Math 317: Homework 2

Due Friday, February 8, 2019

1. (8.2 b,e) For each of the following sequences, determine its limit (usual Calculus reasoning will be helpful here), then prove that the sequence does indeed converge to this limit.

   a. $a_n = \frac{7n-19}{3n+7}$

   b. $s_n = \frac{1}{n} \sin n$

2. (8.6) Let (s_n) be a sequence in $\mathbb{R}$.

   a. Prove that $\lim s_n = 0$ if and only if $\lim |s_n| = 0$. (Hint: This is an "if and only if" statement, so you will need to prove both directions of the statement.)

   b. Prove that if $s_n = (-1)^n $ that $\lim |s_n|$ exists but s_n diverges.

3. (7.4) Give examples of

   a. A sequence of irrational numbers converging to a rational number

   b. A sequence of rational numbers converging to an irrational number

4. For each of the following false statements, give a counterexample.

   a. If s_n and t_n are two divergent sequences, their sum $s_n + t_n$ also diverges.

   b. If a sequence s_n converges, so too does the sequence $t_n = \sum_{i=1}^{n} s_i$.

   c. If the sequence s_n converges and the sequence t_n diverges, the product $s_n t_n$ also diverges.

5. (9.12) Suppose $s_n \neq 0$ for all $n \in \mathbb{N}$ and that the limit $\lim \left| \frac{s_{n+1}}{s_n} \right| = L$ exists.

   a. Show that if $L < 1$ then $\lim s_n = 0$.

   b. Show that if $L > 1$ then $\lim |s_n| = +\infty$.