Homework4 Report – Image Clustering

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1. (1%) 請使用不同的 Autoencoder model，以及不同的降維方式(降到不同維度)，討論其 reconstruction loss & public / private accuracy。

AE1 Encoder: nn.Conv2d(3, 8, 3, 2, 1)

AE1 Decoder: nn.Sequential(nn.ConvTranspose2d(8, 3, 2, 2), nn.Tanh())

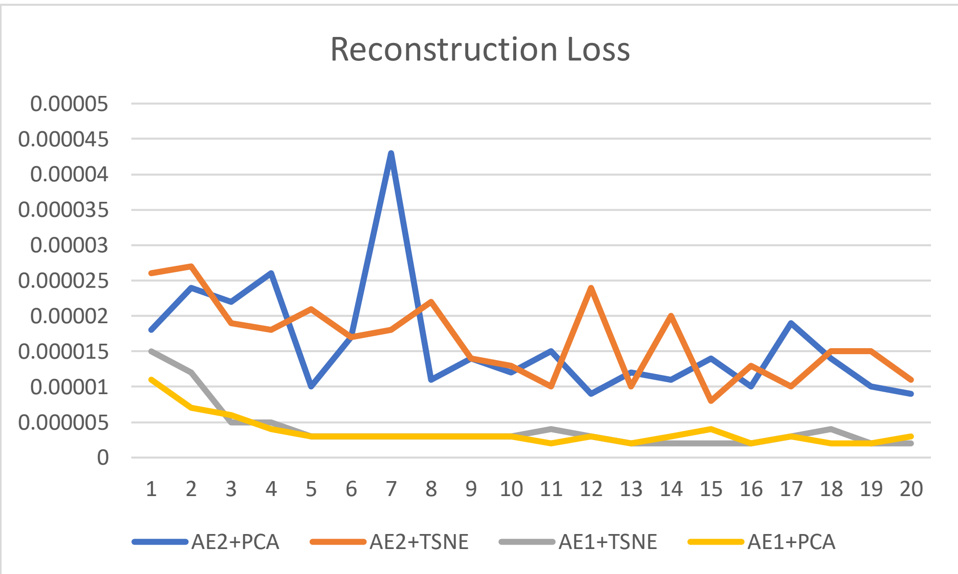
AE2 Encoder: nn.Sequential(

```
            nn.Conv2d(3, 64, 3, 1, 1), nn.ReLU(True),
            nn.BatchNorm2d(64), nn.MaxPool2d(2,2),
            nn.Conv2d(64, 128, 3, 1, 1), nn.ReLU(True),
            nn.BatchNorm2d(128), nn.MaxPool2d(2,2),
            nn.Conv2d(128, 256, 3, 1, 1), nn.ReLU(True),
            nn.BatchNorm2d(256), nn.MaxPool2d(2,2),
            )
```

AE2 Decoder: nn.Sequential(

```
            nn.ConvTranspose2d(256, 128, 2, 2), nn.ReLU(True),
            nn.ConvTranspose2d(128, 64, 2, 2), nn.ReLU(True),
            nn.ConvTranspose2d(64, 3, 2, 2), nn.Tanh(),
            )
```

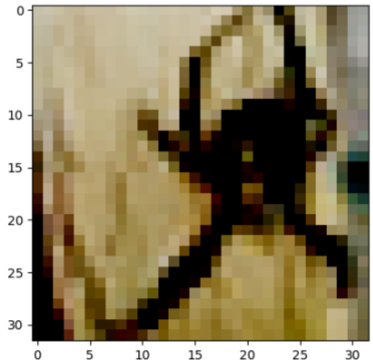
	public score	private score
AE1 + PCA(dim = 32)	0.68259	0.68507
AE1 + TSNE(dim = 2)	0.65000	0.65857
AE2 + PCA(dim = 32)	0.76000	0.76190
AE2 + TSNE(dim = 2)	0.75888	0.75412



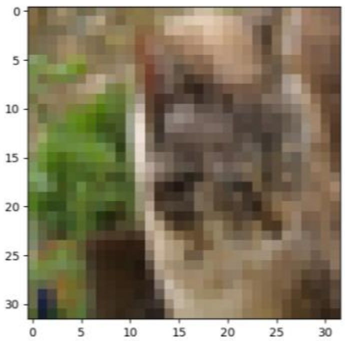
2. (1%) 從 dataset 選出 2 張圖，並貼上原圖以及經過 autoencoder 後 reconstruct 的圖片。

原圖 1:

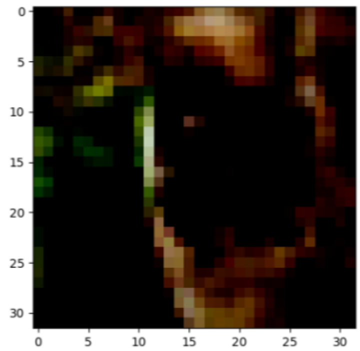
Reconstructed Image1:



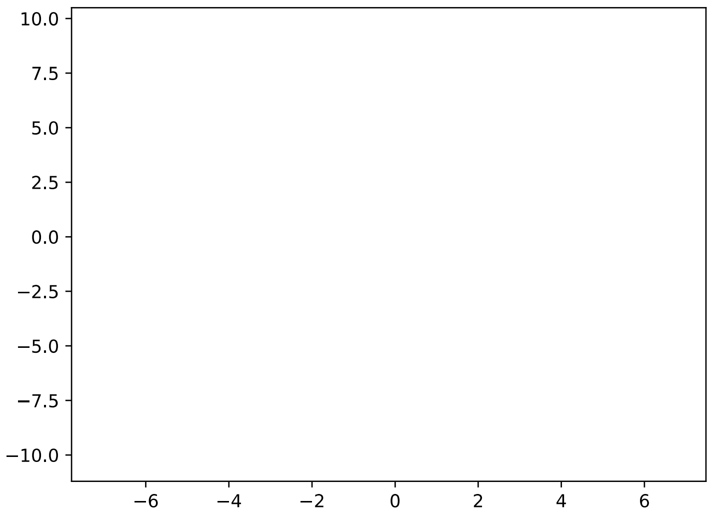
原圖 2:



Reconstructed Image2:



3. (1%) 在之後我們會給你 dataset 的 label。請在二維平面上視覺化 label 的分佈。



4.

(a) 令$X = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 5 \\ 3 & 12 & 9 \\ 1 & 8 & 5 \\ 5 & 14 & 2 \\ 7 & 4 & 1 \\ 9 & 8 & 9 \\ 3 & 8 & 1 \\ 11 & 5 & 6 \\ 10 & 11 & 7 \end{bmatrix}, \mu = [5.4 \quad 8 \quad 4.8] \rightarrow X - \mu = \begin{bmatrix} -4.4 & -6 & -1.8 \\ -1.4 & 0 & 0.2 \\ -2.4 & 4 & 4.2 \\ -4.4 & 0 & 0.2 \\ -0.4 & 6 & -2.8 \\ 1.6 & -4 & -3.8 \\ 3.6 & 0 & 4.2 \\ -2.4 & 0 & -3.8 \\ 5.6 & -3 & 1.2 \\ 4.6 & 3 & 2.2 \end{bmatrix}$

$$\Sigma = \frac{1}{10}(X-\mu)^T(X-\mu) = \begin{bmatrix} 12.04 & 0.5 & 3.28 \\ 0.5 & 12.2 & 2.9 \\ 3.28 & 2.9 & 8.16 \end{bmatrix} = U\Lambda U^T$$

$$= \begin{bmatrix} 0.400 & -0.678 & -0.617 \\ 0.338 & 0.734 & -0.589 \\ -0.852 & -0.027 & -0.523 \end{bmatrix} \begin{bmatrix} 5.472 & 0 & 0 \\ 0 & 11.631 & 0 \\ 0 & 0 & 15.297 \end{bmatrix} \begin{bmatrix} 0.400 & -0.678 & -0.617 \\ 0.338 & 0.734 & -0.589 \\ -0.852 & -0.027 & -0.523 \end{bmatrix}^T$$

Principle Axes =

$[0.400 \quad -0.678 \quad -0.617], [0.338 \quad 0.734 \quad -0.589], [-0.852 \quad -0.027 \quad -0.523]$

(b) $UX^T = \begin{bmatrix} 0.400 & -0.678 & -0.617 \\ 0.338 & 0.734 & -0.589 \\ -0.852 & -0.027 & -0.523 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 5 \\ 3 & 12 & 9 \\ 1 & 8 & 5 \\ 5 & 14 & 2 \\ 7 & 4 & 1 \\ 9 & 8 & 9 \\ 3 & 8 & 1 \\ 11 & 5 & 6 \\ 10 & 11 & 7 \end{bmatrix}^T$

$$= \begin{bmatrix} -2.81 & -6.91 & -12.49 & -8.11 & -8.73 & -0.53 & -7.38 & -4.84 & -2.69 & -7.78 \\ 0.04 & 4.28 & 4.53 & 3.27 & 10.79 & 4.71 & 3.61 & 6.30 & 3.85 & 7.33 \\ -2.47 & -6.24 & -7.59 & -3.68 & -5.69 & -6.60 & -12.59 & -3.30 & -12.65 & -12.48 \end{bmatrix}$$

(c) 令$X^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 5 \\ 3 & 12 & 9 \\ 1 & 8 & 5 \\ 5 & 14 & 2 \\ 7 & 4 & 1 \\ 9 & 8 & 9 \\ 3 & 8 & 1 \\ 11 & 5 & 6 \\ 10 & 11 & 7 \end{bmatrix}, U' = \begin{bmatrix} -0.852 & -0.027 & -0.523 \\ 0.338 & 0.734 & -0.589 \end{bmatrix}, \mu = \begin{bmatrix} 5.4 \\ 8 \\ 4.8 \end{bmatrix}$

$X' = U'X = [x'_1 \quad x'_2 \quad \dots \quad x'_{10}]$

$$= \begin{bmatrix} -2.47 & -6.24 & -7.59 & -3.68 & -5.69 & -6.60 & -12.59 & -3.30 & -12.65 & -12.48 \\ 0.04 & 4.28 & 4.53 & 3.27 & 10.79 & 4.71 & 3.61 & 6.30 & 3.85 & 7.33 \end{bmatrix}$$

Reconstruction error = $\sum_{i=1}^{10} \|(x_i - \mu) - U'^T U' x_i\|_2 = 29.76$

Average reconstruction error = $\frac{1}{10} \times 29.76 = 2.976$

5.

(a) Symmetric:

$(AA^T)^T = (A^T)^T \cdot A^T = AA^T, (A^TA)^T = A^T \cdot (A^T)^T = A^TA$

Positive semi-definite:

Let λ be an eigenvalue of AA^T and x be the eigenvector corresponding to λ

$(AA^T)x = \lambda x \rightarrow x^TAA^Tx = \lambda x^Tx \rightarrow \lambda = \dfrac{(A^Tx)^TA^Tx}{x^Tx}$

$\because (A^Tx)^TA^Tx \geq 0$ and $x^Tx \geq 0$. $\therefore \lambda \geq 0 \rightarrow AA^T$的 eigenvalues 皆非負

$\therefore AA^T$為 positive semi-definite。同理可證 A^TA也為 positive semi-definite。

Share the same nonzero eigenvalues:

Let λ be an eigenvalue of AA^T

$(AA^T)x = \lambda x \rightarrow A^T(AA^T)x = \lambda A^Tx \rightarrow (A^TA)(A^Tx) = \lambda(A^Tx)$

$\therefore \lambda$ is a eigenvalue of A^TA

(b) $\because \Sigma$ 是對稱和半正定，$\therefore \Sigma$ can be written as $U\Lambda U^T$, U 為 eigenvectors 組成的 orthogonal matrix， Λ為對角線由 eigenvalues 遞減排列組成的 diagonal matrix

$$\therefore \Sigma = \sum_{i=1}^{m} \lambda_i u_i u_i^T, where\ \lambda_i = i_{th}\ eigenvalue, u_i = i_{th}\ eigenvector$$

If n = m, $\Sigma = \sum_{i=1}^{n} \lambda_i u_i u_i^T = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)(x_i - \mu)^T$

$\therefore x_i - \mu = \sqrt{\dfrac{\lambda_i}{n}} u_i \rightarrow x_i = \sqrt{\dfrac{\lambda_i}{n}} u_i + \mu$

(c) $min\ {\Phi_1}^T\Sigma\Phi_1,\ s.t\ {\Phi_1}^T\Phi_1 = 1$

By Lagrange Multiplier: $g(\Phi_1) = {\Phi_1}^T\Sigma\Phi_1 - \alpha({\Phi_1}^T\Phi_1 - 1)$

$$\frac{dg(\Phi_1)}{d\Phi_{11}} = 0, \frac{dg(\Phi_1)}{d\Phi_{12}} = 0, \dots, \frac{dg(\Phi_1)}{d\Phi_{1m}} = 0$$

由上式可得$\Sigma\Phi_1 = \alpha\Phi_1, \therefore \Phi_1\ is\ eigenvector, \alpha\ is\ eigenvalue$

$\therefore {\Phi_1}^T\Sigma\Phi_1 = \alpha{\Phi_1}^T\Phi_1 = \alpha$, 要找使$\alpha$最小的$\Phi_1$, 此時$\alpha$為最小的 eigenvalue

$\therefore$取Σ第 k 個 eigenvector 可使${\Phi_1}^T\Sigma\Phi_1$最小化。