Linear Algebra I: Homework 8

Due Friday, October 20, 2017

1. Find the eigenvalues and their corresponding eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

   (*Hint.* One of the eigenvalues is -2, and you can use polynomial long division to factor the characteristic polynomial of A. You are also welcome to use a calculator to find the roots of the characteristic polynomial, but all other work must be shown.)

2. Find the eigenvalues and their corresponding eigenvectors for the matrix

$$B = \begin{pmatrix} 9 & -8 & 6 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 7 \end{pmatrix}.$$

3. Find the eigenvalues and their corresponding eigenvectors for the matrix

$$C = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}.$$

   (*Hint.* They may be complex.)

4. **You can plug a square matrix into any polynomial by also multiplying any constant terms by the matrix I to make them a square matrix too instead of just numbers**. Using this, answer the following questions.

   a. Let R be a 2×2 matrix, and $P_R(\lambda)$ its characteristic polynomial. What is $P_R(R)$ (that is, if you plug the matrix R into all λs)?

   b. **(Bonus)** Let S be a $m \times m$ matrix, and $P_S(\lambda)$ its characteristic polynomial. Based on your answer to part (a), what do you think $P_S(S)$ might be? Can you think of reasons your answer makes sense?

5. Two linear transformations, L and M, have the same eigenvalues with multiplicities. Additionally, every eigenvector of L for eigenvalue λ is **also** an eigenvector of M for eigenvalue λ.

   Are L and M the same linear transformation? Explain or give a counterexample.