Queueing Theory and Simulation, lecture 5

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Contents

1 General things

1.1 What we did

- We tackled the time-dependent analysis of queueing systems, with construction and simulation.
- Time dependent case is too hard: settle for characterization of average behavior
- Distinguish between time and sample (job arrival) averages in KPIs
- For sensible average behavior utilization we need stability, i.e., $\rho = \lambda \mathsf{E}[S]/c < 1$.

1.2 What we are going to do next

- Sakasegawa's formula for $\mathsf{E}[W(G/C/c)]$.
- Application of Sakasegawa's formula to a batch queue.

1.3 Later

- More application of Sakasegawa's formula
- Analytical models for $M/M/c$ and $M/G/1$ queue.

2 Sakasegawa's formula, 4.1

2.1 The formula

$$\rho = \lambda \, \mathsf{E}[S]/c \tag{1}$$

$$\mathsf{E}[W] = \frac{C_a^2 + C_s^2}{2} \frac{\rho^{\sqrt{2(c+1)}-1}}{1-\rho} \frac{\mathsf{E}[S]}{c} \tag{2}$$

$$C_a^2 = \frac{\mathsf{V}[X]}{(\mathsf{E}[X])^2} \tag{3}$$

$$C_s^2 = \frac{\mathsf{V}[S]}{(\mathsf{E}[S])^2} \tag{4}$$

$$\tag{5}$$

- Why is this so useful? (part 1 of 4.1)
- main insights
- How to estimate C_a^2 from other data? (part 2 of 4.1)

3 Queues with setups, 4.2

3.1 why batching?

- Jobs have certain characteristics, e.g., color.
- server needs to be cleaned when changing color.
- Cleaning takes time
- work in batches of jobs with the same characteristics

3.2 What happens logistically?

- time for arrivals to form a batch
- time spent in queue
- time spent at server
- time to regroup?

3.3 time for arrivals to form a batch

$$\mathsf{E}[W_i] = \frac{B-1}{2\lambda_i}, \quad i \in \{r, b\} \tag{6}$$

$$\tag{7}$$

Linear in B.

3.4 Time batch spends in queue

- Use Sakasegawa's formula
- main components λ, $\mathsf{E}[S]$, C_a^2, C_s^2, c.

$$
\begin{aligned}
\lambda &= \lambda_r + \lambda_b && (8)\\
&\mathsf{E}[S_0] && (9)\\
\mathsf{E}[S_B] &= \mathsf{E}[R] + B\,\mathsf{E}[S_0], && (10)\\
\mathsf{E}[S] &= \mathsf{E}[R]/B + \mathsf{E}[S_0], && (11)\\
c &= 1, && (12)\\
\rho &= \lambda\,\mathsf{E}[S] = \lambda\,\mathsf{E}[R]/B + \lambda\,\mathsf{E}[S_0]. && (13)
\end{aligned}
$$

- ρ is decreasing in B!

3.5 SCVs.

$$
\begin{aligned}
C_{a,B}^2 &= \frac{\mathsf{V}\left[\sum_i^B X_i\right]}{(\mathsf{E}\left[\sum_i^B X_i\right]^2)^2} = \frac{B\,\mathsf{V}[X]}{B^2(\mathsf{E}[X])^2} = \frac{C_a^2}{B} && (14)\\
\mathsf{V}[S_B] &= \mathsf{V}[R] + B\,\mathsf{V}[S_0] && (15)
\end{aligned}
$$

We have all we need to fill in Sakasegawa's formula.

3.6 Time at server

- wait for your turn: $\mathsf{E}[R] + (B-1)\,\mathsf{E}[S_0]/2$
- get server: $\mathsf{E}[S_0]$

3.7 Regrouping after service

- If jobs can leave directly, we are done.
- Wait for a batch of size B' to form: $(B'-1)\,\mathsf{E}[S]_0/2$.

3.8 Performance analysis

- Add up all expected times to get $\mathsf{E}[J]$
- Make graph of $\mathsf{E}[J]$ as function of e.g. batch size,