STAT 640: Homework 7

Due **Wednesday, March 23, 11:59pm MT** on the course Canvas webpage. Please follow the homework guidelines on the syllabus.

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Problem 1

For this question, use the same data and model as Problem 4 on Homework 6. Consider the null hypothesis that there is no difference in chick weight at age 6 between chicks on Diets 1, 2, and 4. (Note: this hypothesis does not involve Diet 3.)

a. Provide the form of the linear model for weight as a function of diet for the entire dataset of age 6 chicks. In other words, copy your answer to Problem 4a from Homework 6.

Answer:

b. Find $\boldsymbol{A}$ such that $\boldsymbol{A\beta} = \boldsymbol{0}$ corresponds to this hypothesis.

Answer:

c. Compute $RSS_H - RSS$ using only $\hat{\boldsymbol{\beta}}$, $\boldsymbol{A}$, and $\boldsymbol{X}$.

Answer:

d. Conduct an F-test to test this hypothesis. Provide the test statistic, p-value, and a conclusion statement.

Answer:

e. Check your answer to (c) by fitting a model that corresponds to the null hypothesis and calculating RSS_H.

Answer:

Problem 2

Prove Proposition 5.11. That is, under the conditions of that proposition, show that

$$\frac{RSS_H - RSS}{RSS} = \frac{R^2 - R_H^2}{1 - R^2}$$

Answer:

Problem 3

Consider the two regression lines

$$Y_{ki} = \beta_k x_i + \epsilon_{ki}$$

for $k = 1, 2$ and $i = 1, \ldots, n$. Assume uncorrelated, homoscedastic errors. Find the F-statistic for testing $H : \beta_1 = \beta_2$.

Answer:

Problem 4

Consider the linear model $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ with $\boldsymbol{X} \in \mathbb{R}^{n \times p}$ and $\mathsf{rank}(\boldsymbol{X}) = r$. Let $\boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{0}$ be a testable hypothesis with $\boldsymbol{A} \in \mathbb{R}^{q \times p}$ and $q < r$. Prove that if $\mathsf{rank}(\boldsymbol{A}) = q$, then $\mathsf{rank}(\boldsymbol{A}(\boldsymbol{X}^\mathsf{T}\boldsymbol{X})^{-}\boldsymbol{A}^\mathsf{T}) = q$. (Hint: recall that $\mathsf{rank}(\boldsymbol{B}\boldsymbol{B}^\mathsf{T}) = \mathsf{rank}(\boldsymbol{B})$.)

Answer:

Problem 5

Consider the linear model

$$\begin{aligned} Y_1 &= \theta_1 + \theta_2 + \epsilon_1 \\ Y_2 &= 2\theta_2 + \epsilon_2 \\ Y_3 &= -\theta_1 + \theta_2 + \epsilon_3 \end{aligned}$$

where $\mathrm{E}[\boldsymbol{\epsilon}] = \boldsymbol{0}$ and $\mathrm{Var}(\boldsymbol{\epsilon}) = \sigma^2 \boldsymbol{I}$.

a. Show that $H : \theta_1 = 2\theta_2$ is a testable hypothesis.

Answer:

b. Derive the form of the F-statistic for testing H.

Answer:

c. If we assume the errors are normally distributed and the null hypothesis is true, what is the distribution of F?

Answer: