Linear Algebra I: Homework 2

Due Friday, September 1, 2017

1. a. Rewrite the following system of linear equations as an augmented matrix.

$$\begin{aligned} a - b + 2c - d &= -1 \\ 2a + b - 2c - 2d &= -2 \\ -a + 2b - 4c + d &= 1 \\ -3c &= -3 \end{aligned}$$

   b. Using Gauss-Jordan elimination, find a reduced row echelon matrix which is row equivalent to your answer in part (a). **You have to show your work here for full credit**: Be sure to show me the intermediate matrices you get and which elementary row operations you use at each step.

   c. Using your solution to part (b), describe the solution to the system of linear equations from part (a).

2. For parts (a) – (d), determine whether the system of linear equations each augmented matrix represents has **one unique solution**, **infinitely many solutions**, or **no solution**.

   a.
$$\left(\begin{array}{ccc|c} 1 & 0 & -2.1 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

   b.
$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

   c.
$$\left(\begin{array}{ccc|c} 1 & 7 & 8 & -1 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

   d.
$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

3. a. Suppose you want to find a, b, and c so that the parabola $y = ax^2 + bx + c$ is guaranteed to pass through the points $(1, 2)$, $(2, -3)$, and $(-1, 2)$. Write down (but do not solve) a system of linear equations whose solutions will give values for a, b, and c.

b. How many solutions does your system in part (a) have? Why? *It might help to think about what you know about parabolas.*

4. For parts (a) and (b), find the result of the vector arithmetic.

a.

$$\begin{pmatrix} 1 \\ -2 \\ 5 \\ 9 \\ 0 \\ 10 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ -5 \\ 0 \\ 7 \\ 1 \end{pmatrix}$$

b.

$$-2\begin{pmatrix} 1 \\ 0 \\ 2 \\ 4 \\ 6 \end{pmatrix}$$

5. Compute the matrix product AB of the matrices A and B below.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ -2 & 3 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -1 \\ 3 & -4 \\ 4 & -2 \end{pmatrix}$$