$$\mathcal{L}[g(\mathbf{x})] = \iint_{\mathbf{x},\mathbf{x}'} g(\mathbf{x})\Sigma(\mathbf{x}-\mathbf{x}')g(\mathbf{x}') + \lambda(1 - \int_{\mathbf{x}} g^2(\mathbf{x})) + \int_{\mathbf{x}} \sigma(g(\mathbf{x}))$$
$$= \iint_{\mathbf{x},\mathbf{x}'} g(\mathbf{x})\Sigma(\mathbf{x}-\mathbf{x}')g(\mathbf{x}') + \lambda(1 - \int_{\mathbf{x}} g^2(\mathbf{x})) + \int_{\mathbf{x}} (\sigma_0 + \sigma_1 g(\mathbf{x}) + \sigma_2 g(\mathbf{x})^2 + \sigma_3 g(\mathbf{x})^3)$$

For the first term, we expediate our calculation with the convolution theorem, which states that $\widetilde{(f*g)}(k) = \tilde{f}(k)\tilde{g}(k)$:

$$\begin{aligned}
\iint_{\mathbf{x},\mathbf{x}'} g(\mathbf{x})\Sigma(\mathbf{x}-\mathbf{x}')g(\mathbf{x}') &= \int_{\mathbf{x}} g(\mathbf{x}) \int_{\mathbf{x}'} \Sigma(\mathbf{x}-\mathbf{x}')g(\mathbf{x}') \\
&= \int_{\mathbf{x}} g(\mathbf{x})(\Sigma * g)(\mathbf{x}) \\
&= \int_{\mathbf{x}} g(\mathbf{x}) \int_{\mathbf{k}} \widetilde{(\Sigma * g)}(\mathbf{k}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \\
&= \int_{\mathbf{x}} g(\mathbf{x}) \int_{\mathbf{k}} \tilde{\Sigma}(\mathbf{k})\tilde{g}(\mathbf{k}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \\
&= \int_{\mathbf{k}} \tilde{\Sigma}(\mathbf{k})\tilde{g}(\mathbf{k}) \int_{\mathbf{x}} g(\mathbf{x}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \\
&= \int_{\mathbf{k}} \tilde{\Sigma}(\mathbf{k})\tilde{g}(\mathbf{k})\tilde{g}(-\mathbf{k}) \\
&= \int_{\mathbf{k}} |\tilde{g}(\mathbf{k})|^2 \tilde{\Sigma}(\mathbf{k})
\end{aligned}$$

For the term linear in g, we have

$$\begin{aligned}
\int_{\mathbf{x}} g(\mathbf{x}) &= \int_{\mathbf{x}} \int_{\mathbf{k}} \tilde{g}(\mathbf{k}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \\
&= \int_{\mathbf{k}} \int_{\mathbf{x}} \tilde{g}(\mathbf{k}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \\
&= \int_{\mathbf{k}} \tilde{g}(\mathbf{k})\delta(\mathbf{k}) \\
&= \tilde{g}(0)
\end{aligned}$$

For the term linear in g^2, by Plancheral's theorem, we have

$$\int_{\mathbf{x}} g(\mathbf{x})^2 = \int_{\mathbf{k}} |\tilde{g}(\mathbf{k})|^2$$

For the term linear in g^3, we have

$$\begin{aligned}
\int_{\mathbf{x}} g(\mathbf{x})^3 &= \int_{\mathbf{x}} \Big(\int_{\mathbf{k}} \tilde{g}(\mathbf{k}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \Big)^3 \\
&= \int_{\mathbf{x}} \Big(\int_{\mathbf{k}} \tilde{g}(\mathbf{k}) e^{2\pi i \mathbf{k}\cdot\mathbf{x}} \int_{\mathbf{k}'} \tilde{g}(\mathbf{k}') e^{2\pi i \mathbf{k}'\cdot\mathbf{x}} \int_{\mathbf{k}''} \tilde{g}(\mathbf{k}'') e^{2\pi i \mathbf{k}''\cdot\mathbf{x}} \Big) \\
&= \int_{\mathbf{x}} \Big(\iiint_{\mathbf{k}\mathbf{k}'\mathbf{k}''} \tilde{g}(\mathbf{k})\tilde{g}(\mathbf{k}')\tilde{g}(\mathbf{k}'') e^{2\pi i (\mathbf{k}+\mathbf{k}'+\mathbf{k}'')\cdot\mathbf{x}} \Big) \\
&= \iiint_{\mathbf{k}\mathbf{k}'\mathbf{k}''} \Big(\int_{\mathbf{x}} \tilde{g}(\mathbf{k})\tilde{g}(\mathbf{k}')\tilde{g}(\mathbf{k}'') e^{2\pi i (\mathbf{k}+\mathbf{k}'+\mathbf{k}'')\cdot\mathbf{x}} \Big) \\
&= \iiint_{\mathbf{k}\mathbf{k}'\mathbf{k}''} \tilde{g}(\mathbf{k})\tilde{g}(\mathbf{k}')\tilde{g}(\mathbf{k}'')\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')
\end{aligned}$$

Hence, putting it all together,

$$\tilde{\mathcal{L}}[\tilde{g}(\mathbf{x})] = \int_{\mathbf{k}} |\tilde{g}(\mathbf{k})|^2 \tilde{\Sigma}(\mathbf{k}) + \lambda(1 - \int_{\mathbf{k}} |\tilde{g}(\mathbf{k})|^2) +$$
$$[\sigma_0 + \sigma_1 \tilde{g}(\mathbf{0}) + \sigma_2 \int_{\mathbf{k}} |\tilde{g}(\mathbf{k})|^2 + \sigma_3 \iiint_{\mathbf{k},\mathbf{k}',\mathbf{k}''} \tilde{g}(\mathbf{k})\tilde{g}(\mathbf{k}')\tilde{g}(\mathbf{k}'')\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')]$$