Linear Algebra I: Homework 10

Due: Friday, November 10, 2017

1. For vectors $\vec{v} = \begin{pmatrix} v^1 \\ v^2 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} w^1 \\ w^2 \end{pmatrix}$, $\langle \vec{v}, \vec{w} \rangle = 3v^1w^1 + 2v^2w^2$ is an inner product.
   a. Find **all** unit vectors in $\mathbb{R}^2$ with respect to this new inner product.
   b. Find two different orthonormal bases for $\mathbb{R}^2$ with respect to this new inner product.

2. If $\vec{v} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\vec{u} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, find a decomposition

$$\vec{v} = \vec{v}^{||} + \vec{v}^{\perp}$$

   where $\vec{v}^{||}$ is parallel to $\vec{u}$ and $\vec{v}^{\perp}$ is orthogonal to $\vec{u}$.

3. For two $m \times n$ matrices M, N we can define the inner product,

$$\langle M, N \rangle = tr\left(M^T N\right).$$

   Are the vectors,

$$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$$

   orthogonal? Explain why or why not.