An Optimal Control Strategy for a Multi-Task Whole Body Controller

Henry Cappel

April 15, 2020

1 Introduction

Whole body control is a field well studied and reliant on particular techniques for effective task space realization. Prioritized task space controllers use null space projections to establish a hierarchy of task policies where lower level tasks are performed only in the null space of prioritized tasks. These controllers are effective and provide the user with significant freedom to choose tasks knowing that no two tasks will interfere [1, 2]. However, optimal control strategies are generally not used to realize the task policies. Additionally, dynamic programming approaches may prove to be more efficient for this type of whole-body hierarchical information structure.

For this project I will simulate an optimal control strategy using dynamic programming on a kinematically decoupled task structure. The robot contains a prismatic base with 3 revolute joints capable of locomotion and manipulation.

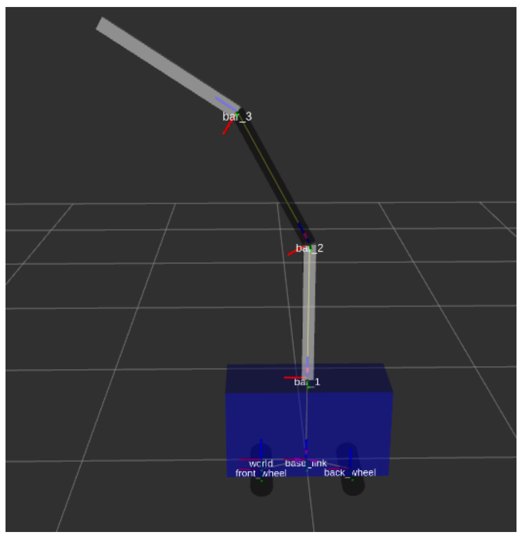

Figure 1: The 4 DOF capbot. The robot contains 1 prismatic and 3 revolute joints and is capable of locomotion and manipulation.

2 Task Space Representation

I will give a brief overview of how the joint space can be mapped into a task space using a series of null space projections. The mapping from a joint displacement to the end effector displacement in cartesian space is described by

$$dx_i = J_i dq \tag{1}$$

where dx is the end-effector displacement of the ith task, J is the jacobian of the ith task, and dq is the joint displacement.

If we define

$$dq = dq_1 + N_1 dq_2 + N_1 N_{2/1} dq_3 + ... + N_{[s]} dq_s \tag{2}$$

where

$$N_{[s]} = \prod_{i=1}^{s} N_{i/i-1} \tag{3}$$

$N_{i/i-1}$ is the null space projector defined to be

$$N_{i/i-1} = I - (J_i N_{i-1/i-2})^\dagger (J_i N_{i-1/i-2}) \tag{4}$$

where

$$N_{1/0} = N_1 = (I - J_1^\dagger J_1) \tag{5}$$

By plugging 2 into 1 for every i in the task space we can establish a set of decoupled task space equations of the form

$$dx_1 = J_1 dq_1$$

$$dx_2 = J_2 N_1 dq_2 + J_2 dq_1$$

$$dx_3 = J_3 N_1 N_{2/1} dq_3 + J_3(dq_1 + dq_2)$$

$$dx_k = (J_k N_{[k-1]}) dq_k + J_k \sum_{n=1}^{k-1} dq_n$$

With this set of task space equations we can generate a discrete time state space model of the form

$$\begin{bmatrix} x_{t+1}^1 \\ x_{t+1}^2 \\ x_{t+1}^3 \\ . \\ . \\ . \\ x_{t+1}^k \end{bmatrix} = I^{k \times k} \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ . \\ . \\ . \\ x_t^k \end{bmatrix} + \begin{bmatrix} J_1 & 0 & 0 & ... & 0 \\ J_2 & J_2 N_1 & 0 & ... & 0 \\ J_3 & J_3 & J_3 N_1 N_{2/1} & ... & 0 \\ . & & & & \\ . & & & & \\ . & & & & \\ J_k & ... & J_k & J_k \prod_{i=1}^{k-1} N_{i/i-1} & \end{bmatrix} \begin{bmatrix} dq_t^1 \\ dq_t^2 \\ dq_t^3 \\ . \\ . \\ . \\ dq_t^k \end{bmatrix} + \begin{bmatrix} w_t^1 \\ w_t^2 \\ w_t^3 \\ . \\ . \\ . \\ w_t^k \end{bmatrix}$$

By defining $\tilde{x}_t^i = x_t^i - \bar{x}_t^i$,$d\tilde{q}_t^i = dq_t^i - d\bar{q}_t^i$, and $w_t^i \sim \mathcal{N}(0, \sigma^2)$ where $\bar{x}_t^i$ and $d\bar{q}_t^i$ are the nominal task and joint states respectively, we can generate a discrete time state space model for the error propagation as

$$\begin{bmatrix} \tilde{x}^1_{t+1} \\ \tilde{x}^2_{t+1} \\ \tilde{x}^3_{t+1} \\ . \\ . \\ . \\ \tilde{x}^k_{t+1} \end{bmatrix} = I^{k\times k} \begin{bmatrix} \tilde{x}^1_{t} \\ \tilde{x}^2_{t} \\ \tilde{x}^3_{t} \\ . \\ . \\ . \\ \tilde{x}^k_{t} \end{bmatrix} + \begin{bmatrix} J_1 & 0 & 0 & ... & 0 \\ J_2 & J_2 N_1 & 0 & ... & 0 \\ J_3 & J_3 & J_3 N_1 N_{2/1} & ... & 0 \\ . & & & & \\ . & & & & \\ . & & & & \\ J_k & ... & J_k & J_k \prod_{i=1}^{k-1} N_{i/i-1} & \end{bmatrix} \begin{bmatrix} d\tilde{q}^1_t \\ d\tilde{q}^2_t \\ d\tilde{q}^3_t \\ . \\ . \\ . \\ d\tilde{q}^k_t \end{bmatrix} + \begin{bmatrix} w^1_t \\ w^2_t \\ w^3_t \\ . \\ . \\ . \\ w^k_t \end{bmatrix}$$

3 Information Structure

Due to the sequential null space projections of tasks it is easy to establish a hierarchy of task objectives defined as end-effector position in cartesian space. The top prioritized task generates a control policy based on information from it's own state, whereas all lower level tasks rely on information from their own state as well as control inputs of all higher level tasks. The information structure is displayed in 2 as well as equations 6,7,8,9.

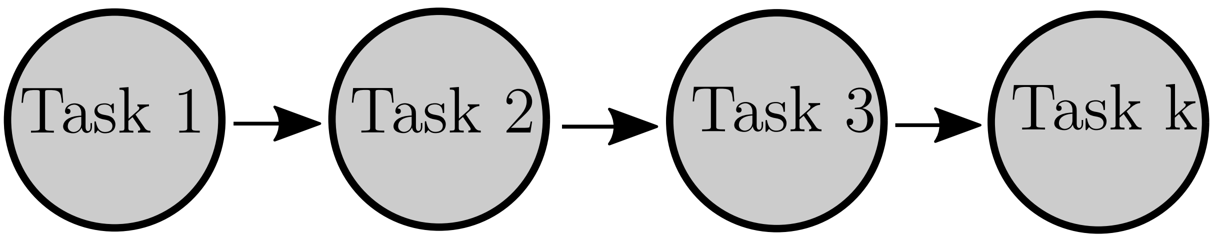

Figure 2: A representation of the information structure for this system. Task 1 represents the prioritized task and all information flows downstream to lower level tasks.

$$u_1(t) = \gamma_{1,t}(x_t^1) \tag{6}$$

$$u_2(t) = \gamma_{2,t}(x_t^2, u_t^1) \tag{7}$$

$$u_3(t) = \gamma_{3,t}(x_t^3, u_t^1, u_t^2) \tag{8}$$

$$u_k(t) = \gamma_{k,t}(x_t^k, u_t^1, u_t^2, u_t^3, ..., u_t^{k-1}) \tag{9}$$

4 Optimal Control Formulation

The aim of the control policies is to minimize the cost function

$$J = \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{E}(\tilde{x}_t^T Q \tilde{x}_t + u_t^T R u_t) \tag{10}$$

Because the information structure contains a hierarchy we can use the dynamic programming approach from [3]. The node structure is shown in 3. There is no time delay in this system therefore the solution reduces to k solutions to the discrete Ricatti equation.

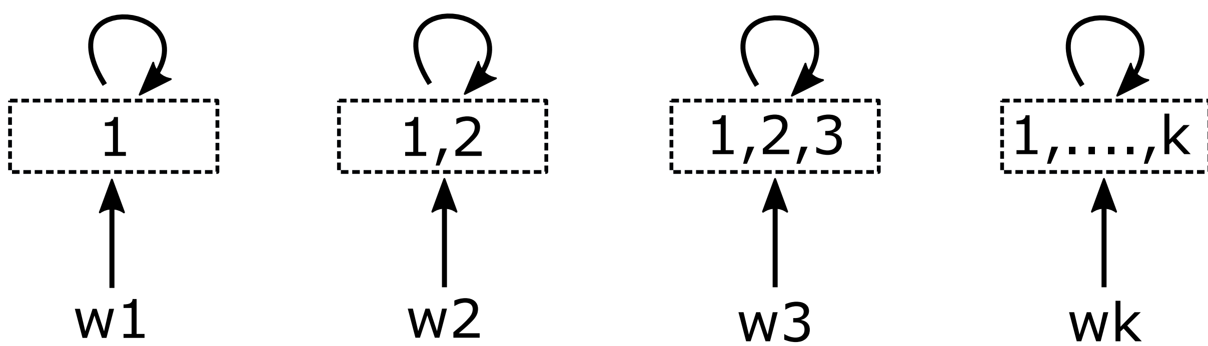

Figure 3: The information graph for this system. Because there are no time delays each node loops to itself for each time iteration.

5 Task Definitions

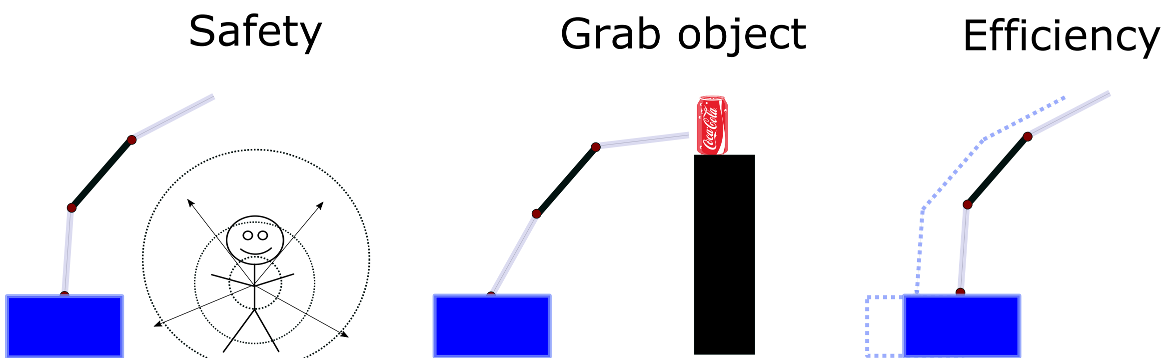

Figure 4: The figure depicts the 3 tasks for the robot to achieve. Safety is defined as a reference position for each link according to a potential field from the objects COM, grab object is defined as the cartesian position of the object to be grasped by the end effector, and efficiency is defined as the reference position as the starting configuration of the robot to minimize overall displacement.

References

[1] L. Sentis, *Synthesis and control of whole-body behaviors in humanoid systems.* stanford university USA, 2007.

[2] S. J. Jorgensen, O. Campbell, T. Llado, J. Lee, B. Shang, and L. Sentis, "Prioritized kinematic control of joint-constrained head-eye robots using the intermediate value approach," *arXiv preprint arXiv:1809.08750*, 2018.

[3] A. Lamperski and L. Lessard, "Optimal decentralized state-feedback control with sparsity and delays," *Automatica*, vol. 58, pp. 143–151, 2015.