Queueing Theory and Simulation, lecture 3

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Contents

1 General things

1.1 General things

- assignments: hope you enjoy them. We do one per week, even though most is on github already
- assignments: making groups works?
- Book: keep up with the exercises.
- book format (main text, comments and references to exercises in the margin.)

2 Notation for queues

2.1 Continuous time, multiserver queue

- Job k sees, upon arrival, a waiting time $w_{k,i}$ at queue i.
- m servers
- $w_k = (w_{k,1}, \ldots, w_{k,m})$,
- I represents here a vector $(1, 1, \ldots, 1)$.
- e_k is kth unit vector, i.e., all zeros with 1 at kth place

$$s_k = \arg\min_i \{w_{k,i}\} \tag{1}$$
$$w_{k+1} = [w_k + S_k e_{s_k} - X_{k+1} I]^+, \tag{2}$$
$$W_k = w_{k,s_k} \tag{3}$$

$\implies$ $\{X_k\}$, $\{S_k\}$ and m suffice construct the queueing process.

2.2 Kendall's notation

- M/M/1
- M/M/c
- G/G/c
- D/D/1

3 Random walk

3.1 Applications

- insurance
- production inventory control
- queueing

It's all the 'same' thing we study.

3.2 Elegant recursion

$$Z_k = Z_{k-1} + a_k - c_k \tag{4}$$
$$L_k = [L_{k-1} + a_k - c_k]^+ = [L_{k-1} + Z_k - Z_{k-1}]^+ \tag{5}$$
$$L_k = Z_k - \min_{1 \le i \le k} Z_k \wedge 0 \tag{6}$$

Nice formula, but still a recursion

3.3 Useful and useless

- Can we find a closed form solution, something like $x_t = x_0 + v_0 t - gt^2/2$ for the height of bullet in a homogeneous gravitational field (and no friction)? No, disappointingly.
- We have to drop the time-dependent analysis
- Henceforth in the course we concentrate on finding formulas compute/estimate time-average waiting times, e.g, $\mathsf{E}[W]$, or $\mathsf{E}[I_{W \ge x}]$.

4 Rest of the course

4.1 Next lecture

- Concept of rate
- Stability
- Convergence to stationary state

4.2 lectures next week

- One of the most beautiful (?) formulas of probability, Sakasegawa's approximation for the average waiting time in an G/G/c queue.
- Application to many different practically useful queueing systems.

4.3 later lectures

- Use sample paths of queueing process to analyze the system; we still use the ideas of simulation, but in a different way!
- Formulas for many queueing models