Linear Algebra I: Homework 6

Due Friday, September 29, 2017

1. Using a formula called **Jacobi's formula**, we can deduce that for any square matrix A,

$$\det(\exp(A)) = e^{\operatorname{tr}(A)},$$

   where $\exp(A)$ is the matrix exponential discussed in homework 5. Suppose that a matrix M is not invertible. Is $\exp(M)$ invertible?

2. Suppose $ad - bc \neq 0$. Let $\vec{u} = \begin{pmatrix} a \\ c \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} b \\ d \end{pmatrix}$.

   a. Can $\vec{v}$ be a multiple of $\vec{u}$? Explain your answer.

   b. Let $\vec{x}$ be a vector in $\mathbb{R}^2$. How many ways are there to write $\vec{x}$ as a linear combination of $\vec{u}$ and $\vec{v}$? Explain your answer.

3. Let $B = (\vec{e}_1, \vec{e}_2)$ be the standard basis for the vector space $\mathbb{R}^2$. Suppose

$$L : \mathbb{R}^2 \to \mathbb{R}^2$$

   is a linear transformation and that $L(\vec{e}_1) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $L(\vec{e}_2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   a. Compute the matrix of L using the basis B.

   b. Compute the trace of your matrix from (a).

   c. If $ad - bc \neq 0$, then

$$B' = \left(\begin{pmatrix} a \\ c \end{pmatrix}, \begin{pmatrix} b \\ d \end{pmatrix} \right)$$

   is a basis for $\mathbb{R}^2$. Compute the matrix of L using the basis B'.

   d. Compute the trace of your matrix from (c).

4. Find the value of a for which

$$\vec{v} = \begin{pmatrix} 6 \\ a \\ -16 \\ -4 \end{pmatrix}$$

   is in the set

$$H = \operatorname{span}\left\{ \begin{pmatrix} -3 \\ 2 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -5 \\ -2 \end{pmatrix} \right\}.$$