$$\left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10} e_L^T & 1 & V_{12} e_F^T \\ \hline 0 & 0 & U_{22} \end{array}\right) \cdot \left(\begin{array}{c|c|c} D_{00} & 0 & 0 \\ \hline 0 & \phi_1 & 0 \\ \hline 0 & 0 & E_{22} \end{array}\right) \cdot \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10} e_L^T & 1 & V_{12} e_F^T \\ \hline 0 & 0 & U_{22} \end{array}\right)^T$$

$$= \left(\begin{array}{c|c|c} L_{00} D_{00} & 0 & 0 \\ \hline \lambda_{10} e_L^T D_{00} & \phi_1 & V_{12} e_F^T E_{22} \\ \hline 0 & 0 & U_{22} E_{22} \end{array}\right) \cdot \left(\begin{array}{ccc} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & V_{12} e_F^T \\ 0 & 0 & U_{22} \end{array}\right)^T$$

$$= \left(\begin{array}{c|c|c} L_{00} D_{00} & 0 & 0 \\ \hline \lambda_{10} e_L^T D_{00} & \phi_1 & V_{12} e_F^T E_{22} \\ \hline 0 & 0 & U_{22} E_{22} \end{array}\right) \cdot \left(\begin{array}{c|c|c} L_{00} & \lambda_{10} e_L & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & e_F V_{12}^T & U_{22} \end{array}\right)$$

$$= \left(\begin{array}{c|c|c} L_{00} D_{00} L_{00} & 0 & 0 \\ \hline \lambda_{10} e_L^T D_{00} L_{00} & \lambda_{10}^2 e_L^T e_L + \phi_1 & V_{12} e_F^T E_{22} U_{22} \\ \hline 0 & U_{22} E_{22} e_F V_{12}^T & U_{22} E_{22} U_{22} \end{array}\right)$$

$$\phi_1 = \alpha_{11} - \lambda_{10}^2 e_L^T e_L$$

b. What is the cost of computing one twisted factorization given that you have already computed the LDL^T and UEU^T factorizations?

$O(n)$ because the elements on the side of the matrix require $O(n)$ computations. All other elements require $O(1)$ because you have already computed them.

c. $O(n^2)$ for n twisted factorizations.