Effective thickness of neutron beam that passing through cylinder target container

Analytic solution

We start with a neutron beam that passing through the target with cylinder container. The top view can be seen in Fig.1. r_{in} and r_{out} are the radii of inner and outer container respectively. s is the shift of incoming beam with respect to the center of the target container and d is the distance of beam lines respect to the central beam line. Assuming that $d + s \leq r_{in}$ so that the entire beam goes through container. Fig.2 is the beam profile with flat intensity. Thus we can rewrite d as $r \cos\phi$. And the interaction length between beam line and target l_t is

$$l_t = 2 \times \sqrt{r_{in}^2 - (r\cos\phi + s)^2} \tag{1}$$

The interaction length between beam line and container wall l_w is

$$l_w = 2 \times \left(\sqrt{r_{out}^2 - (r\cos\phi + s)^2} - \sqrt{r_{in}^2 - (r\cos\phi + s)^2}\right) \tag{2}$$

The effective thickness is the summation of all beams over the area of beam profile.

$$thickness_t = \frac{\int l_t r dr d\phi}{\int r dr d\phi} \tag{3}$$

$$thickness_w = \frac{\int l_w r dr d\phi}{\int r dr d\phi} \tag{4}$$

Monte Carlo Estimator

Definite integral

$$\int_a^b f(x)dx \tag{5}$$

where $x \in$ [a,b] follow the distribution $p(x)$ ($p(x)$ must be nonzero for all x where $f(x)$ is nonzero).

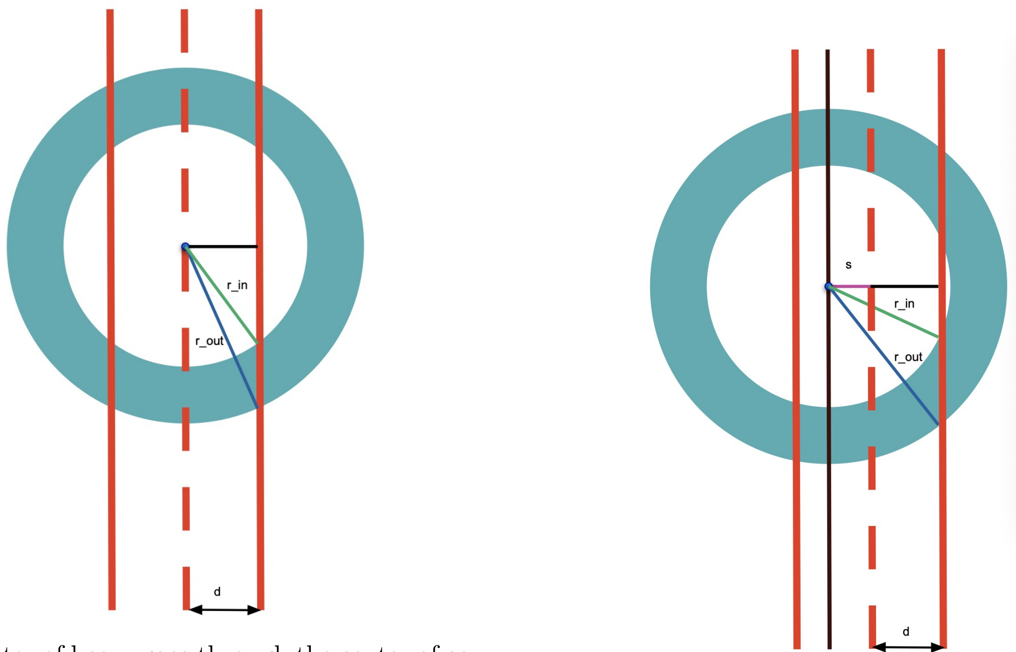
The corresponding Monte Carlo estimator is

$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{p(x_i)} \tag{6}$$

And in 2d dimension, we have

$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i, y_i)}{p(x_i, y_i)} \tag{7}$$

such that X_i,$Y_i \sim p(x, y)$.



(a) The center of beam goes through the center of container

(b) Beam is shifted with distance s

Figure 1: Top view of neutron beam that passing through target container.

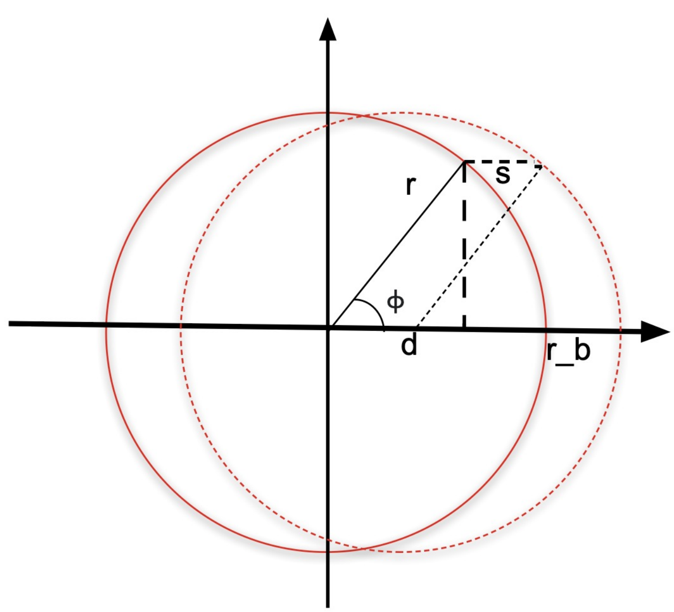


Figure 2: Neutron beam profile. The intensity of beam has not been taken into consideration.

Inverse transform method

To calculate the integral, we need to generate data points first. Since r $\in$ [0,r_b] and $\phi \in$ [0,2π], we uniformly pick r and ϕ from them. The chosen points are uniformly in (r, ϕ) space, while it is non-uniform in (x,y) space, which is not expected. The inverse transform method is introduced here to correct it.

For $X_i \sim p(x)$, the cumulative distribution function (CDF) is

$$F(x) = \int_{-\infty}^{x} p(x)dx \tag{8}$$

The inverse transform method claims that if U is a uniform random variable on (0,1), then $X = F^{-1}(U)$ follows the distribution of F.

Because of the symmetry of ϕ in circle surface, it is uniformly distributed. For r , we can get the CDF

$$F(r) = \pi r^2/\pi r_b^2 = r^2/r_b^2 \tag{9}$$

And the PDF is $2r/r_b^2$. It is invertible and

$$F^{-1}(U) = r_b\sqrt{U} \tag{10}$$

Now we have the chosen points uniformly distributed in (x,y) space. Next, we are going to find the PDF in (r, ϕ) space, which can be obtained from the Jacobian of coordinates transformation.

$$|detJ(r,\phi)| = r \tag{11}$$

And

$$p(r,\phi) = p(x,y)|detJ(r,\phi)| = \frac{r}{\pi r_b^2} \tag{12}$$

So the corresponding integral

$$I_t = \frac{1}{N}\sum_i^N \frac{l_t(r_i,\phi_i)\times r}{r/(\pi r_b^2)} = \frac{\pi r_b^2}{N}\sum_i^N l_t(r_i,\phi_i) \tag{13}$$

$$I_w = \frac{\pi r_b^2}{N}\sum_i^N l_w(r_i,\phi_i) \tag{14}$$

Rejection acceptance method

We start with choose uniformly points in (x,y) space (x,y$\in$ [-r_b,r_b]), whose PDF is $1/4r_b^2$. We accept that points inside of $x^2 + y^2 \leq r_b^2$ and reject the points outside of it.

$$I_t = \frac{4r_b^2}{N}\sum_i l_t(x_i) \tag{15}$$

$$I_w = \frac{4r_b^2}{N}\sum_i l_w(x_i) \tag{16}$$

where x is $r\cos\phi$.