

## Linear Algebra I: Homework 8

Due Friday, October 20, 2017

1. Find the eigenvalues and their corresponding eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

(*Hint.* One of the eigenvalues is  $-2$ , and you can use polynomial long division to factor the characteristic polynomial of  $A$ . You are also welcome to use a calculator to find the roots of the characteristic polynomial, but all other work must be shown.)

2. Find the eigenvalues and their corresponding eigenvectors for the matrix

$$B = \begin{pmatrix} 9 & -8 & 6 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 7 \end{pmatrix}.$$

3. Find the eigenvalues and their corresponding eigenvectors for the matrix

$$C = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}.$$

(*Hint.* They may be complex.)

4.
  - a. Let  $R$  be a  $2 \times 2$  matrix, and  $P_R(\lambda)$  its characteristic polynomial. What is  $P_R(R)$  (that is, if you plug the matrix  $R$  into all  $\lambda$ s)?
  - b. Let  $S$  be a  $m \times m$  matrix, and  $P_S(\lambda)$  its characteristic polynomial. What is  $P_S(S)$ ? Why does your answer make sense?
5. Two linear transformations,  $L$  and  $M$ , have the same eigenvalues with multiplicities. Additionally, every eigenvector of  $L$  for eigenvalue  $\lambda$  is **also** an eigenvector of  $M$  for eigenvalue  $\lambda$ .

Are  $L$  and  $M$  the same linear transformation? Explain or give a counterexample.