

Logical equivalences

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$$\begin{aligned}
P \rightarrow Q &\equiv \neg P \vee Q \\
\neg(P \rightarrow Q) &\equiv P \wedge \neg Q \\
P \rightarrow Q &\equiv \neg Q \rightarrow \neg P \\
P \leftrightarrow Q &\equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \\
P \leftrightarrow Q &\equiv (P \wedge Q) \vee (\neg P \wedge \neg Q) \\
P \wedge Q &\equiv \neg(\neg P \vee \neg Q) \\
P \vee Q &\equiv \neg(\neg P \wedge \neg Q)
\end{aligned}$$

Distribution

$$\begin{aligned}
(P \wedge Q) \vee R &\equiv (P \vee R) \wedge (Q \vee R) \\
P \vee (P \wedge Q) &\equiv (P \vee Q) \wedge (P \vee Q) \\
P \wedge (P \vee Q) &\equiv (P \wedge Q) \vee (P \wedge Q)
\end{aligned}$$

Associativity laws

$$\begin{aligned}
P \wedge (Q \vee R) &\equiv (P \wedge Q) \vee (P \wedge R) \\
(P \vee Q) \wedge R &\equiv (P \wedge R) \vee (Q \wedge R) \\
P \vee (Q \wedge R) &\equiv (P \vee Q) \wedge (P \vee R) \\
P \wedge (Q \vee R) &\equiv (P \vee R) \wedge (Q \vee R) \\
(P \vee Q) \rightarrow R &\equiv (P \rightarrow R) \wedge (Q \rightarrow R) \\
P \rightarrow (Q \wedge R) &\equiv (P \rightarrow Q) \wedge (P \rightarrow R)
\end{aligned} \tag{1}$$

De Morgan's

$$\begin{aligned}
\neg(P \wedge Q) &\equiv (\neg P) \vee (\neg Q) \\
\neg(P \vee Q) &\equiv (\neg P) \wedge (\neg Q)
\end{aligned}$$

Quantifiers

$$\begin{aligned}
\forall x P(x) &\equiv \neg \exists x \neg P(x) \\
\exists x P(x) &\equiv \neg \forall x \neg P(x)
\end{aligned}$$

Absorption

$$\begin{aligned}
P \vee (P \wedge Q) &\equiv P \\
P \wedge (P \vee Q) &\equiv P
\end{aligned}$$