MECHANICS OF SOLIDS

CE 201

DECEMBER 2017

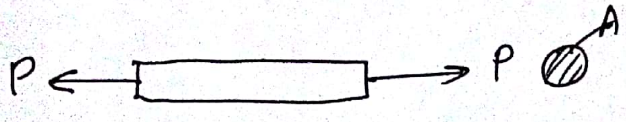
1. a) Define stress and strain. (4)

An external force acting on a body induces an internal resisting force to counter the effect of the external force. The internal resisting force acting on unit area of the body is called stress. Its unit can be N/m^2. Stress produced by a normal force is called normal stress. Stress produced by a shear force is called shear stress. Strain is a measure to quantify deformation. It can be measured as the ratio of change in dimension to original dimension. Natural strain is the ratio of change in length to instantaneous length. Shear strain is measured by change in angle. Strain is dimensionless

b) Fundamental types of stresses and one example for each type. (4)

Normal or Direct stress – The stress produced by a normal force. It can be of two types

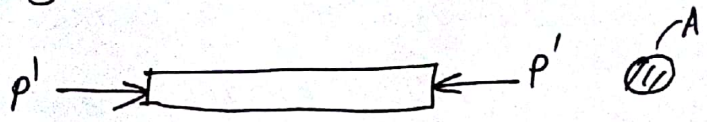
Tensile stress which is produced by tensile force.



Tensile stress $= \dfrac{P}{A}$

Stress formed on a cable carrying a load is tensile in nature.

Compressive stress is produced by a compressive force.

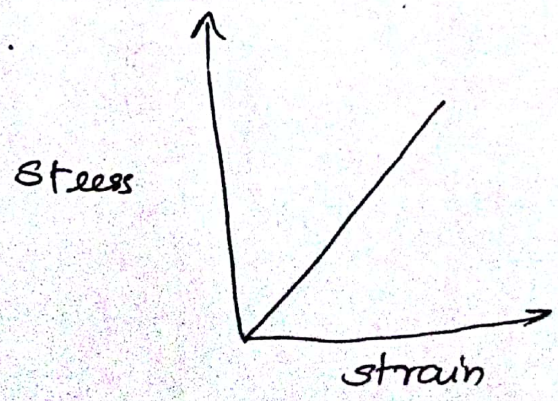


Compressive stress $= \dfrac{P'}{A}$

It produces shortening of the structure. Stress developed on a column or strut is compressive in nature.

c) State and explain Hooke's law. (7)

When an external force is applied within elastic limit, the stress and strain are proportional. Ratio of stress to strain within elastic limit is called Young's modulus.



2 a) Prove that the maximum value Poisson's ratio can have is 0.5.

Poisson ratio $\nu = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

Let V be the volume $= k \cdot a^2 \cdot l$.

On differentiating $dV = k(a^2\,dl + 2al\,da)$

For $dV > 0$, $k(a^2 dl + 2al\,da) > 0$

ie $a^2 dl + 2al\,da > 0$

$a\,dl + 2l\,da > 0$

$a\,dl > -2l\,da$

$\dfrac{dl}{l} > -2\dfrac{da}{a}$

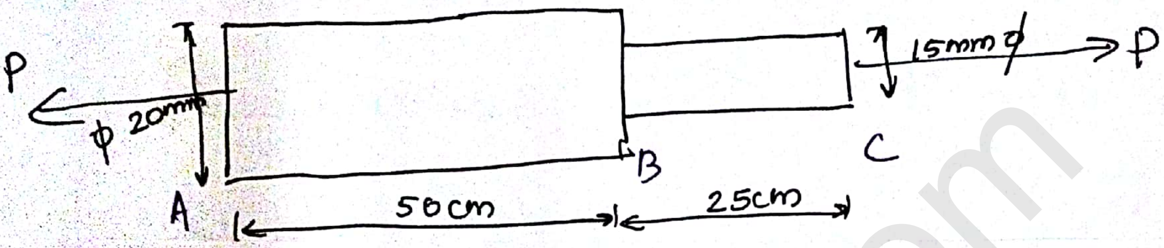
ie $\varepsilon_l > -2\varepsilon_a$

But $\varepsilon_a = -\mu\,\varepsilon_l$

$\therefore$ $\varepsilon_l > -2 \cdot -\mu\,\varepsilon_l$

$\frac{1}{2} > \mu$ Thus it is proved.

2b A cylindrical bar with two sections of lengths 50cm and 25cm, and diameters 20mm and 15mm respectively, is subjected to an axial pull such that the maximum stress is 150 MN/m². Calculate the strain energy stored in bar. $E = 200$ GN/m².



$\sigma_1 A_1 = \sigma_2 A_2$ (Part BC is having less area and max. stress).

$$\sigma_1 = \frac{\sigma_2 A_2}{A_1} = \frac{150 \times \frac{\pi}{4} \times 15^2}{\frac{\pi}{4} \times 20^2} = 84.375 \text{ N/mm}^2$$

$$V_1 = A_1 \times L_1 = \frac{\pi}{4} \times 20^2 \times 500 = 157079.63 \text{ mm}^3$$

$$V_2 = A_2 \times L_2 = \frac{\pi}{4} \times 15^2 \times 250 = 44178.65 \text{ mm}^3$$

$$U_1 = \frac{\sigma_1^2}{2E} \times L = \frac{84.375}{2 \times 10^5} \times \cancel{500}\ 157079.63$$

$$= 662.68 \text{ Nmm}$$

$$U_2 = \frac{\sigma_2^2}{2E} \times L = \frac{150}{2 \times 10^5} \times \cancel{250}\ 44178.65$$

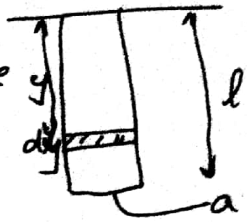
$$= 33.13 \text{ Nmm}$$

$$U = U_1 + U_2 = \underline{695.81 \text{ Nmm}}$$

3 a Find an expression for elongation of bar due to self weight.

unit weight

Let γ be the density per unit volume



$\therefore$ Volume of element $= a \cdot dy$

Weight of element $= \gamma \cdot a \cdot dy$

Elongation of element $= \dfrac{\gamma\, a\, dy\; dy}{a \cdot E}$

$$= \frac{\gamma\, dy^2}{E}$$

Elongation of entire rod $= \dfrac{\gamma}{E}\displaystyle\int_0^l dy^2$

$$= \frac{\gamma}{E} \cdot \left(2y\,dy\right)_0^l$$

$$= \frac{\gamma}{E} 2l$$

$$= \frac{a\,\gamma\, 2l}{aE}$$

$$= 2\frac{Wl}{aE}$$

Elongation due to self weight is half of that due to load W

3. b. A mild steel rod 20mm diameter and 300mm long is enclosed centrally inside a hollow copper tube of ext. dia 30mm and internal dia 25mm. The composite bar is subjected to a pull of 50KN. $E_{steel} = 200\ GN/m^2$. $E_{copper} = 100\ GN/m^2$. Find the stresses in the rod and tube.

$A_s = \frac{\pi}{4} \times 20^2$.

$A_{cu} = \frac{\pi}{4}(30^2 - 25^2)$.

$P = P_s + P_{cu}$

$50 \times 10^3 = \sigma_s A_s + \sigma_{cu} A_{cu}$. $\longrightarrow$ 1.

Deformation of steel = Deformation of copper.

$\Delta_{steel} = \Delta_{copper}$.

$\left(\frac{P_s L}{AE}\right)_{steel} = \left(\frac{PL}{AE}\right)_{copper}$.

$$\frac{\sigma_s \times 300}{200 \times 10^3} = \frac{\sigma_{cu} \times 300}{100 \times 10^3}$$

$\sigma_s = 2\sigma_{cu}$

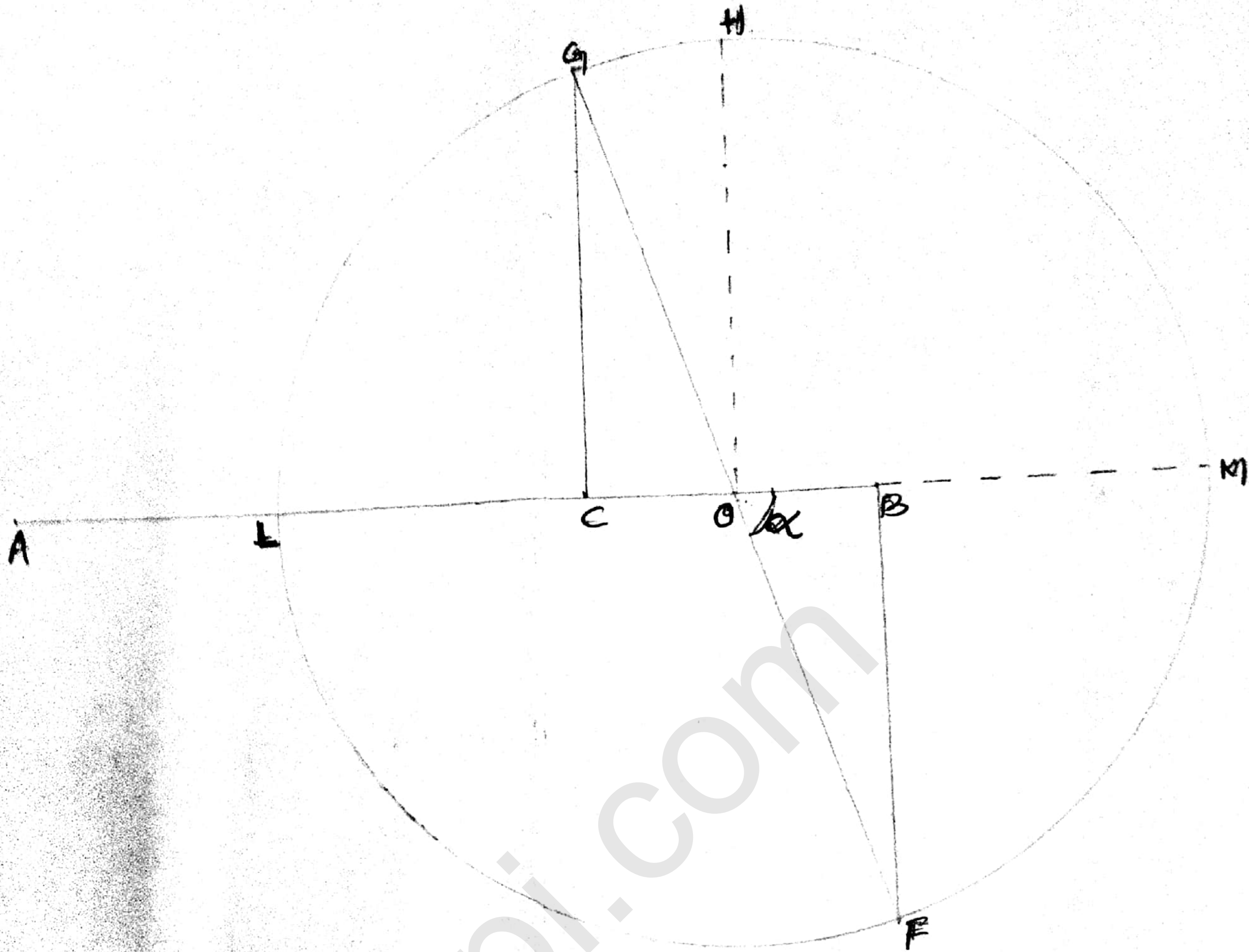
Substitute in 1.

$50 \times 10^3 = 2\sigma_{cu} \times \frac{\pi}{4} \times 20^2 + \sigma_{cu} \times \frac{\pi}{4}(30^2 - 25^2)$

$: 628.318\sigma_{cu} + 392.69\sigma_{cu}$

$\sigma_{cu} = 48.97\ N/mm^2$ $\sigma_s = 97.94\ N/mm^2$

7. a)



Scale

$\sigma_1 = 12 \, N/mm^2 = 12 \, cm$

$\sigma_2 = 8 \, N/mm^2 = 8 \, cm$

$\tau = 6 \, N/mm^2 = 6 \, cm$

$\alpha = 72^\circ$

$\theta = \alpha/2 = 36^\circ$

$\sigma_n \, (max) = 16.3 \times 1 = 16.3 \, N/mm^2$

$\sigma_n \, (min) = 3.65 \, N/mm^2$

$\sigma_t \, (OH) = 6.25 \, N/mm^2$

7. b. A solid circular shaft is to transmit 75 kw Power at 200 rpm. If shear stress is not to exceed 50 MPa, find diameter of shaft $G = 100\,GPa$

$P = 75\,kw = 75 \times 10^6\,N\,mm/sec.$

$N = 200\,rpm$

$q_s = 50\,N/mm^2$

$G = 100 \times 10^3\,N/mm^2$

$$P = \frac{2\pi NT}{60} = 75 \times 10^6\,N\,mm/sec.$$

$$T = \frac{75 \times 10^6 \times 60}{2\pi \times 200} = 3580986.22\,N\,mm$$

$$\frac{T}{J} = \frac{q_s}{R} = \frac{G\theta}{L} \Longrightarrow \text{Here we take } \frac{T}{J} = \frac{q_s}{R}$$

$$\frac{3580986.22}{\frac{\pi d^4}{32}} = \frac{50}{d/2}$$

$$\frac{57295779.51}{50\pi} = d^3$$

$$d = 71.449\,mm$$

8. a. A 2m long thin cylindrical shell (both ends closed), internal diameter 90cm and thickness 12mm, is subjected to internal pressure 2N/mm². Find 1) hoop and longitudinal stresses 2) Changes in diameter and length shell. $E = 2 \times 10^5$ N/mm². Poisson's ratio = 0.3.

$L = 2000$ mm
$d = 900$ mm
$t = 12$ mm
$p = 2$ N/mm²

$E = 2 \times 10^5$ N/mm²
$\mu = 0.3$.

Hoopstress $\sigma_1 = \frac{Pd}{2t} = \frac{2 \times 900}{2 \times 12} = 75$ N/mm²

Longitudinal stress $\sigma_2 = \frac{Pd}{4t} = \frac{2 \times 900}{4 \times 12} = 37.5$ N/mm²

$$\delta d = \frac{P \times d^2}{2t \times E}\left(1 - \frac{1}{2}\mu\right)$$

$$= \frac{2 \times 900^2}{2 \times 12 \times 2 \times 10^5}\left(1 - \frac{1}{2} \times 0.3\right)$$

$$= \underline{\underline{0.287 \text{ mm}}}$$

$$\delta L = \frac{p \times d \times L}{2t \times E}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{2 \times 900 \times 2000}{2 \times 12 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right) = \underline{\underline{0.15 \text{ mm}}}$$