

# Exercise 6.1

a)

$$LDL^T = \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{0e_L}^T & 1 & 0 \\ 0 & \lambda_{2e_F}^T & L_{22} \end{pmatrix} \begin{pmatrix} D_{00} & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & D_{22} \end{pmatrix}$$

$$\begin{pmatrix} L_{00} & \lambda_{0e_L}^T & 0 \\ 0 & 1 & \lambda_{2e_F}^T \\ 0 & 0 & L_{22} \end{pmatrix} = \begin{pmatrix} L_{00}D_{00} & 0 & 0 \\ \lambda_{0e_L}^T D_{00} \delta_1 & 0 & 0 \\ 0 & \lambda_{2e_F}^T \delta_1 & L_{22}D_{22} \end{pmatrix}$$

$$\begin{pmatrix} L_{00} & \lambda_{0e_L}^T & 0 \\ 0 & 1 & \lambda_{2e_F}^T \\ 0 & 0 & L_{22} \end{pmatrix} = \begin{pmatrix} L_{00}D_{00}L_{00} & L_{00}D_{00}\lambda_{0e_L}^T & 0 \\ \lambda_{0e_L}^T D_{00} & \delta_1 + \lambda_{2e_F}^T \lambda_{0e_L}^T & \lambda_{2e_F}^T L_{22} \\ 0 & \lambda_{2e_F}^T \delta_1 & L_{22}D_{22} \end{pmatrix}$$

$$UEU^T = \begin{pmatrix} U_{00} & U_{0e_L} & 0 \\ 0 & 1 & U_{2e_F}^T \\ 0 & 0 & U_{22} \end{pmatrix} \begin{pmatrix} E_{00} & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & E_{22} \end{pmatrix}$$

$$\begin{pmatrix} U_{00} & 0 & 0 \\ e_L^T U_{01} & 1 & 0 \\ 0 & e_F^T U_{12} & U_{22} \end{pmatrix} = \begin{pmatrix} U_{00}E_{00} & U_{01}e_L^T \epsilon_1 & 0 \\ 0 & \epsilon_1 & U_{2e_F}^T E_{22} \\ 0 & 0 & U_{22}E_{22} \end{pmatrix}$$

$$\begin{pmatrix} U_{00} & 0 & 0 \\ e_L^T U_{01} & 1 & 0 \\ 0 & e_F^T U_{12} & U_{22} \end{pmatrix} = \begin{pmatrix} U_{00}E_{00}U_{00} + U_{01}e_L^T \epsilon_1 e_L^T U_{01} & \epsilon_1 & 0 \\ \epsilon_1 e_L^T U_{01} & \epsilon_1 + U_{2e_F}^T E_{22} e_F^T U_{12} & U_{2e_F}^T E_{22} U_{22} \\ 0 & U_{22}E_{22} e_F^T U_{12} & U_{22}E_{22} U_{22} \end{pmatrix}$$

$$\begin{pmatrix} U_{00}E_{00}U_{00} + U_{01}e_L^T \epsilon_1 e_L^T U_{01} & \epsilon_1 & 0 \\ \epsilon_1 e_L^T U_{01} & \epsilon_1 + U_{2e_F}^T E_{22} e_F^T U_{12} & U_{2e_F}^T E_{22} U_{22} \\ 0 & U_{22}E_{22} e_F^T U_{12} & U_{22}E_{22} U_{22} \end{pmatrix}$$