

## Linear Algebra I: Homework 6

Due Friday, September 29, 2017

1. Using a formula called **Jacobi's formula**, we can deduce that for any square matrix  $A$ ,

$$\det(\exp(A)) = e^{\text{tr}(A)},$$

where  $\exp(A)$  is the matrix exponential discussed in homework 5. Suppose that a matrix  $M$  is not invertible. Is  $\exp(M)$  invertible? Explain.

2. Suppose  $ad - bc \neq 0$ . Let  $\vec{u} = \begin{pmatrix} a \\ c \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} b \\ d \end{pmatrix}$ .
  - a. Can  $\vec{v}$  be a multiple of  $\vec{u}$ ? Explain your answer.
  - b. Let  $\vec{x}$  be a vector in  $\mathbb{R}^2$ . How many ways are there to write  $\vec{x}$  as a linear combination of  $\vec{u}$  and  $\vec{v}$ ? Explain your answer.
3. Let  $B = (\vec{e}_1, \vec{e}_2)$  be the standard basis for the vector space  $\mathbb{R}^2$ . Suppose

$$L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

is a linear transformation and that  $L(\vec{e}_1) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $L(\vec{e}_2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ .

- a. Compute the matrix of  $L$  using the basis  $B$ .
- b. Compute the trace of your matrix from (a).
- c. If  $ad - bc \neq 0$ , then

$$B' = \left( \begin{pmatrix} a \\ c \end{pmatrix}, \begin{pmatrix} b \\ d \end{pmatrix} \right)$$

is a basis for  $\mathbb{R}^2$ . Compute the matrix of  $L$  using the basis  $B'$ .

- d. Compute the trace of your matrix from (c).
4. Find the value of  $a$  for which

$$\vec{v} = \begin{pmatrix} 6 \\ a \\ -16 \\ -4 \end{pmatrix}$$

is in the set

$$H = \text{span} \left\{ \begin{pmatrix} -3 \\ 2 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -5 \\ -2 \end{pmatrix} \right\}.$$