UNIVERSITY OF EDINBURGH

DISSERTATION IN LANGUAGE SCIENCES

Understanding Referential Coordination as a Particle Swarm Optimization Task

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Abstract

In "Communicating with Cost-based Implicature: a Game-Theoretic Approach to Ambiguity" (2012), Rohde et al. sought to investigate the effects of ambiguous and unambiguous form costs on referential coordination. To do so, Rohde et al. ran a number of experiments in which participants played an iterated referential coordination language game. This paper models their results using particle swarm optimization (PSO), a general-purpose optimization method for non-differentiable problems with continuous search spaces. Two PSO-based models are presented and shown to perform well against a baseline model; predictions from the more favourable of the two models for several variants of the Rohde et al. language games are also presented.

Contents

1 Introduction

An open question in the field of linguistics is how participants in a conversation coordinate their use of referring expressions. When producing referring expressions, interlocutors must weigh the costs they incur when producing the expression, both in terms of construction and articulation, against the ease with which their conversational partners will be able to infer the intended referent. When speakers employ referring expressions that do not uniquely select a referent within the context of the discourse, they further risk their conversational partner failing to infer the correct referent from those possible. For example, consider a discourse context in which there are three plausible referents, a chocolate Labrador, a black Poodle, and a brown American Water Spaniel/German Longhaired Pointer mix. Given the high cost of producing an unambiguous referring expression for the latter, a speaker might attempt to refer to the American Water Spaniel/German Longhaired Pointer mix as "that brown dog". However, this expression could also plausibly be used to indicate the chocolate Labrador, and should the speaker's communicative partner interpret the referring expression as such, rectifying this misinterpretation is likely to be very costly. While communicating, interlocutors therefore jointly seek to minimize their expended effort while not violating the constraint imposed by their partner's ability to disambiguate the referring expressions used (Benz et al., 2005, p. 65-66). Consequentially, the referential strategy a speaker opts to adopt must be sensitive to the relative costs of producing each reference as well as the evolving state of mappings between referential form and intended referent, as coordinated with their interlocutors.

In "Communicating with Cost-based Implicature: a Game-Theoretic Approach to Ambiguity" (2012), Rohde et al. present an iterated language game in which participants aim to indicate an object to their partner via use of one of several possible referring terms. Participants gain points upon successful communication, but must spend points in order to communicate. Each referent has a corresponding unambiguous form that players may choose to send to their partners; alternatively, players may send an ambiguous form with

a different cost that could potentially indicate a number of referents. The studies run by Rohde et al. demonstrated the likelihood of a pair of participants to successfully coordinate their use of an ambiguous reference to be partially contingent on the relative costs of the unambiguous and ambiguous referring expressions.

This paper seeks to introduce a computational model of Rohde et al.'s findings, and in doing so consider how modeling may be applied to the problem of referential coordination. Such a model would allow simulation of the experimentally infeasible or impossible, as well as enabling extrapolation from the data collected in the lab, the results of which can be used to drive further studies in potentially fruitful directions. Further to this, computational models in general uniquely allow for direct inspection of their parameters and processes, which can shed light on aspects of human cognition. As such, computational modeling has seen extensive use in the field of linguistics, notable examples including the use of iterated learning to demonstrate the spontaneous emergence of syntax (Kirby & Hurford, 2002), the use of incremental probabilistic parsing to model garden-path sentence comprehension (Hale, 2001), and the use of Bayesian statistics to model referential inference (Frank & Goodman, 2012).

When selecting a modeling approach for Rohde et al. (2012), three crucial considerations must be taken into account: first, the chosen approach needs to represent the internal state of a participant with respect to the game, and to allow the participant's actions to be derived from this state, second, changes in a participant's internal state within the model need to be reflective of the participant's communicative success with their partner, even as their partner's internal state itself changes, and third, the modeling approach should be easily extensible to many variants of the Rohde et al. language games; particle swarm optimization is suitable in all three regards. As originally introduced by Kennedy & Eberhart (1995), particle swarm optimization (PSO) can serve not only as a general optimization method, but also as a means of modeling human social behaviour, especially in the context of collaborative problem solving (Kennedy, 1997). These factors make the technique an ideal candidate for modeling the Rohde et al. studies.

If possible, a particle-swarm-based model would allow a more exhaustive exploration of the effects of various form costs on referential coordination, offering a comprehensive picture of the circumstances under which ambiguous form entrainment is possible and likely. Further to this, an accurate particle-swarm-based model of human coordination in a discourse setting might shed light on more fundamental aspects of human cognition and social interaction as represented via the model's parameters. A successful model could also suggest PSO's suitability for the modeling of other linguistic phenomena.

Conversely, if PSO is not a viable means of modeling the results observed in Rohde et al., this could suggest a fundamental difference between human linguistic behaviours and other social behaviours as successfully modeled via PSO. A negative finding might also suggest that specialized methods are required to model more complex human social interactions.

To address these possibilities, this paper models Rohde et al.'s experiments as a PSO task, utilizing a mixed strategy search space to represent form production and comprehension. Two model variants as optimized to the Rohde et al. language games are presented, which differ in their treatment of the search space; these models are shown to compare favorably to a baseline PSO model. Both models respond to changes in form costs in a promisingly similar fashion to the observed experimental data, although neither perfectly replicates human trials.

2   Background

2.1   Referential coordination

2.1.1   Overview

Language is an inherently cooperative endeavour. To successfully understand and be understood by their interlocutors, speakers must actively coordinate their use of referring expressions, and rely on their communicative partners to do the same. The mechanisms by which this coordination can occur with relative facility are an active topic of research, and of special interest is to what extent speakers maintain internal models of their interlocutors

in order to inform their communicative strategies. Research has also been devoted to the processes by which speakers balance the costs they incur when communicating against those incurred by their interlocutors, and how the interplay of these combined factors directs the evolution of referential mappings during discourse. While Rohde et al. is emblematic of a game-theoretic approach to these issues, the psycholinguistics literature has also covered them in detail, and thus it is necessary to provide an overview of previous work done in both communities in order to sufficiently establish a point of departure for this work.

2.1.2   Game theoretic approaches

Game theory provides a methodology for understanding agents' actions by modeling them as strategies within games. When deciding on which actions to choose, agents are said to attempt to maximize their expected utility by leveraging their knowledge of the game's state (Benz et al., 2005). As applied to linguistics, game theoretic concepts can be used to describe a number of phenomena; for instance, Jäger (2008) demonstrates how an evolutionary game-theoretic framework (in which systematic stability is maximized) can be successfully applied to the division of the vowel space in order to predict the vowel systems of modern-day languages.

Lewis (1969) established much of the foundation necessary to frame language in game-theoretic terms, including providing an account of the establishment of conventions as a coordinative (this is to say, positive-sum) game "in which the agents' interests coincide perfectly" (Lewis, 1969, p. 14). His work has been built upon in order to apply game theory to the more specific problem of referential coordination. For example, by convention, the use of more general but costly referring expressions implicitly excludes the referents of easily accessed and more specific forms (e.g. "cutter" versus "knife", or "some" versus "all"). The establishment of this convention has been explained using an evolutionary (but ontogenetic) game-theoretic approach: both speaker and hearer benefit from an interpretation of the former which carries a greater degree of information (e.g. "cutters which are

not knives", "some but not all") (Benz et al., 2005). More recent work by Degen & Franke (2012) has used game theoretic models to predict participant behaviour in referential inference tasks with some success, although the researchers expressed the need for more nuanced, comprehensive models.

2.1.3 Psycholinguistic approaches

- Golland, Liang, Klein (2010) - speaker modeling listener

- Herb Clark - audience design

- cf. Keysar and Horton - audience design without internal audience modeling

2.1.4 Rohde et al. 2012

- When costs are similar, fewer pairs converge

2.2 Particle swarm optimization

2.2.1 Overview

Particle swarm optimization was first formulated by Kennedy & Eberhart (1995) in an attempt to model human social behaviour. The initial model was based off of Heppner & Grenander (1990)'s work in modeling bird flocking and roosting behaviour in two-dimensional space, refined to allow for an arbitrary number of dimensions and for particles to share the same or arbitrarily close positions within said multidimensional space. Kennedy & Eberhart demonstrated that their new optimization method was suitable for use not only in modeling human social behaviour, but also for the general-purpose optimization of non-linear continuous problems. Specifically, PSO was found to be successful both in optimizing the weights of an artificial neural network and in the Schaffer $f6$ problem, a standard benchmark for general-purpose optimization methods (Davis, 1991).

    The concept behind the PSO algorithm is relatively simple. Potential solutions to some problem with n dimensions are represented as particles

existing within an n-dimensional search space. Each particle has both a position within this space and some velocity vector. Each particle also keeps track of the best position it has been in as evaluated by the given objective function, which is known as its "personal" best position; a "global" best position representing the best position found by all particles across a swarm or group is also maintained (Chong & Zak, 2013). A PSO task is run iteratively, until some stopping criterion is reached (Solnon, 2010, p. 80). Every iteration begins by updating the velocities of all particles. In doing so, a particle is acted upon by two forces: the attraction of the particle to its personal previous best known position, as governed by a "cognitive" parameter, and its attraction to the best known position within its group, as governed by a "social" parameter (Chong & Zak, 2013). The particle also maintains some momentum from its previous velocity. Following this, particle positions are updated in accordance with their velocities, with new positions being evaluated via the given objective function and best-known personal and global positions updated where appropriate. Initial particle velocities are assigned randomly; in doing so, exploration of the search space is encouraged, reducing the likelihood of the swarm as a whole becoming caught in a local extremum or local extrema of the objective function (Yang, 2014, p. 32; Solnon, 2010, p. 78, 81).

As noted by Yang (2014, p. 99), PSO is applicable to a large domain of problems, has straightforward conceptual groundings, and is simple to implement; these factors have spurred on its widespread use in a number of fields, and resulted in the development of numerous refinements and derivatives of the original algorithm. This paper uses the well-known inertial variant of PSO, which offers a "noticeable improvement" in speedy convergence on good solutions as compared to standard PSO (Yang, 2014, p. 101). Further to this variant, for the purposes of this paper each particle's position is updated with respect to the best-found solution within a predefined neighborhood or group of particles, as opposed to the global best-found solution, as presented in Solnon (2010, p. 79). When these particle groups do not intersect, this is simply equivalent to running a number of independent PSO tasks equal to the number of groups.

2.2.2 Previous work

While PSO has been applied to a number of problems within the fields of linguistics and psychology, its primary use has been as a means of optimizing parameters for other models, as opposed to direct application as a model in and of itself (e.g. Chatterjee et al. 2005; Mehdad & Magnini 2009). Notable exceptions to this trend include the use of PSO to perform unsupervised phoneme clustering (Ahmadi et al., 2007) and the modeling of human emergency evacuation behaviours via PSO (Cheng et al., 2008).

PSO has likewise been applied to game learning, often using a coevolutionary paradigm in which agents play against one another in order to evaluate their fitness. However, traditionally this method has involved PSO over a search space of neural network weights, where the neural networks are used to choose actions given a game state, or in cases where the "game" is a classical constraint optimization problem, such as the *n*-queens problem (Engelbrecht, 2005, p. 349-351). By contrast, in the new approach presented in this paper, the positions of particles themselves define a mixed strategy (see subsubsection 3.2.1).

2.2.3 Formulation and parameters

In the formulation of PSO employed in this paper, a particle i with position x_i has a velocity v_i at time t such that

$$v_i^t = \theta(t) \cdot v_i^{t-1} + \alpha \cdot \epsilon_1 \cdot (x_{N(i)}^* - x_i^{t-1}) + \beta \cdot \epsilon_2 \cdot (x_i^* - x_i^{t-1}) \qquad (1)$$

, where θ is the inertial scheduling function, α is the cognitive component, β is the social component, $x_{N(i)}^*$ is the global best position known for i's neighborhood or group $N(i)$, x_i^* is i's personal best known position, and ϵ_1 and ϵ_2 are randomly chosen values within $(0.0, 1.0]$. Although θ can take many forms including, most commonly, a constant function (Yang, 2014, p. 101), for the purposes of this paper θ is defined with respect to a base

inertia τ and inertial dampening factor σ such that

$$\theta(t) = \frac{\tau}{\sigma^t} \tag{2}$$

Finally, the position x of i at time t is defined as

$$x_i^t = x_i^{t-1} + v_i^t \tag{3}$$

3   Methods

3.1   Overview

To model referential coordination within the Rohde et al. language game as a PSO task, individual participants were modeled as particles in groups of size 2. In exploring the search space of possible game strategies, each particle sought to maximize its score within the language game in relation to the strategy of its partner. The scoring function of the language game itself was parameterized on the relative costs of unambiguous forms, as in Rohde et al.. Further to this, the parameters of the PSO algorithm (cognitive component, social component, etc.) were optimized to best replicate Rohde et al.'s experimental findings. Two models were ultimately produced, which differed in their handling of the search space; both were evaluated against the experimental data.

3.2   Models

3.2.1   Search space

In modeling the Rohde et al. language game as a PSO task, the form a solution to the game takes must be considered in order to establish a search space. To do so, the game-theoretic notion of a mixed strategy was adopted, in which each possible action a within a game is performed by a participant i with some probability $P_i(a)$ (Benz et al., 2005, p. 22). For the given language game, for each referent r the participant can be said to maintain a

probability of using the associated ambiguous form A, $P_i(A|r)$. Conversely, the probability of a participant using the available unambiguous form for r can be given as $1 - P_i(A|r)$. Therefore, in an instance of the Rohde et al. language game with n possible referents, a participant's strategy was represented with n independent probabilities, yielding an n-dimensional search space.

It is important to note that this does not constitute a traditional mixed strategy, in that a participant's strategy is not a probability distribution. This is to say, the sum of a participant's probability of using the ambiguous referring expression over all referents may not equal 1. As an example, given two referents r_1 and r_2, a participant is capable of opting to use the ambiguous form for neither. In this sense, it may be more accurate to state that a participant i maintains a separate mixed strategy for each referent r, where, for the ambiguous form A and unambiguous form U, $P_i(A|r) + P_i(U|r) = 1$.

It is also important to note that for the purposes of this paper, participants were assumed to seek to optimize their strategies for groups of referents sharing the same ambiguous form independently of other groups. As such, each of the studies presented in Rohde et al. were considered as two independent language games being run concurrently, and the PSO approach presented used 3-dimensional search spaces as opposed to 6-dimensional search spaces. This assumption was justified by the observation that participant pairs in Rohde et al.'s second study were able to coordinate their use of the ambiguous form for one group, but not the other.

Finally, because each dimension in the search space defined above reflects a probability, values outside the interval $[0, 1]$ are invalid. A number of approaches for adapting PSO to constrained optimization problems have been suggested in the literature; two plausible alternatives as presented in Engelbrecht (2005) were considered, resulting in two competing models. In the first model, particles which moved outside the desired search space immediately had a repair method applied to them, whereby they were relocated to the nearest point which did not violate the given constraints. This model will be referred to as the "repair" model. In the second model, particles were allowed to move freely within the search space, including to regions that vio-

lated constraints. However, particles not meeting the given constraints were allowed to update neither their personal best known position nor the global best position. Because all personal best positions and the global best position therefore remained in feasible space, particles were naturally attracted back to the region of the search space which respected the constraints. The model utilizing this technique will be referred to as the "rejection" model. In both models, initial particle positions were assigned randomly within the valid search space.

3.2.2 Objective function

In applying PSO to the Rohde et al. language game, an appropriate representation of the game's goals must be formulated as an objective function. In the game as presented, the expected number of points awarded to participant i given their partner j when asked to communicate referent r can be calculated as follows:

$$EP_i(r|j) = P_i(A|r)(S \cdot P_j(r|A) - cost_A) + (1 - P_i(A|r))(S - cost_r) \quad (4)$$

, where $cost_A$ is the cost of production of the ambiguous form, S is the number of points awarded on successful communication (set at 80 and 85 in Rohde et al., respectively), and $cost_r$ is the cost of production of the unambiguous form for r.

Each round, the actual number of points awarded to participants is dependent on samples from P_i and P_j, as well as the randomly-chosen r. As such, without the strategies of i or j changing, there are for any given round a number of possible scores i might attain. Relying on the game unmodified as the objective function for a PSO task would therefore be imprudent, as an inconsistent objective function would be much harder to optimize. Instead, the objective function f used for both models was chosen as

$$f(i) = \sum_{r \in R} EP_i(r|j) + EP_j(r|i) \quad (5)$$

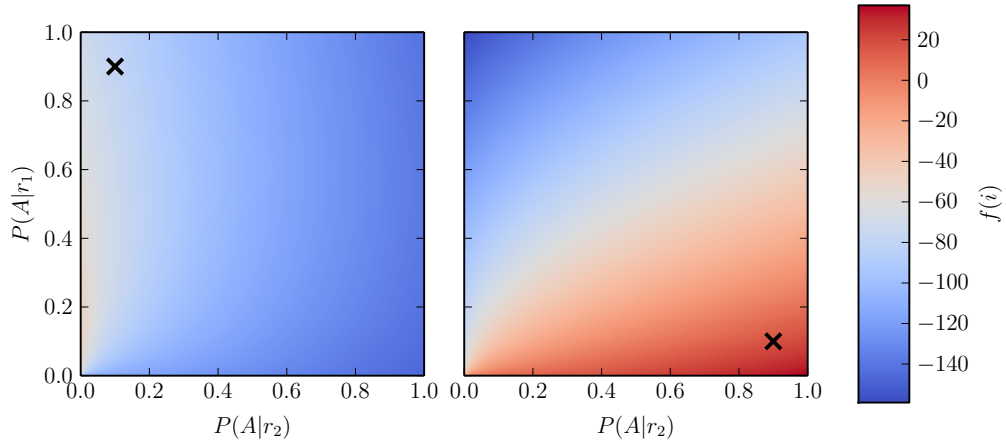There are a few issues to note with this formulation of f. Because the

Figure 1: $f(i)$ with respect to partner j (marked as "X") in a two-referent language game, with $cost_A = 80$, $cost_{r_1} = 60$, $cost_{r_2} = 120$, and $S = 85$. Note that in the scenario presented on the left, with $x_j = (0.1, 0.9)$, f's global optimum is located in the bottom-left region of the search space, whereas in the scenario presented on the right, where $x_j = (0.9, 0.1)$, the global optimum is located at the bottom-right.

value of f is dependent not only on the speaker's strategy, but also on the strategy their partner uses, because these strategies are a direct result of the positions of particles within the particle swarm, and because the positions of particles may change on every iteration of the PSO task, f is *extremely* dynamic. This dynamism is illustrated in Figure 1. PSO is well-known for its resilience to dynamic functions, when accommodations are made by changing the methods by which particle velocities and best positions are updated (Engelbrecht, 2005). However, the established accommodation techniques as presented in Engelbrecht (2005) are unsuitable for use here because they universally assume an objective function which updates periodically, not constantly. Further, many of the techniques proposed make assumptions that would be implausible within the context of the Rohde et al. language game. For example, one technique is to reinitialize all or part of the particle swarm when the objective function changes, however, this would not only result in participants randomly changing strategies every iteration, but would be difficult to justify from a real-world perspective. In another technique, the associated scores for personal best and global best positions are recalculated when the objective function changes. However, in the context of the language game, this would be equivalent to allowing participants to continuously re-evaluate previously held strategies against their partner's current strategy. Because no appropriate adaptations for dynamic fitness functions were found, the implementation of PSO used was not altered to accommodate f.

3.3 Model parameter optimization

To complete the models described above, the model parameters were optimized to best fit the experimental data. These parameters included the standard PSO parameters, namely, the cognitive and social components, which were restricted to the interval $[0, 4]$, an inertial dampening factor, which was restricted to the interval $[1, 1.1]$, and an initial inertia, which was restricted to $[0, 4]$. Additionally, a velocity dampening constant, which was used in the calculation of particle positions and was restricted to $[0, 2]$, and the number of iterations over which the model was to be run, as restricted to $[100, 1000]$,

were also optimized.

Optimization of these parameters was itself performed via PSO over a 6-dimensional search space. The parameter values used for this meta-optimization task were those recommended in Shi & Eberhart (1998) and Solnon (2010). Parameters were optimized over 1255 iterations of the particle swarm algorithm for the "repair" model; these parameters were then also applied to the "rejection" model. The optimization task itself utilized the unbounded "rejection" technique as recommended in Engelbrecht (2005). Initial particle positions were assigned using the randomized nonuniform method presented in Mitchell (1991).

The parameter optimization task sought to minimize the discrepancy in rates of ambiguous form coordination, unambiguous form coordination, and failure to coordinate between the model and the experimental data, across all language game variants. In order to evaluate this, pairs were considered to have coordinated if, when the PSO task had completed, referents could be successfully communicated between the pair $\geq 95\%$ of the time.

4 Results

4.1 Meta-optimization task (PSO parameters)

| Parameter	Optimized	Standard	%Δ
Cognitive component	0.689	2.0	-65.55%
Social component	2.897	2.0	$+44.86\%$
Inertial dampening factor	1.027	1.001	$+2.58\%$
Initial inertia	0.658	1.2	-45.17%
Velocity dampening factor	1.202	N/A	N/A
Iterations	305	N/A	N/A

4.2 Comparison of models to experimental data

Results for both the rejection and repair models are compared against a baseline model, which makes use of the standard parameters used in the meta-optimization task. For each model, 250 simulations of 10 pairs were
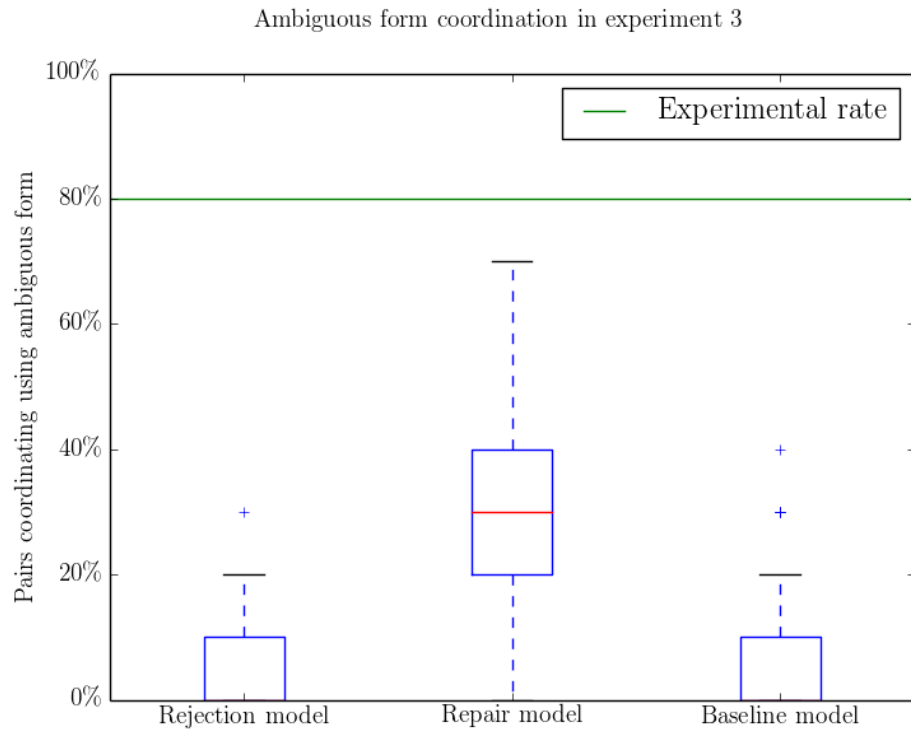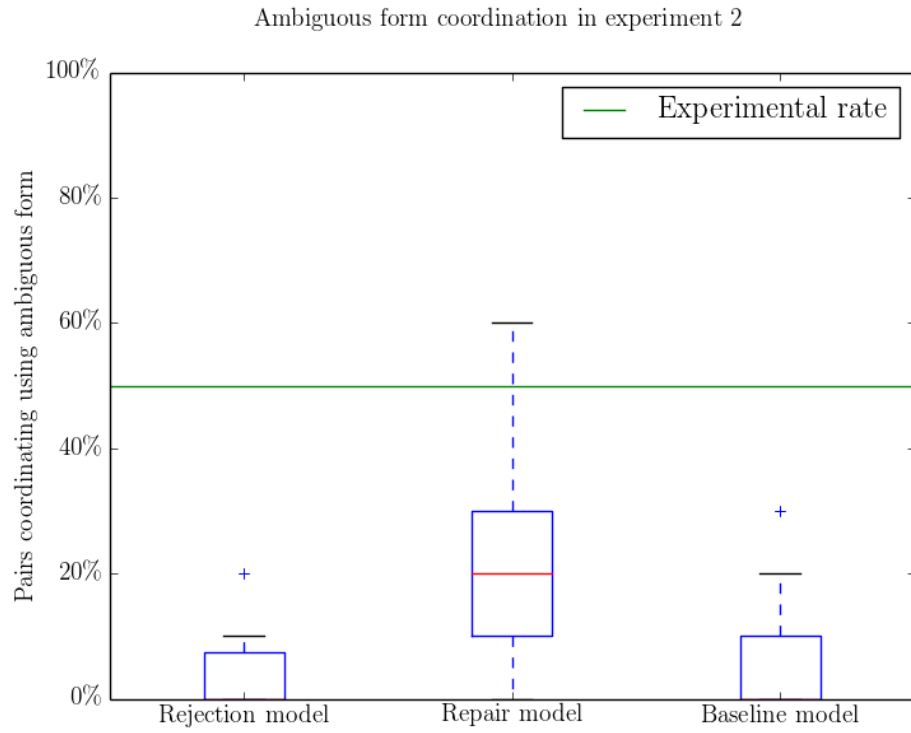
performed.   Costs from each experiment as taken from the Rohde et al.
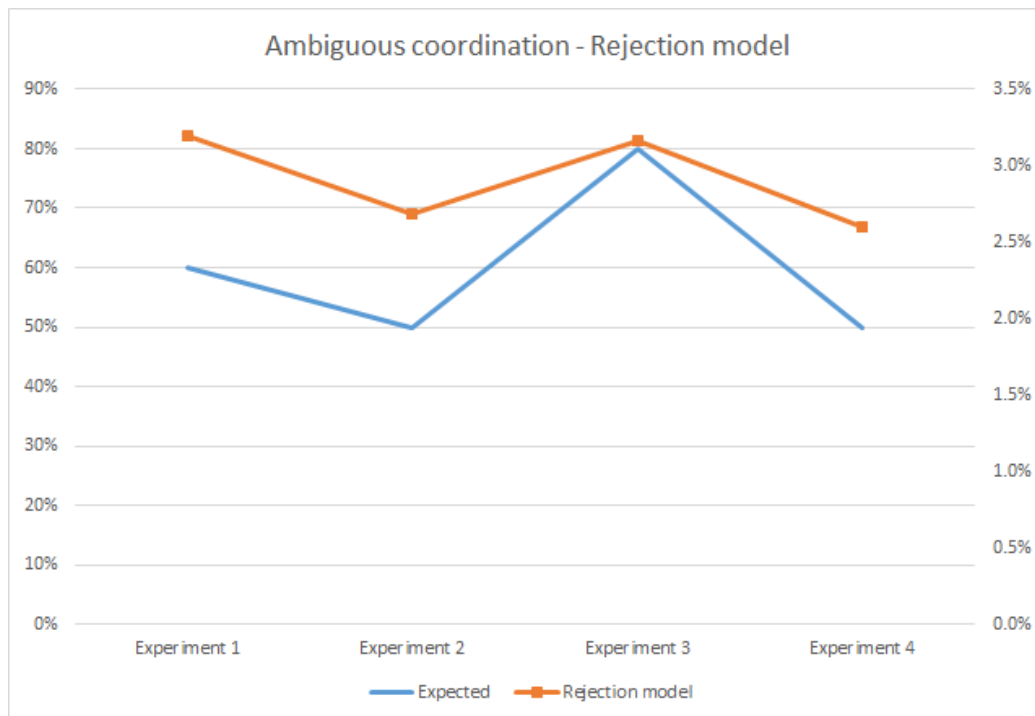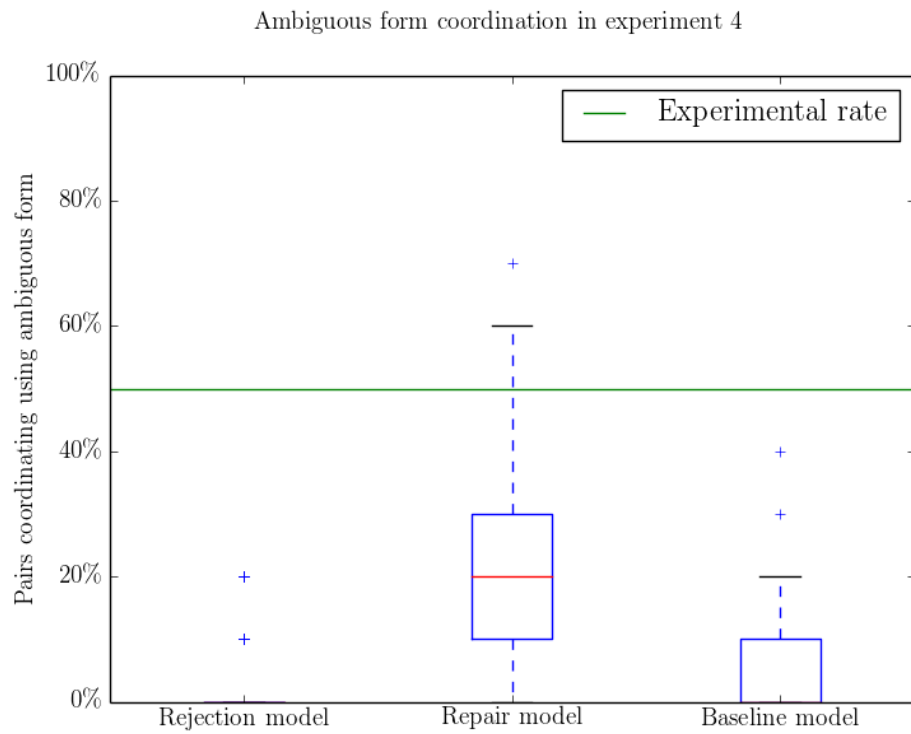studies are presented below.

	$cost_{r_1}$	$cost_{r_2}$	$cost_{r_3}$
Experiment 1	60	120	280
Experiment 2	60	120	250
Experiment 3	80	140	165
Experiment 4	80	135	170

*The results ended up seeming much harder to interpret than when I orig-
inally presented them to you. I'll bring my data for our next meeting, in
the meanwhile, I've included graphs below, along with my questions/concerns
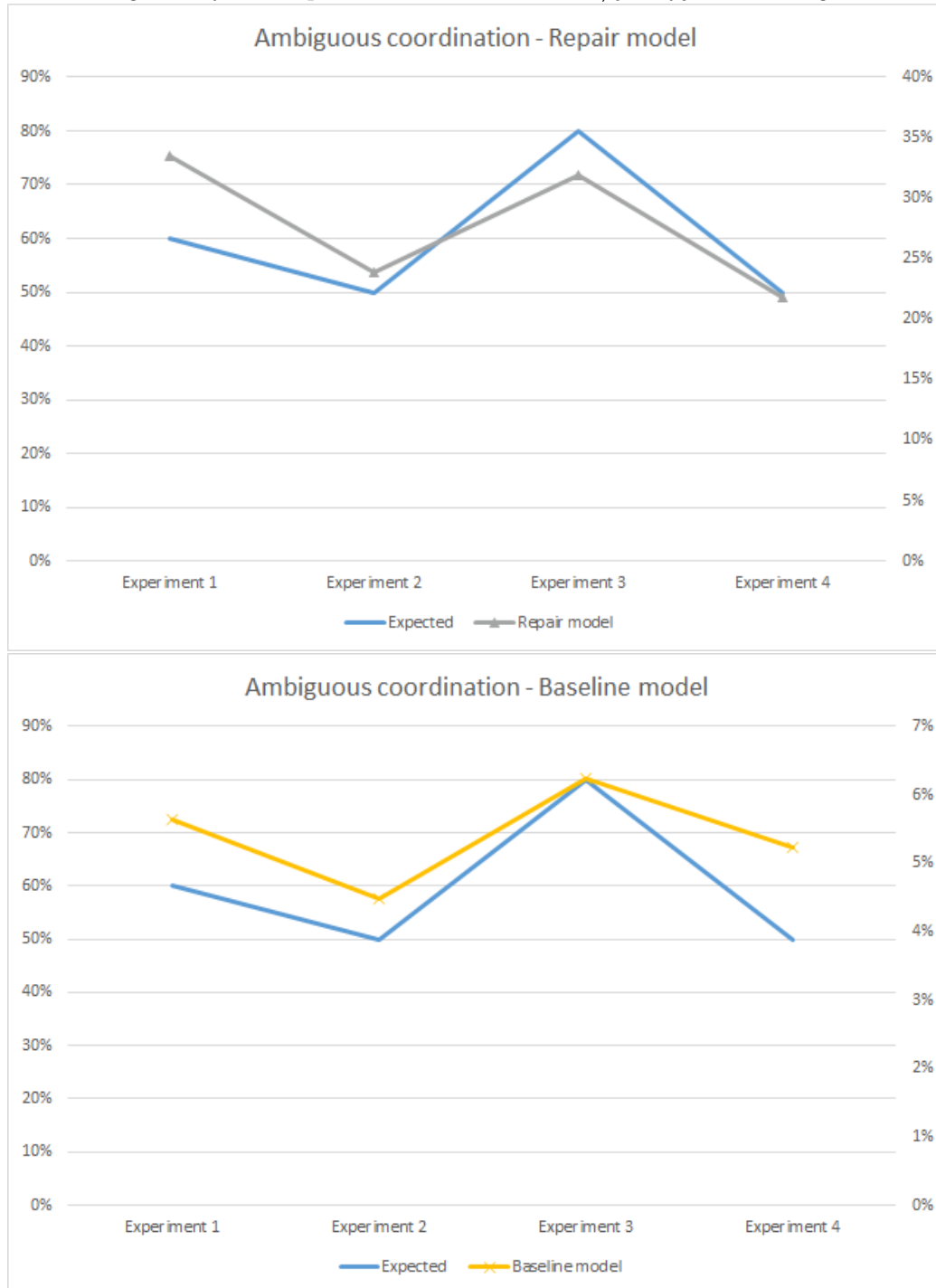where appropriate.*
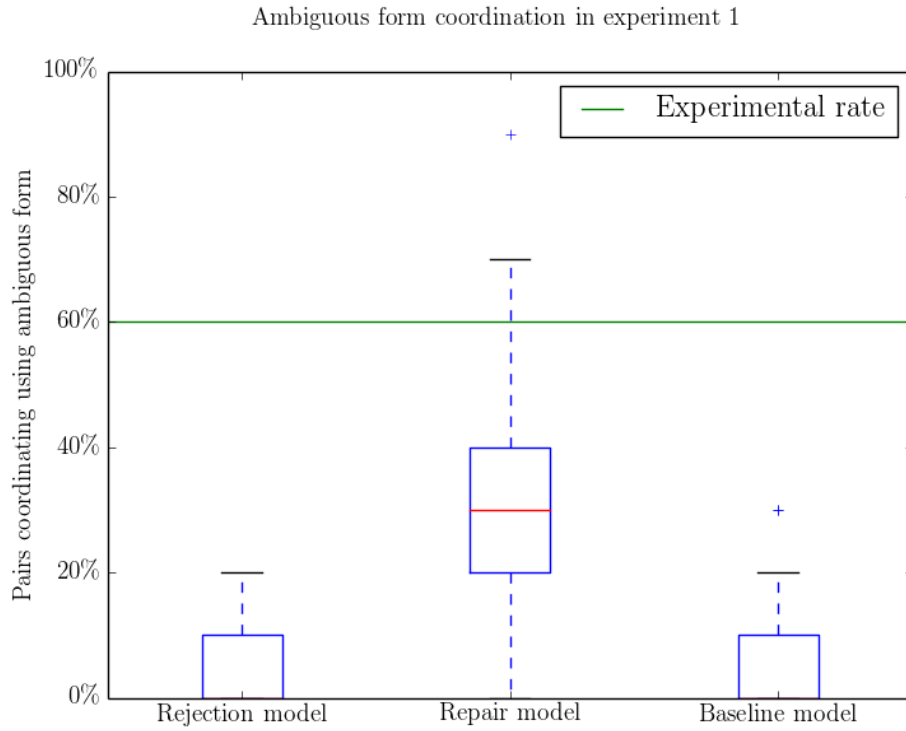
*Is there a more cohesive way of presenting these? Should these box/whisker
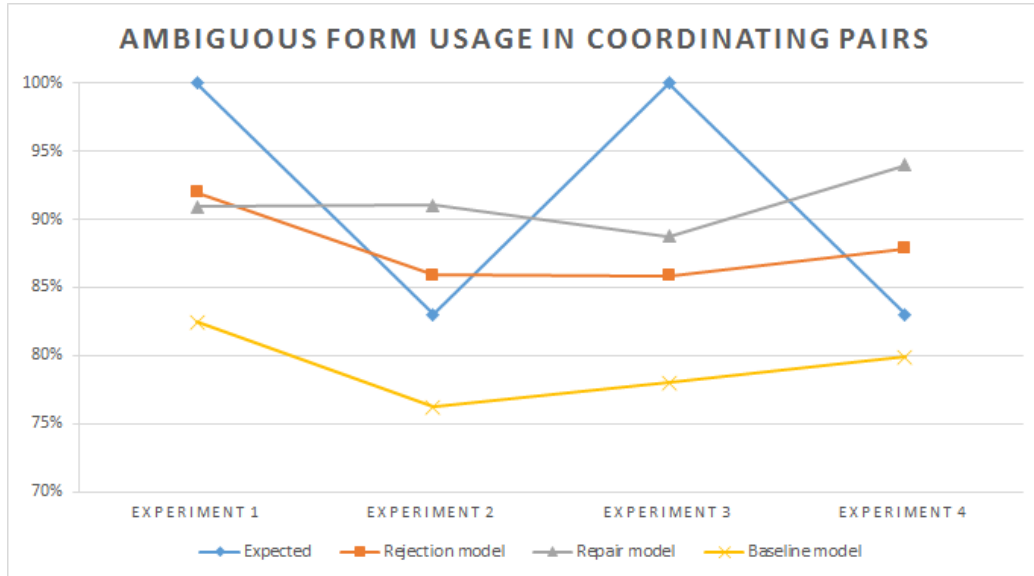plots in particular perhaps be moved to an appendix?*

Ambiguous form coordination in experiment 2

Ambiguous form coordination in experiment 3

Ambiguous form coordination in experiment 4

Given that these are all at different scales, is there a way to present them

together for comparison? How to choose/justify the scaling?

What is the most appropriate way to measure whether either of the two optimized models tracks the experimental data more effectively than the baseline model?

4.3   Predictions from successful model

No longer clear which should be considered "successful".

5   Discussion

- Perhaps mention coevolutionary aspect here?

- Model has response in keeping with experimental data without agents performing complex modeling (i.e. general optimization is sufficient to produce effects)

- Repercussions of predictions

- Possibility of applying PSO to other language games / experiments

6 Conclusion

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