Queueing Theory and Simulation, lecture 6

Nicky van Foreest

June 22, 2021

Contents

1 Overview

1.1 What we did

- Sakasegawa's formula for $\mathsf{E}[W(G/C/c)]$.
- When setups take time, we analyzed the effect of batch sizes B on expected sojourn time.
- Recall, larger batches reduce fraction of time spent on setups, but increase batch forming times. $\mathsf{E}[J]$ is convex in B

1.2 What we are going to do next

- Study effect of other types of interruptions on $\mathsf{E}[J]$:
  - adjustments,
  - failures

1.3 Later

- Simple networks of $G/G/1$ queues.
- Analytical models for $M/M/c$ and $M/G/1$ queue.

1.4 Format of lectures?

- Does this work for you? One hour lectures with overview?
- Assignments? Does plug and play work?

2 Approach to analyze effect of interruptions

2.1 Setups vs adjustments vs failures

- A setup time is a *planned* interruption.
- An adjustment is an *unplanned* interruption, but happens *between* two jobs, i.e., a *non-preemptive outage*
- A failure is an *unplanned* interruption *during* a job service, i.e., a *preemptive outage*.
- We already tackled the setups; now the other two
- For the setup case, can you analyze a model with different batch sizes (depending on the 'color')?

2.2 Procedure to compute $\mathsf{E}[J]$

- Make a logistic model
- Determine the components of Sakasegawa's formula to get $\mathsf{E}[W]$.
- Fill everything
- $\mathsf{E}[J] = \mathsf{E}[W] + \mathsf{E}[S]$ plus other terms (e.g. batching) if relevant.
- Make graphs of $\mathsf{E}[J]$ as a function of the relevant parameters

2.3 Sakasegawa's formula, 4.1

$$\rho = \lambda \mathsf{E}[S]/c \tag{1}$$

$$\mathsf{E}[W] = \frac{C_a^2 + C_s^2}{2} \frac{\rho^{\sqrt{2(c+1)}-1}}{1-\rho} \frac{\mathsf{E}[S]}{c} \tag{2}$$

$$C_a^2 = \frac{\mathsf{V}[X]}{(\mathsf{E}[X])^2} \tag{3}$$

$$C_s^2 = \frac{\mathsf{V}[S]}{(\mathsf{E}[S])^2} \tag{4}$$

$$\tag{5}$$

2.4 What we compute in this lecture

- Specify λ and C_a^2. Often Poisson is reasonable. Then $C_a^2 = 1$.
- In our examples $c = 1$.
- Find model for effective service time S. Compute $\mathsf{E}[S]$ and $\mathsf{V}[S]$
- $\rho = \lambda \mathsf{E}[S]/c$.
- With $\mathsf{E}[S]$ and $\mathsf{V}[S]$ we can compute C_s^2.
- We have all elements to fill in Sakasegawa's formula!
- $\mathsf{E}[J] = \mathsf{E}[W] + \mathsf{E}[S]$, etc.

3 Queues with adjustments, 4.3

3.1 Model

- Adjustments occur geometrically distributed between jobs with probability p.
- Define $B \sim \text{Geo}(p)$. Recall, $\mathsf{E}[B] = 1/p$, $\mathsf{V}[B] = (1-p)/p^2$
- S_0 is the net service time of a job
- S is the time a job spends at the server, i.e., the effective service time of a job as seen by jobs in the queue.
- R is the common rv of the adjustment times.
- $Y = 1$ or 0 if an adjustment is necessary (or not) after a job. Y is independent of S_0 and repair time R.

3.2 Fill in Sakasegawa's formula

- To compute (estimate) $\mathsf{E}[W]$ we only need to compute $\mathsf{E}[S]$ and $\mathsf{V}[S]$, the rest we already have.
- With the (exercises of the) book and the equations we can compute $C_s^2 = \mathsf{V}[S] / (\mathsf{E}[S])^2$.

$$\mathsf{E}[S] = \mathsf{E}[S_0 + RY] = \mathsf{E}[S_0] + p\,\mathsf{E}[R] \tag{6}$$

$$\mathsf{V}[S] = \mathsf{V}[S_0 + RY] = \mathsf{V}[S_0] + \mathsf{V}[RY] \tag{7}$$

$$\mathsf{V}[RY] = \mathsf{E}\left[R^2\right] \mathsf{E}\left[Y^2\right] - (\mathsf{E}[R]\,\mathsf{E}[Y])^2. \tag{8}$$

3.3 Consultancy take away

- Change parameters, and make graphs of $\mathsf{E}[J]$ as a function of the parameters to see the effects.
- Analyze influence of adjustment *rate* on $\mathsf{E}[J]$
- Analyze influence of adjustment *time* on $\mathsf{E}[J]$.
- Perhaps it's possible to do adjustments less often, i.e. $p \downarrow$, but make the adjustments a bit longer $\mathsf{E}[R] \uparrow$. Should we attempt this?
- What if we could plan the adjustments, rather than let them happen at arbitrary moments. Should we attempt to achieve this in our organization?

4 Queues with failures, 4.4

4.1 Model

- Failures arrive as Poisson process with rate λ during a job's net service time S_0.
- Repairs $\{R_i\}$ form an iid set of rvs, independent of S_0.

- N is the number of failures that occur, $N|S_0 \sim \text{Poi}(\lambda S_0)$.
- $S = S_0 + \sum_{i=1}^{N} R_i$

4.2 Fill in Sakasegawa's formula

$$
\begin{aligned}
\mathsf{E}[N] &= \lambda\,\mathsf{E}[S_0] && (9)\\
\mathsf{E}[S] &= \mathsf{E}\left[S_0 + \sum_{i=1}^{N} R_i\right] = \mathsf{E}[S_0] + \mathsf{E}[N]\,\mathsf{E}[R] && (10)
\end{aligned}
$$

- $\mathsf{V}[S]$ is a bit technical, but not hard (you can apply Eve's law if you like).
- The exercises in the book guide you through the derivation step by step. They are important to do.
- The other elements of Sakasegawa's formula are already known.

4.3 Consultancy take away

- The above models and analysis belong to the realm of lean manufacturing.
- With Sakasegawa's formula and the models we discuss here, you understand more of all this than any 50 K Euro management course offers you!

See you next week.