Monte Carlo Simulation

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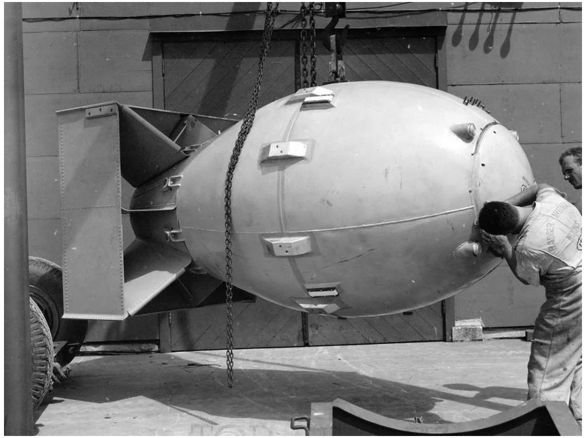
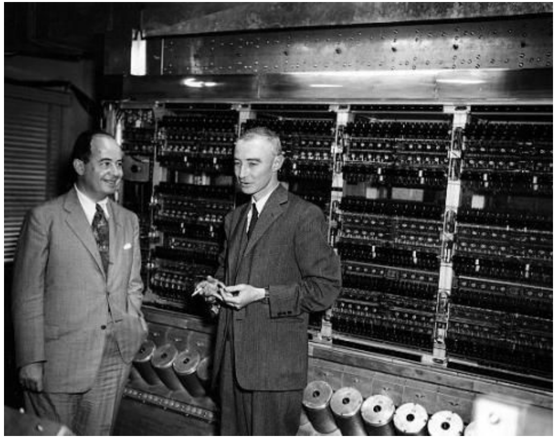
A brief history

- It was developed in tests of the atomic bomb (Manhattan Project) during the Second World War at the Los Alamos National Laboratory in the USA.

- One of the first applications of this method was made to a deterministic problem in 1948 by Enrico Fermi, Ulam, and von Neumann when they considered the singular values of the Schrodinger equation.





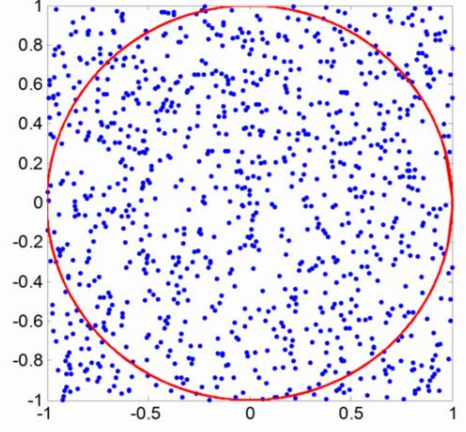
General approach to the problem

- Generate samples of a random variable $X(\{x^n\})$ given its probability density P(x).

- Estimate the expected value of a function φ(x) under a density of probability P(x):

$$< \phi(x) >= \int \phi(x) P(x) dx$$

Example: Approximation to PI number



- We generate random values for points within a square of 2 by 2 with centerpoint (0,0).

- The x-coordinates and the y-coordinates are independent results of a uniform distributed random experiment.

- They range from -1 to 1.

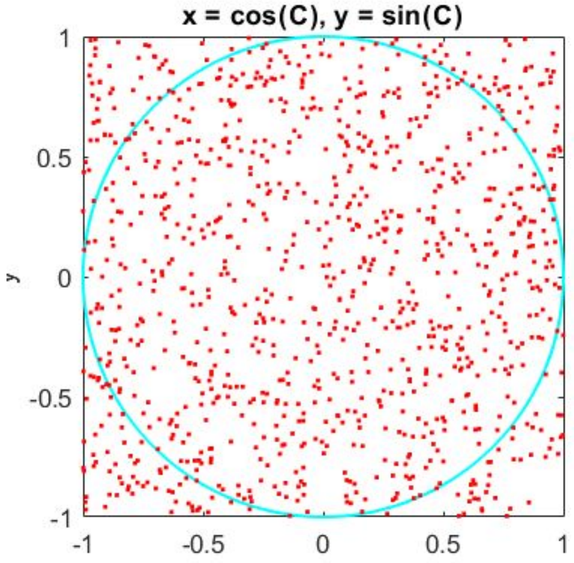
- In the square let's look at a circle with center point (0,0) and radius 1

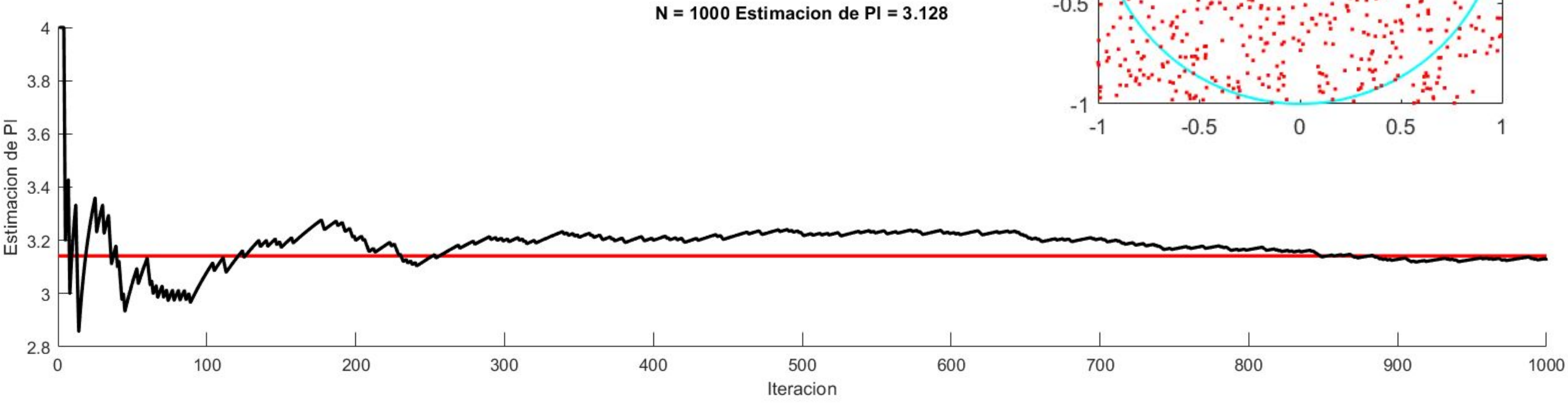
$$\frac{Area\ of\ circle}{Area\ of\ square} = \frac{\pi \cdot r^2}{a \cdot a}$$

$$\frac{\pi}{2 \cdot 2} \approx \frac{PointsInCircle}{N}$$

$$\pi \approx 4 \cdot \frac{PointsInCircle}{N}$$

Example: Approximation to PI number





Example: Boy born on a Tuesday problem (Gary Foshees)

Gary Foshee created a popular probability puzzle that goes like this: "I have two children. One is a boy born on a Tuesday. What is the probability I have two boys?"

In this puzzle, knowing that one of Gary's children was born on a Tuesday is as important as knowing that he has one boy.

Assuming that having boys and girls are equally likely and that births are equally likely on every day of the week, what is the probability that Gary has two boys, given the available information?

https://www.cut-the-knot.org/Probability/BearBornOnTuesday.shtml

Example: Boy born on a Tuesday problem (Gary Foshees)

"I have two children. One is a boy born on a Tuesday. What is the probability I have two boys?"

Simulated N families, let all families have 2 children.

Throw the dice or their sex and the weekday of their birth.

The probability is the number of families with two boys and one boy born on a tuesday divided by the number of families with one boy born a tuesday

To simplify coding:

sex => boy= 1 / girl =2

day => Monday=1, Tuesday=2… Sunday=7

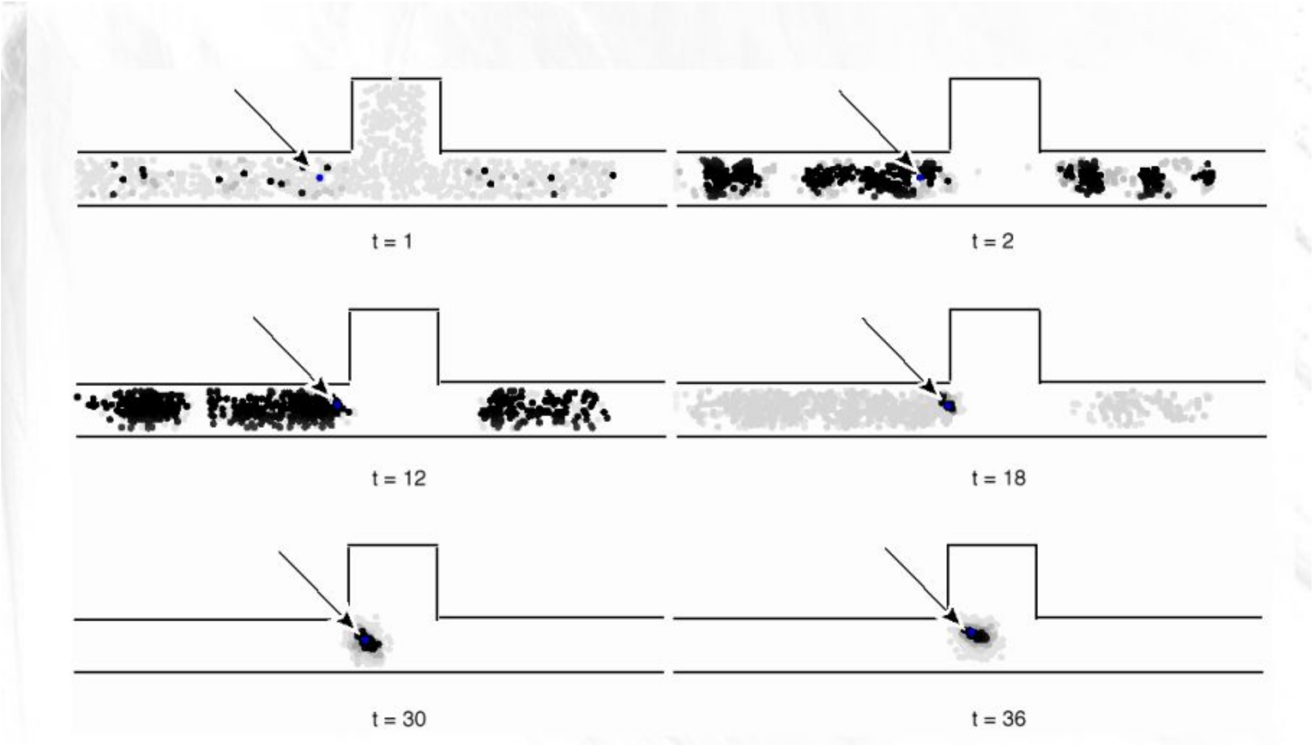
Example: Boy born on a Tuesday problem (Gary Foshees)

"I have two children. One is a boy born on a Tuesday. What is the probability I have two boys?"

```
N=10000000
sex1=randi(2,N,1);
sex2=randi(2,N,1);
day1=randi(7,N,1);
day2=randi(7,N,1);
oneTboy= sex1==1 & day1==2 | sex2==1 & day2==2;
Tboys= sex1==1 & sex2==1;
oneTboyAnd2Boys= oneTboy & Tboys;
P=sum(oneTboyAnd2Boys) / sum(oneTboy)
```

Example: applications on robotics (localization indoor)

While the robot advances, it measures with its laser sensor and obtains the probability of being at that point, the greatest probabilities are accumulating the points around it.



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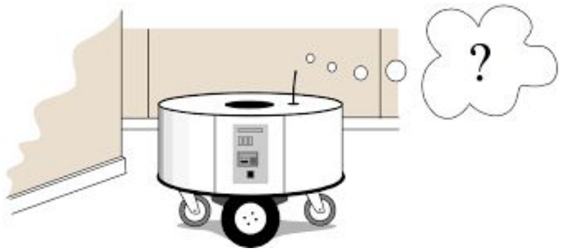
METHOD Montecarlo Localization

1. Model and simulate the movement of the robot and its sensors
2. Distribute points randomly throughout the map, each point is a simulation of a possible location of the robot.
3. Compare the sensor measurement of the robot with the measurements of the simulated robots (points) and obtain a probability
4. Higher odds stay, lower odds are re-sampled
5. This is done until the algorithm converges (until high probabilities remain)



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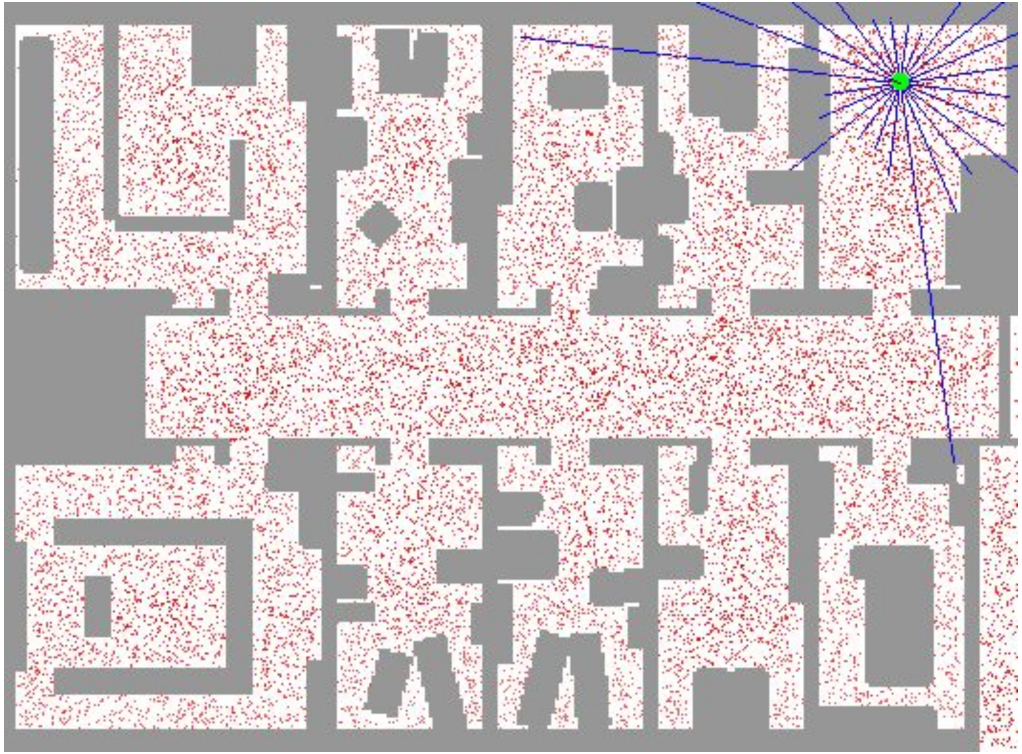

Conclusions

The consistent and reasonable results obtained confirm that the Monte Carlo method provides an efficient tool to estimate the results of different problems.