Applications of Belief Revision

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1 author:

Mary-Anne Williams
University of Technology Sydney

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Mary-Anne Williams

Information Systems, School of Management
The University of Newcastle
NSW 2308, Australia
maryanne@infosystems.newcastle.edu.au

1 Introduction

Belief revision provides support for intelligent information processing and decision making. The objective of this paper is to illustrate how theoretical ideas in belief revision have laid the foundation for practical implementations and real world applications.

Belief revision operators are often based on one of the following constructive definitions: an epistemic entrenchment ordering of sentences, or a system of spheres which is essentially a total preorder over possible world states. We outline each of these constructions, and show how they can be implemented and used for real world applications. To illustrate the epistemic entrenchment construction we use an application in software engineering [18], and for the system of spheres construction, an application in marketing research [35].

In section 2 we introduce the AGM paradigm for belief revision. In section 3 we describe the epistemic entrenchment construction for revision functions and show how it can be implemented using a standard theorem prover. An application in requirements engineering is used to illustrate its applicability. In section 4 we describe the systems of spheres construction and illustrate how it can be implemented using relational database technology [32]. The construction is then used to model changes in consumer preferences in a marketing research application.

2 Belief Revision

Belief revision provides mechanisms for changing repositories of information in the light of new information. These mechanisms can be used to incorporate new information into a knowledge base without compromising its integrity.

If the new information to be incorporated is consistent with the knowledge base then this process is straightforward; simply add the new information. On the other hand, if the new information contradicts the knowledge base then great care must be exercised, otherwise the introduction of inconsistency will compromise the integrity of the knowledge base.

In order to incorporate new information which is inconsistent with the knowledge base, the system or agent must *decide* what information it is prepared to give up. Belief revision attempts to model decisions concerning modifications to a knowledge base. The guiding principles are that the changes should be both rational and minimal in some sense.

Consider the situation where a systems analyst is assessing a software module that performs electronic commerce transactions called e-comm. His brief from management is to redesign inefficient mission critical modules, and he subsequently learns that the module *e-comm is inefficient* and *mission critical*. He then concludes that *e-comm must be redesigned*. Shortly after coming to this conclusion the analyst is informed by management that the module *e-comm does not need to be redesigned*. How should he go about modifying his knowledge? Not only must the inferred information *e-comm must be redesigned* be retracted, but other beliefs which conflict with the new information must also be removed. There appears to be several possible choices, for example: (i) retract the module *e-comm is inefficient*, (ii) retract the module *e-comm is mission critical*, (iii) retract *inefficient mission critical modules must be redesigned*, or (iv) any combination of all three. The alternative adopted will depend upon the relative importance attributed to his background information. If the analyst has less confidence in the fact that *e-comm is inefficient* than both *e-comm is mission critical* and *inefficient mission critical modules must be redesigned* he would probably prefer the solution that (i) offers, conversely if he believes *inefficient mission critical modules must be redesigned* with the least confidence then he might prefer (iii). If he is unable to decide which to prefer, or if all three beliefs are causally linked then he might relinquish them all to make way for the acceptance of the new information. From this intuitive discussion it is obvious that a preference ordering of our knowledge base can be used to resolve the nontrivial problem of choosing what information to surrender in order to avoid inconsistency. This is essentially how an epistemic entrenchment ordering is used to uniquely determine a revision function.

Another approach uses a (plausibility) relation over possible states of the world to help make a decision. For example, if the analyst believes that the most plausible world state in which the e-comm module does not require redesign it is not inefficient, then he could justify adopting (i) above. This is the general approach taken by constructions of belief revision operators based on a system of spheres.

The framework for belief revision adopted herein is known as the AGM paradigm, so-called after its founders Alchourrón, Gärdenfors and Makinson [1]. It is a formal framework for modeling ideal and rational changes to repositories of information under the principle of Minimal Change. In particular, it provides mechanisms for modeling the coherent retraction and incorporation of information.

Technically, the framework models corpora of information as logical theories, and changes in information content as functions that take a theory (the current knowledge base) and a logical sentence (the new information) to another theory (the new knowledge base). There are several types of change functions: *contraction, withdrawal, expansion*, and *revision*. Contraction and withdrawal functions model the retraction of information, whilst expansion and revision model various ways of incorporating information. All four functions are interrelated.

Change functions can be described either *axiomatically* using rationality postulates, or *constructively* using certain preference relations or selection functions. The rationality postulates are properties that we would expect rational change functions to satisfy and they characterise various classes of change functions. Moreover they may be satisfied by more than one function. An individual function can be singled out using extralogical information such as an *epistemic entrenchment ordering* or a *system of spheres* which help to make necessary choices concerning what information should be given up.

The belief revision framework restricts itself to modeling changes to logical theories that involve the addition and removal of facts. Therefore, we do not consider the possibility of explicitly modifying individual facts, such as transforming *inefficient mission critical modules must be redesigned* to *inefficient mission critical modules except e-comm must be redesigned*, as a primitive operation. Modifying individual facts is often seen in machine learning for example, and can be modelled in the belief revision framework by observing that *inefficient mission critical modules must be redesigned* entails *inefficient mission critical modules except e-comm must be redesigned*, therefore removing *inefficient mission critical modules must be redesigned* and retaining *inefficient mission critical modules except e-comm must be redesigned* achieves the same result.

Belief revision models *rational modifications* to knowledge bases guided by the principle of *Minimal Change*; unfortunately, the notions of *rationality* and *minimality*, in the sense we would like to capture, defy explicit definition. Intuitively, by rational we mean things like: the reasoner realises that inconsistency is problematical and thus actively seeks to avoid it, and that given our example it is not sufficient to retract only the fact that *e-comm does not need to be redesigned* because it is derivable from the remaining information. The principle of Minimal Change says that, as much information should be conserved as is possible in accordance with an underlying preference relation. The underlying preference relation is used to capture the information *content* of the knowledge base, the reasoning agent's *commitment* to this information, and how the information should behave under *change*.

It has often been incorrectly argued that the choices (i), (ii) and (iii) are more minimal than (iv), because only *one* basic fact is jettisoned as opposed to *two* or more. The problem with this argument is that the interdependencies among our beliefs might force us to discard more than the minimal *number* of beliefs. The web of *causal* interdependencies is enmeshed in the preference relation, and cardinality measures are not the only, nor necessarily the most appropriate, when it comes to measuring the *magnitude of change*. For instance, to take our example a little further, it might have been that the only reason for believing that *inefficient mission critical modules must be redesigned* is that *e-comm is inefficient* and *mission critical*, so if we contract *the modules must be redesigned* then it should be permissible to retract *e-comm is inefficient* and *mission critical* at the same time. Clearly, cardinality is not the only allowable measure of change. Sometimes the most rational response is to forfeit more than the minimal number of beliefs. For instance, it may be better to remove several weakly held beliefs rather than a single strongly held belief.

2.1 Expansion

Expansion models the simplest change to a knowledge base, it involves the acceptance of information without the removal of any previously accepted information, and as a consequence it may lead to an inconsistent knowledge base.

The *expansion* of a theory T with respect to a sentence φ is the logical closure of T and φ. The set of all theories of a language $\mathcal{L}$ is denoted by $\mathcal{K}_{\mathcal{L}}$, and formally, an expansion function $^+$ is a function from $\mathcal{K}_{\mathcal{L}} \times \mathcal{L}$ to $\mathcal{K}_{\mathcal{L}}$, mapping (T, φ) to T^+_φ where $T^+_\varphi = \mathrm{Cn}(T \cup \{\varphi\})$. We denote the inconsistent theory by $\perp$.

Expansion is a monotonic operation, since $T \subseteq T^+_\varphi$, and if $\neg\varphi \in T$, then T^+_φ is inconsistent. Certainly if $\neg\varphi \notin T$, then we can accept that the principle of Minimal Change is at work, since T^+_φ would be the *smallest* change we can logically make to T in order to incorporate φ.

In contradistinction to expansion, it turns out that both contraction and revision are nonunique operations and cannot be realised using logical and set theoretical notions alone.

2.2 Contraction

Contraction of a knowledge base involves the removal of information, the difficulty, as we noted in the introduction, is in determining those sentences that should be given up. We are normally presented with a choice. For example, perhaps the analyst erroneously believed that the module *e-comm was mission critical* and subsequently wished to contract it, this may or may not involve the removal of other beliefs, such as *e-comm is mission critical when the network link between Sydney and Melbourne fails*, depending on how he views the causal dependencies, etc.

A *contraction* of T with respect to φ involves the removal of a set of sentences from T so that φ is no longer implied, provided φ is not a tautology. Formally, a contraction function $^-$ is any function from $\mathcal{K}_{\mathcal{L}} \times \mathcal{L}$ to $\mathcal{K}_{\mathcal{L}}$, mapping (T, φ) to T^-_φ which satisfies the following postulates. For any $\varphi,\ \psi \in \mathcal{L}$ and any $T \in \mathcal{K}_{\mathcal{L}}$:

($^-$1) $T^-_\varphi \in \mathcal{K}_{\mathcal{L}}$
($^-$2) $T^-_\varphi \subseteq T$
($^-$3) If $\varphi \notin T$ then $T \subseteq T^-_\varphi$
($^-$4) If $\not\vdash \varphi$ then $\varphi \notin T^-_\varphi$
($^-$5) $T \subseteq (T^-_\varphi)^+_\varphi$ (*recovery*)
($^-$6) If $\vdash \varphi \leftrightarrow \psi$ then $T^-_\varphi = T^-_\psi$
($^-$7) $T^-_\varphi \cap T^-_\psi \subseteq T^-_{\varphi\wedge\psi}$
($^-$8) If $\varphi \notin T^-_{\varphi\wedge\psi}$ then $T^-_{\varphi\wedge\psi} \subseteq T^-_\varphi$

The postulates embody the principle of *Minimal Change*, and act as integrity constraints for change functions. They do not uniquely determine a change function, rather their purpose is to identify the set of possible new knowledge bases that might reasonably result when information is contracted from the current knowledge base, T. The postulates themselves are explained in Gärdenfors [7] and are motivated via the criterion of informational economy.

The first postulate $(^{-}1)$ simply says that the result of a contraction is a theory, so that contracting information results in a theory, i.e. the new knowledge base is logically closed. Essentially the change functions described in the AGM paradigm model the processes of an ideal reasoning agent. Since theories are typically infinite structures the first postulate will not usually be satisfied by a computer-based implementation. Postulate $(^{-}5)$ together with the previous four, says if $\varphi \in T$ then $T = (T^{-}_{\varphi})^{+}_{\varphi}$. In other words, no more information is lost than can be reincorporated by an expansion with respect to the explicit information contracted, that is, if we contract φ and then immediately replace it using expansion then we obtain the theory we started with. Intuitively then, this postulate forces a *minimal* amount of information to be lost during a contraction.

The postulates for contraction identify a class of functions for a knowledge base and for each one of these functions Gärdenfors and Makinson [8] showed that there is a preference criterion that can be used to construct it.

The most controversial of the contraction postulates is *recovery*, $(^{-}5)$, because one can argue that it is not always an appropriate requirement, especially for a limited reasoning agent. A withdrawal function [13] is similar to a contraction function with the exception that it may not satisfy recovery. In particular, a *withdrawal function* satisfies $(^{-}1)$ – $(^{-}4)$ and $(^{-}6)$ – $(^{-}8)$, but not necessarily $(^{-}5)$.

2.3 Revision

Revision attempts to change a knowledge base as *little as possible* in order to incorporate newly acquired information. This new information may be inconsistent with the knowledge base. In order to maintain consistency some old information may need to be retracted. Thus revision functions are nonmonotonic in nature, and related to withdrawal and contraction functions.

The process of revision was discussed in the introduction where the analyst, if you recall, was mistaken about the module's mission criticalness, and had to revise his knowledge.

Formally, a revision function * is any function from $\mathcal{K}_{\mathcal{L}} \times \mathcal{L}$ to $\mathcal{K}_{\mathcal{L}}$, mapping (T, φ) to T^{*}_{φ} which satisfies the following postulates. For any $\varphi,\ \psi \in \mathcal{L}$ and any $T \in \mathcal{K}_{\mathcal{L}}$:

(*1) $T^{*}_{\varphi} \in \mathcal{K}_{\mathcal{L}}$
(*2) $\varphi \in T^{*}_{\varphi}$
(*3) $T^{*}_{\varphi} \subseteq T^{+}_{\varphi}$
(*4) If $\neg\varphi \notin T$ then $T^{+}_{\varphi} \subseteq T^{*}_{\varphi}$
(*5) If $T^{*}_{\varphi} = \perp$ then $\vdash \neg\varphi$

(*6) If $\vdash \varphi \leftrightarrow \psi$ then $T^*_\varphi = T^*_\psi$
(*7) $T^*_{\varphi\wedge\psi} \subseteq (T^*_\varphi)^+_\psi$
(*8) If $\neg\psi \notin T^*_\varphi$ then $(T^*_\varphi)^+_\psi \subseteq T^*_{\varphi\wedge\psi}$

The revision postulates attempt to encapsulate the principle of Minimal Change. Several explicit relationships exist between the various change functions. In the theorems below Alchourron, Gärdenfors and Makinson [1] demonstrated that contraction and revision functions are interdefinable.

Theorem 1. *If $^-$ is a contraction function and $^+$ the expansion function, then * defined by the* Levi Identity *below defines a revision function.*

$$T^*_\varphi = (T^-_{\neg\varphi})^+_\varphi$$

Theorem 2. If * is a revision function, then $^-$ defined by the *Harper Identity* below defines a contraction function.

$$T^-_\varphi = T \cap T^*_{\neg\varphi}$$

3 Epistemic Entrenchment

As noted earlier the rationality postulates for contraction and revision merely describe classes of functions; they do not provide a mechanism for defining a particular function. For any theory there might be a vast number of functions that satisfy the postulates for contraction and revision (see Theorem 9). So in order to single out a unique one, additional structure is necessary; this usually takes the form of a preference relation such as an epistemic entrenchment ordering.

Gärdenfors' epistemic entrenchment orderings [7] are based on a relative ranking of information according to importance in the face of change. This ordering can be used to uniquely determine a change function by providing a selection criteria that can be used to identify those sentences to be retracted, those to be retained, and those to be acquired during changes. Intuitively, when faced with a choice, sentences having the lowest degree of epistemic entrenchment are shed.

Definition 3. *Given a theory T of $\mathcal{L}$, an* epistemic entrenchment related to T *is any binary relation $\leq$ on $\mathcal{L}$ satisfying the conditions below:*

(EE1) If $\varphi \leq \psi$ and $\psi \leq \chi$, then $\varphi \leq \chi$.
(EE2) For all $\varphi,\ \psi \in \mathcal{L}$, if $\varphi \vdash \psi$ then $\varphi \leq \psi$.
(EE3) For all $\varphi,\ \psi \in \mathcal{L}$, $\varphi \leq \varphi \wedge \psi$ or $\psi \leq \varphi \wedge \psi$.
(EE4) When $T \neq \perp$, $\varphi \notin T$ iff $\varphi \leq \psi$ for all $\psi \in \mathcal{L}$.
(EE5) If $\psi \leq \varphi$ for all $\psi \in \mathcal{L}$, then $\vdash \varphi$.

If $\varphi \leq \psi$, then we say ψ is at least as entrenched as φ. *We define $\varphi < \psi$, as $\varphi \leq \psi$ and not $\psi \leq \varphi$. If $\varphi \leq \psi$ and $\psi \leq \varphi$, then we say φ and ψ are* equally entrenched.

The condition (EE1) requires that an epistemic entrenchment ordering be transitive. (EE2) says that if φ is logically stronger than ψ, then ψ is at least as entrenched as φ. For example the sentence $\varphi \vee \psi$ is entailed by φ, and (EE2) tells us that $\varphi \vee \psi$ is at least as entrenched as φ, in other words we believe in $\varphi \vee \psi$ at least as much as φ. It certainly would not make any sense to believe in φ *more* strongly than $\varphi \vee \psi$; a rational agent could hardly be less certain of $\varphi \vee \psi$ than it is of φ. The condition (EE3) together with (EE1) and (EE2) implies that a conjunction is ranked at the same level as its least ranked conjunct. For example, if $\varphi \leq \psi$ then $\varphi \wedge \psi \leq \varphi$ and $\varphi \leq \varphi \wedge \psi$, so $\varphi \wedge \psi$ and φ are equally entrenched. The condition (EE4) tells us that sentences not in the theory T are minimal, and (EE5) says that the tautologies are maximal.

Definition 4. *For an epistemic entrenchment ordering $\leq$ and a sentence φ, define* $\mathrm{cut}_{\leq}(\varphi) = \{\psi : \varphi \leq \psi\}$.

The set $\mathrm{cut}_{\leq}(\varphi)$ contains all those sentences that are at least as entrenched as φ. An important property of an epistemic entrenchment is that it is a total preorder of the sentences in $\mathcal{L}$ such that the following theorem from [27] holds.

Theorem 5. *If $\leq$ is an epistemic entrenchment, then for any sentence φ,* $\mathrm{cut}_{\leq}(\varphi)$ *is a theory.*

Since a subtheory of a finitely axiomatizable theory may not be finitely axiomatizable we give the following result from [27] concerning a *finite description* of an epistemic entrenchment ordering. In the next section we develop a computational model for change functions using a finite description of an epistemic entrenchment ordering, and the following theorem provides the theoretical basis for the one we adopt.

Theorem 6. *An epistemic entrenchment ordering $\leq$ is* finitely representable *if and only if it has a finite number of natural partitions, and for all $\varphi \in \mathcal{L}$,* $\mathrm{cut}_{\leq}(\varphi)$ *is finitely axiomatizable.*

The following result of Gärdenfors and Makinson [8] provides us with a constructive method for building change functions from an epistemic entrenchment ordering. Theorem 7 gives a condition that can be used for constructing a contraction function, and Theorem 8 provides a similar one for constructing a revision function.

Theorem 7. *Let T be a theory of $\mathcal{L}$. For every contraction function $^-$ for T there exists an epistemic entrenchment $\leq$ related to T such that (E^-), below, is true for every $\varphi \in \mathcal{L}$. Conversely, for every epistemic entrenchment $\leq$ related to T, there exists a contraction function $^-$ such that (E^-) is true for every $\varphi \in \mathcal{L}$.*

$$(\mathbf{E^-}) \qquad T^-_\varphi = \begin{cases} \{\psi \in T : \ \varphi < \varphi \vee \psi\} & \textit{if } \nvdash \varphi \\ T & \textit{otherwise} \end{cases}$$

Given an epistemic entrenchment related to T, the condition (E$^-$) explicitly determines the information to be retained, and retracted in a contraction operation. Furthermore, every contraction function can be constructed from some epistemic entrenchment ordering.

Returning to our example, if $\neg$*e-comm to be redesigned* $\vee$ *e-comm is mission critical* is strictly more entrenched than $\neg$*e-comm to be redesigned* , then *e-comm is mission critical* will remain after the contraction of *e-comm not to be redesigned* . Conversely, if instead they are equally entrenched then *e-comm is mission critical* will be retracted.

It is easy to show using (E$^-$) that if ψ is at least as entrenched as φ then under the contraction with respect to $\varphi \wedge \psi$, the conjunct φ is removed, and if φ and ψ are equally entrenched then both are retracted. This mirrors our intuition that sentences with the lowest epistemic entrenchment are given up.

An analogous result derived from [8, 11] is given in [21] for revision, and is provided in the theorem below.

Theorem 8. *Let T be a theory of $\mathcal{L}$. For every revision function * for T there exists an epistemic entrenchment $\leq$ related to T such that (E^*), below, is true for every $\varphi \in \mathcal{L}$. Conversely, for every epistemic entrenchment $\leq$ related to T, there exists a revision function * for T such that (E^*) is true for every $\varphi \in \mathcal{L}$.*

$$(E^*) \qquad T^*_\varphi = \begin{cases} \{\psi \in \mathcal{L} : \neg\varphi < \varphi \rightarrow \psi\} & \text{if } \not\vdash \neg\varphi \\ \\ \perp & \text{otherwise} \end{cases}$$

Gärdenfors and Makinson [8] showed that for a finite language, an epistemic entrenchment related to T is determined by the ordering of the dual atoms (maximal disjunctions) in T. They also showed that there is a one-to-one correspondence between epistemic entrenchment orderings and revision (and contraction) functions, consequently the number of revision (and contraction) functions for T is equal to the number of epistemic entrenchment orderings on the dual atoms[1] in T. Therefore it is straightforward to determine the number of possible revision (and contraction) functions for T. Indeed it was calculated [34] to be the number of ways we can place a total preorder on the dual atoms in T. But first note that if there are k atoms in the language, then there are 2^k dual atoms. For instance, if the only atoms are φ and ψ then the set of dual atoms is $\{\varphi \vee \psi, \varphi \vee \neg\psi, \neg\varphi \vee \psi, \neg\varphi \vee \neg\psi\}$.

Theorem 9. *For a theory T in a finite language, if n is the number of dual atoms in T, then the number of revisions (and contraction) functions is given by:*

$$p(n) = \Sigma_{m=1}^{n} m! S(m, n)$$

where $S(m, n)$, are Stirling numbers of the second kind*, that is, the number of partitions of an $n-$element set into m parts.*

[1] If the language consists of the atoms $p_1, \ldots, p_n$, then a dual atom is a sentence $L_1 \vee \ldots \vee L_n$, where for each i, L_i is either p_i or $\neg p_i$.

The growth of $p(n)$ is more rapid than exponential; $p(1) = 1$, $p(2) = 3$, $p(3) = 13$, $p(4) = 75$, $p(5) = 541 \cdots$, hence the class of change functions described by the rationality postulates is colossal in n, the number of dual atoms!

3.1 Implementing Entrenchment Based Revision

Several problems arise when one attempts to implement the process of belief revision using an epistemic entrenchment ordering. First, because AGM change functions take an epistemic entrenchment ordering together with a sentence, to be contracted or accepted, and produce a theory, the entrenchment ordering is lost. Consequently, iterated revision is not naturally supported. Most applications where belief revision capabilities would be most valuable require iterated revision capabilities. So from a practical perspective modeling the iteration of change is essential. Second, because epistemic entrenchment orderings typically rank an infinite number of sentences in a logical language, there is a serious *representation* problem for computer-based implementations.

In this section we define a *finite partial entrenchment ranking* which can be used to generate a finitely representable epistemic entrenchment ordering overcoming the representation problem. In order to overcome the iteration problem we describe two simple computational models that transmute a finite partial ranking using an *absolute measure of minimal change.* Any theorem prover can be used to realise them. Along the way we demonstrate the explicit relationships between change functions constructed from partial entrenchment rankings, and epistemic entrenchment orderings. Somewhat surprisingly despite the apparent syntax dependence it turns out that standard AGM revision and contraction functions can be constructed from partial entrenchment rankings [27].

Finite Partial Entrenchment Rankings.
A *finite partial entrenchment ranking* grades the content of a finite knowledge base according to its epistemic importance, and as such it can be used to specify a finitely representable epistemic entrenchment ordering. Formally defined below, this ranking maps a finite set of sentences to rational numbers. Intuitively, the higher the value assigned to a sentence the more firmly held it is, or the more entrenched it is.

Definition 10. *A* finite partial entrenchment ranking *is a function* $\mathbf{B}$ *from a finite subset of sentences into the interval* $[0, 1]$ *such that the following conditions are satisfied for all* $\varphi \in \mathrm{dom}(\mathbf{B})$*:*

(PER1) $\{\psi \in \mathrm{dom}(\mathbf{B}) : \mathbf{B}(\varphi) < \mathbf{B}(\psi)\} \not\vdash \varphi$.
(PER2) *If* $\vdash \neg\varphi$, *then* $\mathbf{B}(\varphi) = 0$.
(PER3) $\mathbf{B}(\varphi) = 1$ *if and only if* $\vdash \varphi$.

(PER1) says sentences assigned a value higher than an arbitrary sentence φ, do not entail φ, (PER2) says inconsistent sentences are assigned zero, and (PER3) says that tautologies are assigned 1. An example of a finite partial entrenchment ranking is given below.

Example 11. Let $\mathbf{B}$ be given by

$\mathbf{B}$(module(e-comm,mission_critical)$\vee\neg$module(e-comm,mission_critical)) $= 1$,
$\mathbf{B}$(($\forall$X(module(X,inefficient)$\wedge$(module(X,mission_critical))$\rightarrow$redesign(X)))$= 0.8$
$\mathbf{B}$(module(e-comm,inefficient)) $= 0.6$,
$\mathbf{B}$(module(e-comm,mission_critical)) $= 0.4$,
$\mathbf{B}$(module(e-comm,mission_critical)$\wedge\neg$module(e-comm,mission_critical)) $= 0$.

A finite partial entrenchment ranking can be used to represent a finitely representable epistemic entrenchment ordering. Of course, a finite ranking does *not* imply that the language is finite.

The numerical assignment can be viewed in two distinct ways: (i) qualitatively, where the relative ordering of sentences is used, or (ii) quantitatively, where the numerical value assigned to sentences possesses some extra meaning, such as probability, necessity [5] and a calculus based on their numerical value adopted.

The intended interpretation of a finite partial entrenchment ranking is that sentences mapped to numbers greater than zero represent the *explicit* beliefs, and their logical closure represents its *implicit* beliefs.

Definition 12. *Define the* explicit information content *represented by* $\mathbf{B}$ *a finite partial entrenchment ranking to be* $\{\varphi \in \mathrm{dom}(\mathbf{B}) : \mathbf{B}(\varphi) > 0\}$*, and denote it by* $\exp(\mathbf{B})$*. Similarly, define the* implicit information content *represented by* $\mathbf{B} \in \mathcal{B}$ *to be* $Cn(\exp(\mathbf{B}))$*, and denote it by* $\mathrm{content}(\mathbf{B})$*.*

A finite partial entrenchment ranking usually represents an *incomplete specification* of an agent's preferences from which an epistemic entrenchment ordering can be generated. Note there is no single way to generating an epistemic entrenchment ordering from a partial specification in general. For example, if $\mathbf{B}(\varphi) = 0.2$ and $\mathbf{B}(\psi) = 0.4$ then all compatible epistemic entrenchment orderings will have $\varphi < \psi$, however one compatible epistemic entrenchment ordering will have $\psi < \varphi \vee \psi$, whilst another will have $\psi = \varphi \vee \psi$. The epistemic entrenchment ordering we generate for our computational model gives sentences the *minimum* possible degree of entrenchment. For the example above, the generated epistemic entrenchment ordering would have $\psi = \varphi \vee \psi$ because $\varphi \vee \psi$ must be at least as entrenched as ψ by (EE2) but need not be strictly more entrenched.

In order to describe epistemic entrenchment orderings generated from a ranking it will be necessary to rank implicit information. In the definition below we assign a minimal degree of acceptance to implicit information under the constraint of (PER1).

Definition 13. *Let* φ *be a nontautological sentence. Let* $\mathbf{B}$ *be a finite partial entrenchment ranking. We define the* degree of acceptance *of* φ *to be*

$$\mathrm{degree}(\mathbf{B},\varphi) = \begin{cases} \textit{largest } j \textit{ such that } \{\psi \in \exp(\mathbf{B}) : \mathbf{B}(\psi) \geq j\} \vdash \varphi \\ \qquad\qquad\qquad\qquad\qquad\qquad \textit{if } \varphi \in \mathrm{content}(\mathbf{B}) \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\quad \textit{otherwise} \end{cases}$$

One can design a simple procedural algorithm to calculate the degree of acceptance of a sentence φ given the information encoded in a finite partial entrenchment ranking. Tautologies have degree 1, and to determine the degree of a nontautological sentence φ we can adopt a straightforward top down procedure. Attempt to prove φ using the sentences assigned the largest value in the range of $\mathbf{B}$, say n, if we are successful then φ is assigned degree n, otherwise try to prove it using sentences assigned the next largest degree, say m, and those assigned n, if φ is successfully proven then it is assigned degree m, etc. A bottom-up procedure could also be used, or a high performance binary search [12].

Example 14. Using $\mathbf{B}$ from the previous example we calculate the minimal degree of acceptance of the following implicit information.

degree($\mathbf{B}$,module(e-comm,inefficient)$\vee$ module(e-comm,mission_critical)) $= 0.6$,
degree($\mathbf{B}$,module(e-comm,inefficient)$\wedge$module(e-comm,mission_critical)) $= 0.4$,
degree($\mathbf{B}$,redesign(e-comm)) $= 0.4$,
degree($\mathbf{B}$,$\neg$redesign(e-comm)) $= 0$,

Theorem 15, below, shows how a finite partial entrenchment ranking can generate an epistemic entrenchment ordering using degrees of acceptance.

Theorem 15. *Let* $\mathbf{B}$ *be a finite partial entrenchment ranking, and* φ, ψ *be sentences. Define* $\leq_{\mathbf{B}}$ *by* $\varphi \leq_{\mathbf{B}} \psi$ *iff* $\vdash \psi$*, or* $\mathrm{degree}(\mathbf{B},\varphi) \leq \mathrm{degree}(\mathbf{B},\psi)$*. Then* $\leq_{\mathbf{B}}$ *is an epistemic entrenchment ordering related to* $\mathrm{content}(\mathbf{B})$*.*

We refer to $\leq_{\mathbf{B}}$ as *the minimal epistemic entrenchment ordering generated from* $\mathbf{B}$. From Theorem 15 we see that the tautologies are maximal, and sentences not in $\mathrm{content}(\mathbf{B})$ are minimal with respect to $\leq_{\mathbf{B}}$. Since $\mathrm{dom}(\mathbf{B})$ is finite, the minimal epistemic entrenchment ordering generated is finite, that is, $\leq_{\mathbf{B}}$ possesses a finite number of natural partitions.

Back to our simple example, for sentences φ and ψ if $\mathbf{B}(\varphi) = 0.2$ and $\mathbf{B}(\psi) = 0.4$ then the minimal epistemic entrenchment ordering generated by $\mathbf{B}$ is given by the following ordering on dual atoms: $\neg\varphi\vee\neg\psi <_{\mathbf{B}} \varphi\vee\neg\psi <_{\mathbf{B}} \neg\varphi\vee\psi =_{\mathbf{B}} \varphi\vee\psi$. From this ordering we can derive $\varphi \wedge \psi =_{\mathbf{B}} \varphi <_{\mathbf{B}} \psi =_{\mathbf{B}} \varphi \vee \psi$

Recall that an epistemic entrenchment ordering, $\leq$, is *finitely representable* if and only if every $\mathrm{cut}_{\leq}(\varphi)$ is finitely axiomatizable. Consequently, $\leq$ is finitely representable if and only if there exists a finite partial entrenchment ranking $\mathbf{B}$ such that $\leq=\leq_{\mathbf{B}}$.

It is not hard to show that given a finitely representable epistemic entrenchment ordering there exists a finite partial entrenchment ranking that can generate it. In fact, a partial entrenchment ranking can be considered to be a (possibly minimal) specification of a finite epistemic entrenchment ordering. However, going from a finitely representable epistemic entrenchment to a finite partial entrenchment ranking is a nonunique process. For example, consider the epistemic entrenchment ordering: $\neg\varphi\vee\neg\psi < \varphi\vee\neg\psi < \neg\varphi\vee\psi = \varphi\vee\psi$. Any of the following three finite partial entrenchment rankings could be used to generate it using the function degree, that is, $\leq=\leq_{\mathbf{B}_1}=\leq_{\mathbf{B}_2}=\leq_{\mathbf{B}_3}$.

(a) $\mathbf{B}_1(\neg\varphi \vee \neg\psi) = 0, \mathbf{B}_1(\varphi \vee \neg\psi) = 0.2, \mathbf{B}_1(\neg\varphi \vee \psi) = 0.4, \mathbf{B}_1(\varphi \vee \psi) = 0.4.$
(b) $\mathbf{B}_2(\neg\varphi \wedge \neg\psi) = 0, \mathbf{B}_2(\varphi \vee \neg\psi) = 0.2, \mathbf{B}_2(\psi) = 0.4.$
(c) $\mathbf{B}_3(\varphi) = 0.2$, $\mathbf{B}_3(\psi) = 0.4.$

We call finite partial entrenchment rankings *equivalent* if they generate the same minimal epistemic entrenchment ordering. In this sense, the rankings $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{B}_3$ above are equivalent.

Iterated Revision.
As noted earlier from a practical perspective the AGM paradigm does not provide a policy to support the iteration of its theory change functions. Not specifying the resultant epistemic entrenchment is attractive in a theoretical context because it allows the resultant theory to adopt any of many possible epistemic entrenchment orderings depending on the desired dynamic behaviour. In practice, however, a *policy for change* is necessary.

In this subsection we describe two strategies for *transmuting* a finite partial entrenchment ranking: adjustment and maxi-adjustment.

The information input for the standard AGM change functions is a sentence. If our aim is to modify a ranking then not only do we need a sentence but we also need a degree of acceptance to assign it in the transmuted ranking. In other words, we need to know where in the new ranking the sentence should reside.

We define the *adjustment of a partial entrenchment ranking*, below, the new information is a contingent sentence φ and a number $0 \leq i < 1$ where φ is the information to be accepted, and i is the degree of acceptance φ is to be assigned in the adjusted ranking.

Adjustments are closely related to the standard epistemic entrenchment construction given in Theorem 7, and use a *policy for change* based on an *absolute minimal measure*; they transmute a finite partial entrenchment ranking so as to incorporate the desired new information using an absolute minimal measure of change. The set of contingent sentences is denoted $\mathcal{L}^{\bowtie}$.

Definition 16. *Let* $\varphi \in \mathcal{L}^{\bowtie}$ *and* $0 \leq i < 1$. *We define the* adjustment of a finite partial entrenchment ranking $\mathbf{B}$ *to be an function* * *such that*

$$\mathbf{B}^\star(\varphi, i) = \begin{cases} (\mathbf{B}^-(\varphi, i)) & \textit{if } i \leq \mathrm{degree}(\mathbf{B}, \varphi) \\ \\ (\mathbf{B}^-(\neg\varphi, 0))^+(\varphi, i) & \textit{otherwise} \end{cases}$$

where

$$\mathbf{B}^-(\varphi, i)(\psi) = \begin{cases} i & \textit{if } \mathrm{degree}(\mathbf{B}, \varphi) = \mathrm{degree}(\mathbf{B}, \varphi \vee \psi) \textit{and } \mathbf{B}(\psi) > i \\ \mathbf{B}(\psi) & \textit{otherwise} \end{cases}$$

for all $\psi \in \mathrm{dom}(\mathbf{B})$, *and*

$$\mathbf{B}^+(\varphi, i)(\psi) = \begin{cases} \mathbf{B}(\psi) & \textit{if } \mathbf{B}(\psi) > i \\ i & \textit{if } \varphi \leftrightarrow \psi \textit{ or } \mathbf{B}(\psi) \leq i < \mathrm{degree}(\mathbf{B}, \varphi \rightarrow \psi) \\ \mathrm{degree}(\mathbf{B}, \varphi \rightarrow \psi) & \textit{otherwise} \end{cases}$$

for all $\psi \in \mathrm{dom}(\mathbf{B}) \cup \{\varphi\}$.

Adjustments define change functions for *theory bases*, rather than logically closed sets of sentences as in the previous section. Note that an adjustment is not defined for inconsistent, or tautological sentences. Accepting inconsistent information will compromise the integrity of the system, and thus ought to be avoided, whilst tautologies must always be assigned 1. So the definition focuses on the principal case, and can easily be extended to include both tautological and inconsistent sentences, if desired.

Intuitively, an (φ, i)–adjustment of $\mathbf{B}$ involves minimal changes to $\mathbf{B}$ such that φ is accepted with degree i. In particular, each sentence $\psi \in \mathrm{dom}(\mathbf{B})$ is reassigned a number closest to $\mathbf{B}(\psi)$ in the adjusted partial entrenchment ranking $\mathbf{B}^\star(\varphi, i)$ under the guiding principle that if we reduce the degree of an accepted sentence φ, to i, say, then we also reduce the degree of each sentence that would be retracted in φ's contraction to i as well.

There are essentially two processes at work in an adjustment; sentences migrate *up* or *down* the ranking. Migration downwards is related to contraction, whilst movement upwards is related to expansion; movement upwards must ensure that the new ranking satisfies (PER1).

Adjustments use the *relative ranking* of information encoded in a partial entrenchment ranking, and they *preserve finiteness*; adjusting a finite partial entrenchment ranking results in a finite ranking, and if $\exp(\mathbf{B})$ is finite then $\exp(\mathbf{B}^\star(\varphi, i))$ is finite, in fact the size of the knowledge base is never increased by more than a single sentence.

The following theorem [27] illustrates the interrelationships between theory base revision and theory base contraction based on adjustments. In particular, Theorem 17(i) is analogous to the Harper Identity and it captures the dependence of contraction on the information content of the theory base, that is, $\exp(\mathbf{B})$. Similarly Theorem 17(ii) is analogous to the Levi Identity.

Theorem 17. *Let* $\mathbf{B} \in \mathcal{B}$, *let* * *be an adjustment, and let* $0 < i < 1$. *Then*

(i) $\exp(\mathbf{B}^\star(\varphi, 0)) = \exp(\mathbf{B}^\star(\neg\varphi, i)) \cap \exp(\mathbf{B})$, *and*
(ii) $\exp(\mathbf{B}^\star(\varphi, i)) = \exp(\mathbf{B}^\star(\neg\varphi, 0)) \cup \{\varphi\}$.

For the purpose of developing real world applications, a major shortcoming of adjustment is that in order to achieve desired behaviour the user must specify when information is independent with respect to change. For many applications it is more natural and easier to specify dependence. The maxi-adjustment strategy was introduced in [30] as a means to address this problem. Maxi-adjustment, defined below, allows the user to specify dependencies among beliefs using Spohnian reasons [22]. An algorithm for maxi-adjustment is given in [31], and an implementation of both adjustment and maxi-adjustment is available at world wide web address http://infosystems.newcastle.edu.au/webworld/saten.

We use maxi-adjustment for the software engineering application in section 3.2 to perform the three most common types of design revision. Maxi-adjustments change a partial entrenchment ranking under *maximal inertia* in accordance with the principle of Minimal Change. In other words, maxi-adjustments modify a ranking in a minimal way while maintaining as much of the content of the original ranking as possible. In contrast to adjustments, maxi-adjustments assume by default that information is *independent* with respect to change, unless dependence is derivable from the explicitly specified information.

The system designer specifies all the known information dependencies using the notion of one sentence being the *reason* for another. Maxi-adjustment uses these explicit dependencies to determine the information to be retracted. Essentially, φ is a reason for ψ with respect to $\mathbf{B}$ whenever $\mathrm{degree}(\mathbf{B}, \psi) < \mathrm{degree}(\mathbf{B}, \varphi \rightarrow \psi)$.

A maxi-adjustment involves the absolute minimal change of a partial entrenchment ranking required to incorporate the desired new sentence such that currently held information is retained unless there is an explicit reason to retract it. The maxi-adjustment of $\mathbf{B}$ defined below is suitable for modeling changes to finite knowledge bases where the systems designer specifies all the dependencies, and independence can be assumed otherwise. It involves successive calls to the function degree for each sentence in the domain of $\mathbf{B}$.

Definition 18. *Let $\mathbf{B} \in \mathcal{B}$ be finite. We enumerate the range of $\mathbf{B}$ in ascending order as $j_0, j_1, \ldots, j_{\mathcal{R}_{\max}}$. Let φ be a contingent sentence, $j_m = \mathrm{degree}(\mathbf{B}, \varphi)$ and $0 \leq i < \mathcal{R}_{\max}$. Then the (φ, i)*-maxi-adjustment of $\mathbf{B}$ *is $\mathbf{B}^\star(\varphi, i)$ defined by:*

$$\mathbf{B}^\star(\varphi, i) = \begin{cases} (\mathbf{B}^-(\varphi, i)) & \textit{if } i \leq \mathrm{degree}(\mathbf{B}, \varphi) \\ \\ (\mathbf{B}^-(\neg\varphi, 0))^+(\varphi, i) & \textit{otherwise} \end{cases}$$

where for all $\psi \in dom(\mathbf{B})$, we define $\mathbf{B}^-(\varphi, i)$ as follows:

1. *For ψ with $\mathbf{B}(\psi) > j_m$ we have $\mathbf{B}^-(\varphi, i)(\psi) = \mathbf{B}(\psi)$.*
2. *For ψ with $i < \mathbf{B}(\psi) \leq j_m$, suppose we have defined $\mathbf{B}^-(\varphi, i)(\psi)$ for ψ with $\mathbf{B}(\psi) \geq j_{m-k}$ for $k = -1, 0, 1, 2, \ldots, n-1$, then for ψ with $\mathbf{B}(\psi) = j_{m-n}$ we have*

$$\mathbf{B}^-(\varphi, i)(\psi) = \begin{cases} i & \text{if } \ \varphi \vdash \psi \ \text{ or} \\ & \quad \varphi \not\vdash \psi \ \text{ and } \ \psi \in \Gamma \\ & \quad \text{where } \ \Gamma \ \text{ is a minimal subset of} \\ & \quad \{\psi : \mathbf{B}(\psi) = j_{m-n}\} \ \text{ such that} \\ & \quad \{\psi : \mathbf{B}^-(\varphi, i)(\psi) > j_{m-n}\} \cup \Gamma \vdash \varphi \\ \mathbf{B}(\psi) & \text{otherwise} \end{cases}$$

3. *For ψ with $\mathbf{B}(\psi) \leq i$ we have $\mathbf{B}^-(\varphi, i)(\psi) = \mathbf{B}(\psi)$.*

and for all $\psi \in dom(\mathbf{B}) \cup \{\varphi\}$ we define $\mathbf{B}^+(\varphi, i)$ as follows:

$$\mathbf{B}^+(\varphi, i)(\psi) = \begin{cases} \mathbf{B}(\psi) & \text{if } \ \mathbf{B}(\psi) > i \\ i & \text{if } \ \varphi \equiv \psi \ \text{ or} \\ & \quad \mathbf{B}(\psi) \leq i < \mathrm{degree}(\mathbf{B}, \varphi \rightarrow \psi) \\ \mathrm{degree}(\mathbf{B}, \varphi \rightarrow \psi & \text{otherwise.} \end{cases}$$

It is shown in [30] that if i is greater than zero then content($\mathbf{B}^\star(\varphi, i)$) satisfies the postulates for revision, and similarly content($\mathbf{B}^\star(\varphi, 0)$) satisfies all bar the recovery postulate for contraction.

It can be shown that both adjustment and maxi-adjustment retain all sentences with higher rank when we retract a sentence, i.e. $\{\psi \in \exp(\mathbf{B}) : \text{degree}(\mathbf{B}, \varphi) < \mathbf{B}(\psi)\} \subseteq \text{content}(\mathbf{B}^\star(\varphi, 0))$.

Thus, when considering the desirable behaviour of contractions and revisions (more precisely transmutations) in the design process, we only need to consider the effects on sentences at an equal or lower rank than the retracted sentence (or equivalently the negation of the newly introduced sentence).

3.2 A Software Engineering Application

The ability to correctly analyse the impact of changes to system designs is an important goal in software engineering. It has been estimated [25] that in safety-critical systems such as those found in the aerospace industry, as much as 80% of software development costs are consumed in modifications rather than initial development. Without automated assistance, the complexity of these systems makes it difficult to assess the effects of changes in advance.

In [6] Duffy *et al* propose a framework called *Goal-structured Analysis* (GSA) which addresses this problem. The framework promotes the development of logical representations of requirements and design decisions alongside traditional representations, providing a semantic link between requirements that can be used to support automated analysis. The use of natural language processing tools to help in the development of these logical expressions is considered in [20].

From a technical point of view the main result of [6] is the formalisation of conditions under which goals are *supported* by the underlying model, and more particularly the conditions for *local support* which enable a goal structure to be built up incrementally. The latter requires a partial order on sentences, which we will call a *support order*, in which goals must be entailed by statements that strictly precede them. Duffy *et al* briefly discuss changes to goal structures by way of an example involving aircraft landing control but do not provide any formal machinery for changing the structure.

In this section we summarize the framework developed in [18] which demonstrated the usefulness of belief revision as a way of providing a formal mechanism for the revision of user specifications in top-down designs and, in particular, goal structures.

GSA is a design and analysis methodology based on goal decomposition. In order to give our presentation of goal structures based on epistemic entrenchments, we will require some terminology from [6]. However, we will have space here only to briefly introduce some aspects of the framework which are pertinent to the discussion as described in [18]: for a fuller motivation and discussion the reader is referred to [6].

A goal structure consists of a directed acyclic graph of *frames*, each of which contains a number of fields of information. This will include an *assertion* or statement and may include additional information such as the system to which the assertion refers, stakeholders in the assertion, a decomposition of the assertion and so on. As well as information expressed in natural language, the frame will contain corresponding fields which express some of the information as logical sentences, and it is this representation that will interest us in this paper.

There are four different classes of frames and corresponding logical assertions:

goals are statements of what the system should achieve, and can be decomposed into subgoals which will bring about their achievement.

effects are similar to goals, and identical from a logical point of view — the difference is that although they still require decomposition, they may be undesirable.

facts are statements that require no further decomposition — they may for example be always true, be trivially implementable, or be passed as goals to the design team of a different subsystem.

conditions are like facts in that they form the “leaves” of the structure, however, they may be true in some scenarios and false in others, representing, for example, environmental conditions.

Formally a *scenario* is a set of conditions. In a complete structure the satisfaction of a goal (or effect) should follow from just facts and conditions. In this case the goal is said to be *(globally) supported*. During the process of top-down decomposition, however, there is no way of knowing whether a goal will eventually be globally supported. One of the aims of [6] is to identify constraints on incremental development which, if followed by the designer, will ensure global support in the final structure. Goals satisfying these constraints are said to be *locally supported*.

One of the main results in [6] is a proof that local support leads to global support. In order to achieve this a partial order on goals, $\prec_G$, is developed in order to prevent support “loops”. For a goal to be locally supported, it must be entailed by assertions lower in the partial order.

Each stage in the development of a goal structure consists of the decomposition of a goal or effect into goals, effects, facts or conditions which are strictly lower than that goal or effect in the partial order. This is illustrated by the following two examples taken from [6]. This example is derived from accounts of an actual incident in which a plane over-ran a runway when landing in poor conditions in Warsaw. The strategies outlined below are simplifications of alternative strategies which have been used in aircraft.

In the first example, the strategy for satisfying the goal is based on existing facts. For example, assume our goal is to bring an aircraft to a stop within 1000m. We know from information about the dynamics of the aircraft, or perhaps tests on previous models of similar weight, that applying the wheel brakes is sufficient providing the speed of the aircraft is less than 154km/h. This is embodied in the following fact:

Fact	
Label	Fact1
System	Aircraft dynamics
Assertion	The aircraft will stop within 1000m if wheel brakes are applied when speed $\leq$ 154km/h.
Assertion*	$(\texttt{speed} \leq 154) \wedge \texttt{applied}(\texttt{wheel.brakes}) \rightarrow (\texttt{stop.length} < 1000)$
Rationale	Engineering tests

The logical assertion in this frame represents a *causal rule*. On the basis of this we can develop a goal frame such as the following:

Goal	
Label	Goal1
System	Landing control
Assertion	Stop aircraft within 1000m
Assertion*	$\texttt{stop.length} < 1000$
Selection	Apply wheel brakes when speed $\leq$ 154 km/h
Rationale	Based on Fact1

The 'selection' field refers to a selected strategy — this, in conjunction with the fact speed $\leq$ 154 km/h, ensures that the goal is satisfied. The selection will in turn be treated as a goal and decomposed.

In the second example the designer of the control system has more scope — it is up to him to decide upon the conditions under which the reverse thrusters will fire. Two alternatives - wheel.loads > 12, and (altitude <10)$\wedge$ (wheel.speed > 72) - are shown.

Goal	
Label	Goal2
Assertion*	$\texttt{applied}(\texttt{reverse.thrust})$
Alternatives	$\texttt{wheel.loads} > 12$; $(\texttt{altitude} < 10) \wedge (\texttt{wheel.speed} > 72)$
Selection	$\texttt{wheel.loads} > 12$

The selection will be implemented and therefore lead to the following causal rule:

Fact	
Label	Fact2
Assertion*	$(\texttt{wheel.loads} > 12) \rightarrow \texttt{applied}(\texttt{reverse.thrust})$

Notice that both of the fact examples include a causal rule which makes explicit the relationship between assertions and selected strategies in the goals. This will always be the case, and is in fact necessary from a deductive point of view. In [6] a causal rule is not treated differently to any other fact. In our treatment it will be necessary to treat causal rules as a special case. We argue, however, that this is a natural thing to do. Causal rules do clearly perform a different, more "implicit", role to other facts.

From a practical point of view we do not want to think explicitly about the causal rule at all. Rather we would like to enter alternative strategies through a template such as that illustrated in Figure 1 [6], and automate the process of generating appropriate causal rules. In the example shown in Figure 1, the second alternative for goal Goal2 above is highlighted, while the underline indicates that the first alternative has been selected.

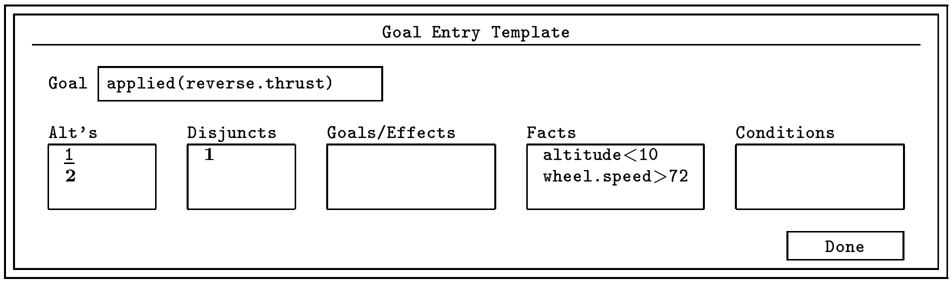

Fig. 1. A template for entering goals and strategies for their decomposition.

If we give special status to causal rules, then a finite entrenchment ranking can be quite naturally applied. In the following, $\mathcal{R}_{\mathrm{c}}$, $\mathcal{R}_{\mathrm{f}}$, $\mathcal{R}_{\mathrm{g}}$ and $\mathcal{R}_{\mathrm{r}}$ represent such that

$$0 < \mathcal{R}_{\mathrm{c}} < \mathcal{R}_{\mathrm{f}} < \mathcal{R}_{\max}.$$

Conditions are ranked in the interval $(0, \mathcal{R}_{\mathrm{c}}]$, facts are ranked in the interval $(\mathcal{R}_{\mathrm{c}}, \mathcal{R}_{\mathrm{f}}]$, goals and effects are ranked in the interval $(\mathcal{R}_{\mathrm{f}}, \mathcal{R}_{\mathrm{r}})$, and causal rules are ranked in the interval $[\mathcal{R}_{\mathrm{r}}, \mathcal{R}_{\max})$.

Definition 19. *Let* $\mathbf{B}$ *be a partial entrenchment ranking. An assertion* g *is* globally supported *(with respect to a scenario) iff there exists a set of assertions* $S = \{s_1, \ldots, s_m\}$ *such that:*

GS1 *For each* $s_i \in S$*, either* $0 < \mathrm{degree}(\mathbf{B}, s_i) \leq \mathcal{R}_{\mathrm{f}}$ *or* $\mathrm{degree}(\mathbf{B}, s_i) \geq \mathcal{R}_{\mathrm{r}}$.
GS2 $S \models g$.

We say that g is globally supported by the set S. Of course a goal or effect may be supported by more than one assertion, or set of assertions.

Definition 20. *Let* $\mathbf{B}$ *be a partial entrenchment ranking. An assertion* g *is* locally supported *(in a scenario) iff there exists a set of assertions* $S = \{s_1, \ldots, s_m\}$ *such that:*

LS1 *For each* s_i*, either* $0 < \mathrm{degree}(\mathbf{B}, s_i) < \mathrm{degree}(\mathbf{B}, g)$ *or* $\mathrm{degree}(\mathbf{B}, s_i) \geq \mathcal{R}_{\mathrm{r}}$.
LS2 $S \models g$.

We say that g is locally supported by the set S.

In [6] it is shown that local support implies global support. This is stronger than we need. In general we will not require that all goals are (locally or globally) supported in a particular scenario, but rather that there is a "tree" of support for certain high-level goals.

From a practical vantage, we would like a simple syntactic characterisation of support. That is, we would like to be able to decompose goals in an intuitive way — by coming up with alternative strategies, selecting one, and placing its component assertions in designated positions in the entrenchment ranking — without the need to show entailment. We would also like to be able to develop tools that aid in this process which avoid the computational complexity normally associated with theorem proving.

GSA allows one to do this in a straightforward way if we are willing to place tighter constraints on the form of causal rules. Given the above intuition — that we would like to form alternative strategies and select one — a natural constraint is to require that the antecedents of causal rules are expressed in disjunctive normal form. That is, our causal rules will be of the form

$$\text{alternative1} \vee \cdots \vee \text{alternative n} \rightarrow \text{goal}$$

where each alternative is a conjunction of literals. Conditions for goal support can then be defined syntactically as follows.

Definition 21. *An assertion g is* causally supported *in a partial entrenchment ranking* $\mathbf{B}$ *iff there exists a dnf sentence $d = c_1 \vee \cdots \vee c_m$ such that:*

CS1 *$d \rightarrow g$ is asserted as a causal rule — therefore $\mathcal{R}_{\mathrm{r}} \leq \mathrm{degree}(\mathbf{B}, d \rightarrow g)$.*
CS2 *For some disjunct $c_i = s_1 \wedge \cdots \wedge s_l$ appearing in d, each s_j appearing in c_i is asserted such that $\mathrm{degree}(\mathbf{B}, s_j) < \mathrm{degree}(\mathbf{B}, g)$.*

We say that the goal or effect g is causally supported by the set $\{s_1, \ldots, s_l, d \rightarrow g\}$. As is the case for local support, an assertion may be causally supported by more than one set.

It is a simple matter to show that these conditions are sufficient to guarantee local support and hence global support. MacNish and Williams [18] showed that if g is causally supported in $\mathbf{B}$ then g is locally supported.

Finally, although it is not necessary for the above arguments, for convenience conditions are separated from facts by placing them lower in ranking. This accords with the original notion of epistemic entrenchment, in which the least entrenched sentences are those that we are most willing to give up.

3.3 An Aircraft Landing Control System Example

Consider the example in Figure 2 [18]. It is easy to see that goal G1 is locally supported under the conditions shown, since it is causally supported by R1, G3 and G4. Similarly G3 is supported by R4, F1 and F2; and G4 is supported by R7, C1 and C3. G1 is therefore globally supported. G5 on the other hand is not causally supported under these conditions.

Note that it is only the relative ranking of sentences that is important. The absolute values of $\mathcal{R}_{\mathrm{c}}$, $\mathcal{R}_{\mathrm{f}}$ and so on are unimportant, and the assertions between any pair of horizontal lines may be equally ranked or ordered arbitrarily.

$\mathcal{O}_{\max}$		
	Causal Rules	
	speed $\leq$ 154 $\wedge$ applied(wheel.brakes) $\rightarrow$ stop.length $<$ 1000	R1
	154 $<$ speed $\leq$ 170 $\wedge$ applied(wheel.brakes) $\wedge$ applied(reverse.thrust) $\rightarrow$ stop.length $<$ 1000	R2
	wheel.loads $>$ 12 $\rightarrow$ applied(reverse.thrust)	R3
	wheel.loads $>$ 12 $\vee$ altitude $<$ 10 $\wedge$ wheel.speed $>$ 72 $\rightarrow$ applied(wheel.brakes)	R4
	cross.wind $\wedge$ banked $\rightarrow$ wheel.loads $>$ 12	R5
	tail.wind $\wedge$ level $\rightarrow$ wheel.loads $>$ 12	R6
	cross.wind $\wedge$ throttle(high) $\rightarrow$ speed $\leq$ 154	R7
	tail.wind $\wedge$ throttle(high) $\rightarrow$ 154 $<$ speed $\leq$ 170	R8
$\mathcal{O}_{\mathrm{r}}$		
	Goals and Effects	
	stop.length $<$ 1000	G1
	applied(reverse.thrust)	G2
	applied(wheel.brakes)	G3
	speed $\leq$ 154	G4
	154 $<$ speed $\leq$ 170	G5
	wheel.loads $>$ 12	G6
$\mathcal{O}_{\mathrm{f}}$	**Facts**	
	altitude $<$ 10	F1
	wheel.speed $>$ 72	F2
$\mathcal{O}_{\mathrm{c}}$	**Conditions**	
	cross.wind	C1
	banked	C2
	throttle(high)	C3
0		

Fig. 2. A partial entrenchment ranking for the landing control system.

Having replaced the partial order on assertions with an entrenchment ranking, we now turn to design revision and show that the entrenchment ranking supports design revision using maxi-adjustment. In particular, we illustrate three types of revisions: testing alternative scenarios, revising goals and effects, and retracting causal rules.

Testing Alternative Scenarios

The most common, and simplest change to a goal structure results from a "what if?" analysis — testing different scenarios to see whether goals are still supported. This can be illustrated by the landing control example from [6].

In this (simplified) account, the pilot is informed of a cross-wind and takes the appropriate action of banking the aircraft and increasing throttle, thus giving the conditions shown in Figure 2. As we saw earlier, the main goal, that the aircraft stop within 1000m, is supported under these conditions.

Just as the aircraft is making its approach, however, the wind swings around to a tail wind. In terms of the content of the ranking, condition C1 is retracted (given rank 0) and `tail.wind` is asserted. This causes a coincidence of two things. First, the tail wind and high throttle lead to a greater speed (R8), which in turn means that reverse thrusters are required (R2). Secondly, because the aircraft is banked without a cross-wind, the weight is skewed and one of the wheels does not register a weight of 12 tonnes. As a result, the reverse thrusters are not applied (R3).

From belief revision point of view, the conditions are revised by performing maxi-adjustments $\mathbf{B}^{\star}(\varphi, i)$, where a is a condition and $0 \leq i \leq \mathcal{R}_{\mathrm{c}}$. In the above example, for instance, we are making the adjustments

$$[\mathbf{B}^{\star}(\texttt{cross.wind}, 0)]^{\star}(\texttt{tail.wind}, \mathcal{R}_{\mathrm{c}}).$$

Using maxi-adjustment we can vary conditions without affecting the rest of the design: that is, the causal rules, goals, effects or facts. Information ranked equally or less than the information to be revised will be retracted *only if* it is a reason for the information to be revised. Thus other unrelated conditions will not be affected.

Revising Goals and Effects

The second type of revision we consider is the revision of goals and effects. This will occur when part of a design is considered to be inadequate and an alternative top-down decomposition path is sought.

A typical example would be where a causal rule with disjunctive antecedents has been used (such as rule R4 in Figure 2), the decomposition from treating one of the disjuncts as a goal has proved unsuccessful, and an alternative disjunct is to be treated as a goal.

Retracting the old goal should satisfy two constraints. First, neither the causal rule in which it appears, nor the goal representing the consequent of that rule, should be retracted. This follows once again from conditions **CS1** and **CS2**, respectively. They ensure that the retracted assertion appears lower in the entrenchment ranking than either the causal rule or its consequent.

Second, we would like to retract that part of the design (if any) which resulted from decomposing the disjunct. In the simplest case this is handled automatically by the maxi-adjustments. This is illustrated in Figure 3 [18], where goal χ has been decomposed to a subgoal ψ, which in turn is decomposed to φ. If χ is retracted, then ψ is also eliminated since $\mathrm{degree}(\mathbf{B}, \chi) < \mathrm{degree}(\mathbf{B}, \psi \rightarrow \chi)$. Although $\varphi \rightarrow \chi$ is not in $\exp(\mathbf{B})$, it follows from (PER1), along with the fact that $\{\varphi \rightarrow \psi, \psi \rightarrow \chi\} \models \varphi \rightarrow \chi$, that it must be ranked at least as highly as $\varphi \rightarrow \psi$. Therefore φ is eliminated also.

This "transitive" behaviour will continue so that the subtree in the design rooted at goal c is retracted. Note that this transitive behaviour does not necessarily occur with other transmutation strategies.

The case where the retracted goal is the consequent of a causal rule with disjunctive antecedents will not cause a problem, since that causal rule will be equivalent to the conjunction of two or more rules with just the individual

$\mathcal{O}_{\max}$

Causal Rules

$\psi \rightarrow \chi$

$\varphi \rightarrow \psi$

$\vdots$

$\mathcal{O}_{\mathrm{r}}$

Goals and Effects

χ

ψ

φ

$\vdots$

Fig. 3. A simple example of "transitivity" in retraction of goals.

disjuncts as antecedents. Again (PER1) will ensure that these new rules have rank at least as high as the original causal rule.

There are two cases, however, which do require further development of the maxi-adjustment strategy. First, in the case where the retracted goal is the consequent of a causal rule with conjunctive antecedents, the maxi-adjustment strategy is not sufficient on its own. The maxi-adjustment ensures only that the subtree rooted at *at least one* of the conjuncts is retracted. Secondly, since the design is strictly speaking a directed acyclic graph rather than a tree, we need to consider what happens when assertions are shared by multiple design "trees".

Returning to the landing example used earlier, Duffy *et al* go on to show that an alternative choice of strategy for the reverse thrust goal re-supports the top level goal. In particular, the coincidence of events leading to failure can be traced back to the fact that the reverse thrusters did not fire when required. This in turn was caused by the choice of wheel loads as the impetus for making the reverse thrusters fire. The designers may wish to examine the alternative strategy of firing on the basis of altitude and wheel speed.

Revising Causal Rules

Choosing an alternative strategy requires the retraction of a causal rule, followed by expansion with respect to a new causal rule. Typically no further goals or facts will be retracted with a causal rule, since this would require a further implication at a higher rank than the causal rule. Since only tautologies are ranked higher this would indicate redundancy in the design. The designer has the option of removing the subtree related to the causal rule, however, simply by retracting the consequent (goal or effect) as described above prior to retracting the causal rule.

4 System of Spheres

An epistemic state can be represented by a total preorder on possible worlds, and Grove [11] and Spohn [22] developed mechanisms for building revision functions based on this representation. In essense, the preorder on possible worlds captures the agent's degrees of belief where world W_1 resembles the real world more than W_2 if W_1 is lower in the preorder than W_2. All of the least worlds must be consistent with the agent's knowledge, and it is understood that the agent expects that any of the least worlds could represent the actual world (according to their present knowledge). Typically more than one possible world is consistent with the agent's knowledge, in fact the less the agent knows the greater the number of possible worlds that are consistent with it. This representation based on possible worlds or world states is called a *system of spheres* [11]. If instead of using a simple preorder we map worlds to ordinals then the representation is referred to as an *ordinal conditional function* (OCF) [22].

Spohn [22] introduced the process of conditionalisation on OCF's as a means of performing iterated belief revision. During a conditionalisation an OCF is rearranged so as to accept new, possibly inconsistent, incoming information. It was shown in [32] that conditionalisation can be implemented using standard relational database technology, and we illustrate the ideas using a real world application in Marketing Research. In particular, belief revision can be used to model changes in consumer preferences, and hence used to predict consumer behaviour effortlessly. In particular, it was shown in [32] that belief revision can be implemented elegantly and efficiently using a relational database to represent the system of spheres, and revision is performed using standard SQL. As a consequence, marketers can use hypothetical reasoning to generate numerous scenarios based on alternative types of new information and thus experiment with changes in consumer preferences. In this way they will be able to determine the most effective information to provide the consumer so as to obtain desired consumer behaviour.

Extensive work has been carried out in the literature that focuses on the static nature of consumer preferences, however it is becoming increasingly obvious that consumer preferences *change* over time. For instance, Bolton and Drew [2]conducted a study to investigate how customers' evaluations of service quality are influenced by changes in service offerings. Their study concentrated on temporal changes in individual consumer attitudes. Similarly Moschini [15] explored changing preferences for meat products due to consumers' awareness of the health hazards of cholesterol and saturated-fat intake.

Conjoint analysis focuses on a static set of consumer preferences and is not designed to be longitudinal in nature. Thus firms must *recollect* preferences if they want to take consumer's preference changes into consideration. In practice they almost never recollect the data because it is an expensive process. Consequently, applying the belief revision process to consumer attitudes and preferences enables marketers to make effective informed decisions more efficiently by short circuiting the recollection process.

Representing epistemic *attitude* and *preferences*, and modeling consumer *behaviour* are fundamental issues for Marketing Research. According to Myers and Reynolds [16] attitudes directly effect purchase decisions which in turn affect attitudes through the experience of the product. Therefore organisational decisions that consider the development of new products or the modification of attributes of existing product offerings must be cognizant of consumer epistemic attitudes and preferences.

Consumer preferences are constantly changing. For example, changes in preference can occur when a consumer acquires new information about a product, and when their desires, intentions, beliefs, expectations, or needs change.

In the current context we represent the consumer as a reasoning agent in a particular *epistemic state*. More precisely, we model consumer epistemic attitudes using a preference relation over possible product profiles. The more firmly a consumer prefers a product profile, the lower in the ranking it is. The most preferred product profiles are ranked zero. Belief revision techniques allow us to model the changing preferences of a consumer, thereby enhancing the decision making process, and in turn supporting more effective marketing.

A ranking of all possible product profiles fully characterizes a consumer's epistemic state with respect to the product offerings and prescribes its desired dynamic properties.

Conditionalisation was introduced by Spohn [22], it is based on a *relative* measure of minimal change.

A consumer possesses a set of beliefs about the products he ranks. A *consistent* belief set is deductively closed and does not contain both φ and $\neg\varphi$, for any sentence φ of $\mathcal{L}$. A *complete* belief set contains either φ or $\neg\varphi$ for all sentences φ. A belief set is *finite* if the consequence relation $\vdash$ partitions its elements into a finite number of equivalence classes. We introduce the following notation:

- Θ is the set of all possible world states. For the purpose of this paper a possible world will be represented by a full product profile, hence Θ is the set of all product profiles.
- If φ is a sentence of $\mathcal{L}$, define $[\varphi]$ to be the set of all consistent complete product profiles containing φ. If φ is inconsistent, then $[\varphi] = \emptyset$, and if $\vdash \varphi$, then $[\varphi] = \Theta$.

Ordinal conditional functions are preference rankings, and map possible world states to ordinals. For the application under consideration possible world states correspond to possible product profiles, we will use these terms interchangably for the remainder of the discussion. Intuitively, they define a ranking of the respective domains providing a response schema for all possible consistent information [22]. We use ordinal conditional functions as the formal representation of consumer's preference ranking over product profiles.

We define an *ordinal conditional function*, OCF, as a function C from Θ, the set of all possible world states into the ordinals such that there is some element of Θ assigned the smallest ordinal 0.

We denote the family of all ordinal conditional functions by $\mathcal{C}$. Intuitively $\mathrm{C} \in \mathcal{C}$ represents a *preference* grading of possible product profiles; the profiles that are assigned zero are the most preferred by an individual consumer.

The ordinal assigned to a set of consistent complete theories $\Delta \subseteq \Theta$ is the smallest ordinal assigned to the elements of Δ, and it is denoted by $\mathrm{C}(\Delta)$, i.e. $\mathrm{C}(\Delta) = \min\{\mathrm{C}(K) : K \in \Delta\}$. For a set of states $\Delta \subseteq \Theta$ we define $\overline{\Delta}$ to be the complement of Δ, that is $\Theta \backslash \Delta$. Hence $\mathrm{C}(\overline{[\varphi]}) = \mathrm{C}([\neg\varphi])$.

We refer to $\mathrm{C}([\neg\varphi])$ as the *degree of preference* of φ. Intuitively, the degree of preference of φ is given by the smallest rank assigned to a possible world that does not contain φ, i.e the least ranked profile that is inconsistent with φ.

The set of beliefs determined by an epistemic state consists of all the information whose negation has a degree of preference greater than zero. More formally, we define the *belief set* represented by $\mathrm{C} \in \mathcal{C}$ to be $\{\varphi : \mathrm{C}([\neg\varphi]) > 0\}$, and denote it by content(C). Another way of defining content(C) is to take the intersection of all the profiles ranked at zero, i.e. the intersection of the most preferred profiles.

The information input for iterated revision is a sentence φ which represents the new information to be accepted, and the result of a revision is a theory containing φ, and *not* a new preference ranking.

We now define a conditionalisation [22] of OCF's where information inputs are composed of an ordered pair, $([\varphi], i)$, namely a sentence φ and an ordinal i. The interpretation of this is that φ is the information to be accepted by the ranking, and i is the degree of preference with which this new information is incorporated into the conditionalised ranking. The result of a conditionalisation is a new OCF.

$$\mathrm{C}^*(\Delta, i)(K) = \begin{cases} -\mathrm{C}(\Delta) + \mathrm{C}(K) & \text{if } K \in \Delta \\ \\ -\mathrm{C}(\overline{\Delta}) + \mathrm{C}(K) + i & \text{otherwise.} \end{cases}$$

We say $\mathrm{C}^*(\Delta, i)$ is a (Δ, i) – conditionalisation of C. Note that the definition excludes transmutations with respect to contradictory and tautological information. In this way we restrict our attention to the principal cases.

It can be easily shown that if $\Delta \subset \Theta$, $\Delta \neq \emptyset$, and $i > 0$ then content$(\mathrm{C}^*(\Delta, i))$ is the result of an AGM revision, and if $i = 0$ then an AGM contraction.

Spohn has argued [22]that conditionalisation is a desirable transmutation, for instance it is reversible and commutative. Intuitively, conditionalisation means that becoming informed about Δ does not change the grading of preference restricted to either Δ or $\overline{\Delta}$, rather the worlds in Δ and $\overline{\Delta}$ are shifted relative to one another.

4.1 Conjoint Analysis

Conjoint analysis is used to estimate the willingness of consumers to trade off varying levels of product attributes on the basis of their preferences. The technique has been shown to be valuable in the design of products, the design of advertisements, the redesign of products, and for supporting pricing and branding decisions. Firstly information concerning consumer preferences is obtained via some form of data collection process which results in the determination of a preference ranking. Conjoint analysis is then used to find utility functions that represent the willingness of consumers to trade off product attibutes. These functions relate attribute levels to global multiattribute evaluations.

Conjoint analysis techniques are applicable to consumer preferences regarding nonhabitual products that can be adequately described using a set of attributes. An underlying assumption behind conjoint analysis is that the attributes under study are more or less independent as perceived by the consumers.

The *data collection process* is straightforward, relatively simple, and usually takes one of two forms; *pairwise* attribute ranking, or *full profile* rankings. Care must be exercised to ensure that data collection devices and consumers do not confuse attribute meaning, attribute importance and attribute determinance [23]. There has been extensive debate and discussion regarding the various tools that can be used to undertake conjoint analysis. Whilst these are important they do not influence the applicability of belief revision to conjoint analysis and will not be discussed in this paper. Data collection on the other hand is important for belief revision. Indeed the utility of the resulting preference ranking is directly proportional to the quality of the information gathered about consumer attitudes during the data collection phase.

4.2 A Marketing Research Example

A firm intends introducing a new cat food designed specifically to met the needs of mother cats with kittens [24]. They determine that the following attributes affect cat owner's willingness to use the product: *form* (dry, moist, canned), *brand name* (MamaCat, Formula 9, Plus), *price* (55c, 65c, 75c), *endorsement* from a veterinarian association (yes, no), money-back *guarantee* if not satisfied (yes, no).
Management is interested in determining:

(i) Which combination of these attributes will produce the highest level of trial?
(ii) Which attributes are the most important?
(iii) Will consumer's be willing to pay 75c rather than 65c if the brand has an endorsement, all other things being equal?

The pairwise approach to conjoint data collection involves the ranking of all pairwise combinations by consumers. For N attributes there are $N(N-1)/2$ possible pairs.

Full profile conjoint data collection involves presenting consumers with a set of product descriptions which contain information about each attribute to be ranked. Ranking all potential product descriptions is prohibitive in practice, and a fractional factorial orthogonal table of profiles is used [10]. There exists several computer software packages that design a set of orthogonal tables.

Conjoint analysis measurement procedures take the consumer preference rankings and calculate part-worth utility functions, the details of which can be found in numerous sources, for example Green *et al* [9]. Decisions concerning product attributes and marketing strategies are then made based on an analysis of various utility function combinations.

Based on preliminary experiments conditionalisation is a reasonable approximation to the process consumers use when they transmute their preferences over a *small* set of *consecutively ranked* product profiles. To illustrate conditionalisation let us consider the scenario where a cat owner who initially prefers moist cat food, changes his attitude because he has discovered the convenience of dry cat food. In particular, he would like to change his preferences so that dry food is preferred with degree 2. Let us consider the case where the current lowest rank of a dry cat food product is 6, therefore we can say that $\mathrm{C}([form = \text{“}dry\text{”}]) = 6$. Since the cat owner previously prefered moist over dry food we know that the current degree of $form = \text{“}dry\text{”}$ is 0, i.e. $\mathrm{C}([form \neq \text{“}dry\text{”}]) = 0$.

In order to conditionalise the ranking so as to achieve the goal of accepting $form = \text{“}dry\text{”}$ with degree 2 we must (i) reduce the rank of all products in which the form is dry by 6, and (ii) increase the rank of all products in which the form is *not* dry by 2.

Conditionalisation makes the most preferred dry product profile become the most preferred product, indeed all the products with dry form are adjusted by subtracting 6. The smallest rank assigned to nondry forms is 2, and conditionalisation adjusts all the nondry profiles in the same way, i.e it adds 2.

Belief revision can be efficiently implemented using standard relational database technology. To see how one needs only make the following observations:

- A world state (or product profile) can be represented as a single tuple in a relation. For example, cat_food(brand_name, form, price, veterinary_endorsement, guarantee)
- A ranking of world states can be represented using a single numerical attribute in each tuple. For example, cat_food(brand_name, form, price, veterinary_endorsement, guarantee, **rank**)
- The reranked tuples provide a means to achieve a standard AGM revision, or standard AGM contraction. In particular, tuples can be reranking independently of one another. How each tuple is reranked depends entirely on its relationship with the incoming information, i.e. the goal of the transmutation.

Once the conceptual framework above has been established it becomes a straightforward task to define belief revision in terms of relational database SQL update. Let C be a relation with a distinguished attribute labelled **rank**, such that at least one tuple has **rank** = 0. Let φ be a standard database (in)equality statement such as ‘form = “dry”’, or ‘price < \$3.00’. More precisely, φ can be any standard compound relational database selection criteria which may include conjunctions and/or disjunctions. Finally, let i be an integer representing the degree of acceptance.

Conditionalisation of the relation C can then be defined using the following SQL transaction composed of two update queries. The first query lowers the rank of world states that are consistent with φ, and the second query raises the rank of those world states which are inconsistent with φ.

```
UPDATE rank = rank - (  SELECT rank
                        FROM C
```

```
                        WHERE φ
                        GROUP BY rank
                        HAVING minimum(rank))
FROM C
WHERE φ


UPDATE rank = rank + i
FROM C
WHERE not φ
```

5 Discussion

We outlined the AGM paradigm by describing the axiomatic approach to belief revision, and two constructive approaches, namely the epistemic entrenchment construction, and the the systems of spheres construction of belief revision functions.

We described how transmuting an epistemic entrenchment ranking can be used as a sound basis for implementing iterated revision using a standard theorem prover, and described how belief revision can be used for requirements analysis in software engineering [18] using the Goal Structure Analysis of [6].

We also described how transmuting an ordinal conditional function can be used to implement iterated revision using standard relational database technology, and we illustrated this approach by using it to assist market researchers in product design decisions by modeling changes to consumer preferences [35].

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