

STAT 530 Homework 3

Hannah Butler

2/18/2022

(1) (8 pts) Problem 7.7, Casella & Berger:

Let X_1, \dots, X_n be iid with one of two pdfs: If $\theta = 0$, then

$$f(x|\theta) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

while if $\theta = 1$, then

$$f(x|\theta) = \begin{cases} 1/2\sqrt{x} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the MLE of θ .

(2) (12 pts) Problem 7.11, Casella & Berger:

Let X_1, \dots, X_n be iid with pdf

$$f(x|\theta) = \theta x^{\theta-1}, \quad 0 \leq x \leq 1, \quad 0 < \theta < \infty.$$

(a) Find the MLE of θ , and show that its variance $\rightarrow 0$ as $n \rightarrow \infty$.

(b) Find the method of moments estimator of θ .

(3) (10 pts) Problem 7.12(a), Casella & Berger:

Let X_1, \dots, X_n be a random sample from a population with pmf

$$P_\theta(X = x) = \theta^x(1 - \theta)^x, \quad x = 0 \text{ or } 1, \quad 0 \leq \theta \leq \frac{1}{2}.$$

(a) Find the method of moments estimator and MLE of θ .

(b) Find the mean squared errors of each of the estimators.

(c) Which estimator is preferred? Justify your choice.

(4) (18 pts) Problem 7.14, Casella & Berger. (*Hint: what is the joint distribution of (Z, W) ? For $F(z, w|\lambda, \mu) = P(Z \leq z, W = w|\lambda, \mu)$, $f(z, w|\lambda, \mu) = dF(z, w)/dz$ depends on w .*)

Let X