i + \epsilon_{ki}$$

for  $k = 1, 2$  and  $i = 1, \dots, n$ . Assume uncorrelated, homoscedastic errors. Find the F-statistic for testing  $H : \beta_1 = \beta_2$ .

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**Answer:**

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## Problem 4

Consider the linear model  $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$  with  $\mathbf{X} \in \mathbb{R}^{n \times p}$  and  $\text{rank}(\mathbf{X}) = r$ . Let  $\mathbf{A}\boldsymbol{\beta} = \mathbf{0}$  be a testable hypothesis with  $\mathbf{A} \in \mathbb{R}^{q \times p}$  and  $q < r$ . Prove that if  $\text{rank}(\mathbf{A}) = q$ , then  $\text{rank}(\mathbf{A}(\mathbf{X}^\top \mathbf{X})^- \mathbf{A}^\top) = q$ . (Hint: recall that  $\text{rank}(\mathbf{B}\mathbf{B}^\top) = \text{rank}(\mathbf{B})$ .)

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**Answer:**

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## Problem 5

Consider the linear model

$$Y_1 = \theta_1 + \theta_2 + \epsilon_1$$

$$Y_2 = 2\theta_2 + \epsilon_2$$

$$Y_3 = -\theta_1 + \theta_2 + \epsilon_3$$

where  $E[\boldsymbol{\epsilon}] = \mathbf{0}$  and  $\text{Var}(\boldsymbol{\epsilon}) = \sigma^2 \mathbf{I}$ .

a. Show that  $H : \theta_1 = 2\theta_2$  is a testable hypothesis.

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**Answer:**

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**b.** Derive the form of the F-statistic for testing  $H$ .

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**Answer:**

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**c.** If we assume the errors are normally distributed and the null hypothesis is true, what is the distribution of  $F$ ?

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**Answer:**

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