Instantaneous lateral velocity estimation of a vehicle using Doppler radar

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***Abstract*— High-resolution image radars open new opportunities for estimating velocity and direction of movement of extended objects from a single observation. Since radar sensors only measure the radial velocity, a tracking system is normally used to determine the velocity vector of the object. A stable velocity is estimated after several frames at the earliest, resulting in a significant loss of time for reacting to certain situations such as cross-traffic. The following paper presents a robust and model-free approach to determine the velocity vector of an extended target. In contrast to the Kalman filter, it does not require data association in time and space. An instant (~ 50 ms) and bias free estimation of its velocity vector is possible. Our approach can handle noise and systematic variations (e.g., micro-Doppler of wheels) in the signal. It is optimized to deal with measurement errors of the radar sensor not only in the radial velocity, but in the azimuth position as well. The accuracy of this method is increased by the fusion of multiple radar sensors.**

Keywords— intelligent vehicles, dynamic driving environment perception, extended object tracking, Doppler radar, motion estimation, velocity profile

I. INTRODUCTION

Today most of the automotive radar based systems such as adaptive cruise control (ACC) are designed for highway applications. Because of traffic direction, highway scenarios are dominated by longitudinal motion since vehicles approach or move away mainly in that dimension. Lateral motion only appears during lane changes. Hence, there is a minor focus on lateral traffic in radar-based driving assistant systems. Radar systems measure the radial distance and radial velocity of an object very accurately. The detection and tracking of lateral motion appears to be an important handicap [1]. With the advent of driving assistant systems in city traffic applications, the lateral velocity estimation is growing in importance. It is desirable to detect all targets surrounding the ego-vehicle and measure their position and direction of movement.

The time available to detect a potential hazardous situation is exceedingly short. On highways, potential dangerous situations normally arise at a large distance near the end of the field of view of the sensor. Compared to city scenarios, other vehicles can appear on a random side of the field of view, are obscured by other vehicles or emerge from a hidden side road. A faster determination of velocity and orientation from a single observation for lateral traffic opens new possibilities for advanced driver assistance systems.

With the development of high resolution radar sensors, the traditional point-source assumption is no longer valid. The radar receives more than one reflection from an extended object (further called target). Tracking gets more challenging, due to the need for a stable reference point which is difficult to extract out of multiple targets and in the event of partial occlusions. By analyzing the measured radial velocity over the azimuth position, a so-called velocity profile of the object is estimated without need of a reference point. The parameters of the velocity profile are used to estimate the velocity vector of the object in a single frame.

The paper is organized as follows: Section 2 reviews related work. The problem formulation is presented in Section 3. Section 4 gives an overview of the velocity estimation algorithm and describes in detail the three main steps. Section 5 presents the fusion of multiple radar sensors. Simulation results and the influence of crucial system parameters are discussed in Section 6. Section 7 provides experimental results for the fusion of two radar sensors and for a comparison of our algorithm with state-of-the-art approaches.

II. RELATED WORK

In this paper only methods based on a single frame and for a linear motion of the observed object are considered. The orientation and velocity can be estimated in two different ways, using its geometric extension or the analysis of the radial velocity over the azimuth angle. Both methods can be applied only on high-resolution radar, since at least two targets of the same object are required.

A. Geometric extension

Laser scanners receive an almost deterministic and detailed image of the vehicle contour and consequently the estimation of the orientation is a solvable problem, as done e.g. in [3]. In contrast radar sensors receive less detailed reflections of the objects, depending on a various number of parameters like vehicle shape, orientation of vehicle, viewing angle etc.

In [4] an accurate vehicle radar response model is proposed which is able to fully exploit the information in the detections. It describes the radar response of a vehicle as a fixed number of point- and plane reflectors. The radar receives a reflection only if the sensor is within a reflector-specific visibility region. Using this model, the orientation of the vehicle can be estimated from the received reflection centers. Knowing the orientation, the velocity vector can be simply extracted out of the radial velocity of one target.

This model was originally developed for simulation purposes. Real radar sensors have a limited resolution, hence there is an uncertainty regarding which vehicle reflections are resolved and which are clustered. Furthermore, the targets are not equally distributed due to a different range and azimuth resolution. These effects and the presence of clutter lead to a difficult data association problem with a large number of hypotheses. For a good overview of common association methods dealing with this problem the reader is referred to [5] or [6].

B. *Radial velocity analysis*

This method analysis the measured radial velocity over the azimuth angle and was first published in [2]. A 24 GHz Doppler radar sensor is used to receive more than one target of a crossing vehicle in approximately 20m distance. A Least-Square (LSQ) approach is used to determine the velocity vector of the vehicle in one measurement based on four targets. In [7] a second measurement is presented, in which the lateral velocity of a delivery van is estimated based on six targets. Due to a missing reference system and the analyzing of only one single frame, it is impossible to make a statement about the accuracy and robustness of the proposed method.

III. PROBLEM FORMULATION

The aim of the work is to design a robust and universal algorithm for instantly analyzing the velocity vector (orientation and absolute value) of a vehicle (linear motion) in a single frame with one Doppler radar sensor. To deal with any kind of vehicles (trucks, vans etc.), the algorithm cannot be based on a model assumption. Further properties should be a high sampling rate and a time-independent signal processing without any initialization period. Radar is well-known to be capable of detecting accurate the radial velocity and azimuth angle of a target.

The algorithm presented in [2] is able to show the general functionality of the idea and meets most of the presented requirements, but is not suitable for a real world application. During a test on a couple of sequences for crossing vehicles, several problems occurred. The main points are shortly summarized and discussed later in detail.

A simple LSQ approach is not robust enough to analyze a complete sequence. The main reasons are clutter and micro-Doppler effects of the wheels. Depending on the exact position of a target on the wheel, the received Doppler-velocity is usually inside 0 … 2v, with v the velocity of the vehicle assuming no wheel slippage. Another error source is the previous cluster process. This effect is discussed in detail for example in [8] where the effect is used to classify vehicles. If there are other vehicle or stationary objects in proximity to the considered vehicle, wrong velocity information is included.

Another point is that the LSQ estimator is biased, resulting in a constant error for the velocity and orientation estimation. The main reason is that “the presence of errors of measurement in the independent variables in univariate linear regression makes the ordinary least square estimators inconsistent and biased” [9]. That is the case, since there is not only a measurement uncertainty for the radial velocity estimation (depended variable), but also for the azimuth angle estimation (independent variable).

IV. VELOCITY ESTIMATION ALGORTIHM

The algorithm described in this paper is based on an analysis of the Doppler values over the azimuth angle. From an undefined number of reflection centers, different radial velocities are measured by a 76 Ghz pre-series radar. Taking their azimuth angle into consideration, they form a cosine, further called velocity profile.

After receiving the signal, a cluster algorithm identifies targets belonging to the same object due to their close position proximity. A simple cluster algorithm like dbscan [10] can be used for this purpose. This step is not considered in this paper and it is assumed that the input parameters of the algorithm are the targets of one cluster.

For each cluster, three independent steps are performed. The targets of one cluster are cleared from clutter, micro-Dopplers and cluster-errors without any model assumptions and based on a single frame. After that, a bias-free estimator is used to calculate the exact velocity profile. In the last step the velocity parameters are extracted directly out of the determined velocity profile. The three steps are shown in Fig. 1 and described in detail in the following sections.

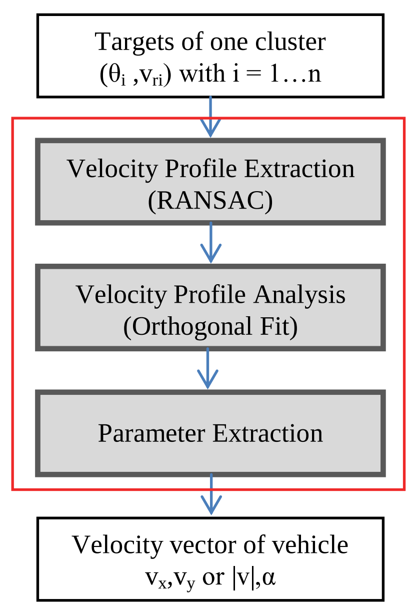

Fig. 1. Overview of the different steps of the system

A. *Theoretical Background*

The theoretical background is that all points on the vehicle (linear motion) have the identical velocity vector (v_x,v_y). For all received targets (i = 1…N), the Doppler radar gets the measured radial velocity (v_r) depending on its azimuth angle (θ). This is shown in Fig. 2 and can be formulated as in (1).

$$\begin{bmatrix} v_{r,1} \\ v_{r,2} \\ \vdots \\ v_{r,N} \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) \\ \cos(\theta_2) & \sin(\theta_2) \\ \vdots & \vdots \\ \cos(\theta_N) & \sin(\theta_N) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} \tag{1}$$

Equation (1) is a cosine with the parameters v_x and v_y, called velocity profile. The velocity profile is a linear system which is over-determined for more than two targets and a regression analysis can be used to determine its parameters.

Equation (1) can be transformed in a general cosine form with orientation α and absolute velocity v:

$$\begin{gathered} v_{r,i} = v(\cos(\alpha)\cos(\theta_i) + \sin(\alpha)\sin(\theta_i)) \\ \text{using } v_x = v\cos(\alpha) \text{ and } v_y = v\sin(\alpha) \\ v_{r,i} = v\cos(\theta_i - \alpha) \end{gathered} \tag{2}$$

As can be seen, the phase-shift of the velocity profile is identical to the negative vehicle orientation and the amplitude is identical to the absolute velocity.

A crossing car (α = 0°) and a car, which is heading towards the sensor (α = 90°) differ significantly in their velocity profile (Fig. 3). The velocity profile of a crossing car contains a high velocity variation and has a zero-velocity point at the position where the azimuth angle is orthogonal to the moving direction of the vehicle. The orientation of the vehicle can be interpolated, whereas the absolute velocity can only be extrapolated.

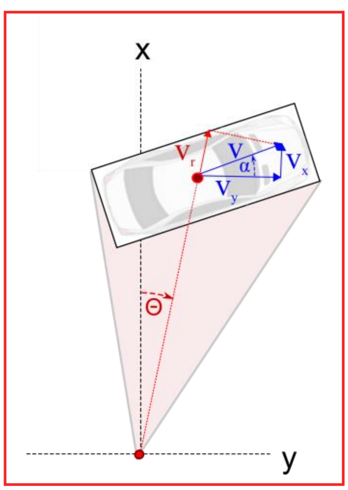

Fig.2 Measurement of one target on a vehicle, with the measurement values (red) and the motion parameters of the vehicle (blue)

For a car heading towards the ego-vehicle, the situation is reversed. The velocity of the vehicle is nearly parallel to the position vector of the targets, so that the measured radial velocity is almost equal to the velocity of the vehicle. Its velocity profile hardly changes, since it contains only the peak of the cosine. It usually covers a smaller azimuth area, because a vehicle has a smaller width than length. The absolute velocity can directly be interpolated, whereas the orientation is extrapolated.

B. *Velocity Profile extraction*

In the preprocessing step targets are identified, belonging to the bulk motion of the vehicle. Micro-Dopplers and clutter are excluded, since they would strongly influence the estimation of the velocity profile. This is performed on a single frame just using the radial velocity and the azimuth angle. The exact position of the targets and their radar cross section (RCS) are not required, but can be integrated to improve the result. With the usage of the raw target data, the algorithm avoids data association and saving of previous scans.

A Random Sample and Consensus (RANSAC) algorithm is used to identify the velocity profile. RANSAC was first published in [11] and is normally used to eliminate outliers in a dataset. RANSAC is used for “fitting a model to experimental data” and is “capable of interpreting/smoothing data containing a significant percentage of gross errors” [11].

In each iteration, two targets (θ_i, $v_{r,i}$) are randomly chosen and the parameters of their velocity profile (v_x, v_y) are calculated using (1). If both targets differ in their parameter values, the equation has a unique solution. The error of the current fit is estimated by the sum of the radial velocity errors of all targets to the determined velocity profile. In this process, a corridor threshold is used to identify outliers and limit their error to the corridor threshold. All targets inside the corridor are called inliers and their error is added up directly. After a certain number of iterations (result-driven), the best fit is chosen according to the calculated error or/and number of inliers.

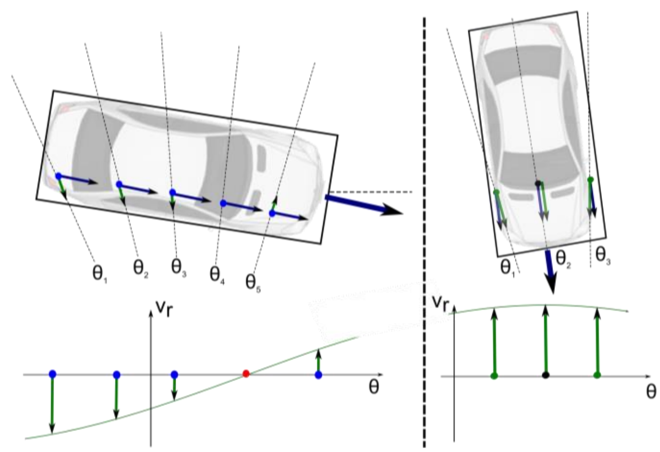

Fig. 3. Velocity profile of a linear moving vehicle (blue vector). For each observation the radial velocity component is measured by the radar sensor (green vector). Considering one radial cell the velocity describes a cosine (green curve), for a crossing car (left) and a car heading towards the sensor (right)

C. *Velocity Profile Analysis*

With the identification of all targets belonging to the bulk motion (inliers of RANSAC), a precise determination of the velocity profile is performed using an orthogonal regression analysis.

Orthogonal regression is used to correct the effects of measurement errors in predictors, also called errors-in-variables regression. Fuller [12] is now the standard reference. Beside the radial velocity, the azimuth angle is measured by the radar sensor, so that both parameters contain measurement errors. It is assumed that both are independent zero-mean variables with variances σ_{vr} and σ_θ and the error variance ratio is known:

$$\eta = \frac{\sigma_{vr}}{\sigma_\theta} \tag{3}$$

The measurement errors are shown in Fig. 4. Both variances are known and specified in the sensor specification. An Orthogonal Distance Regression (ODR) is used. It is associated with finding the maximum likelihood estimators of parameters in measurement error models in the case of normally distributed errors when the underlying model is assumed to be nonlinear in the independent variable (in our case θ). [13]

Another assumption is that there is no coupling between the measurement errors of the radial velocity and azimuth angle, resulting in a diagonal covariance matrix. [14] provides a brief overview of the method. The present algorithm defines the "true" predictor as X (of θ), which is unobservable. The orthogonal regression estimator is then obtained by minimizing:

$$\sum_{i=1}^{N} \left(\mathrm{v}_{r,i} - v_x \cos(X_i) - v_y \sin(X_i)\right)^2 + \eta\left(\theta_i - X_i\right)^2 \tag{4}$$

Solving (4) directly results in an optimizing problem containing not only the two unknown system parameters (v_x, v_y), but N unknown estimates of X_i. The result is a large and complex optimization problem. Therefore, the problem is separated into two solution steps by alternating projection, discussed, e.g., in [15]. Using this method, both system parameters are solved in one step and then all X_i are solved separately and parallel. Fig. 5 shows the distance error of the LSQ and ODR-solution.

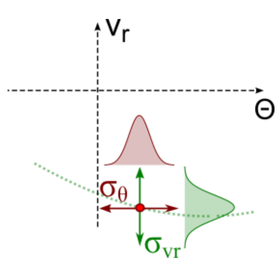

Fig. 4. Modeling of measurement errors – angle accuracy (red) and radial velocity accuracy (green)

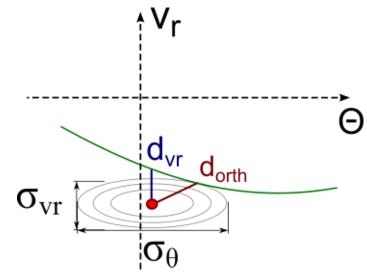

Fig. 5. Comparison of ODR-error (d_{orth}) and LSQ-error (d_{vr})

It begins with an initial solution of the system parameters by a standard LSQ-Fit (v_{x0}, v_{y0}). Based on the first guess the following steps are performed j-times:

1. Optimization of all $X_{i,j}$ separately, considering each term of the sum of (4) by itself. This can be done, since all measurement points are independent of each other and the system parameters are held fixed. Due to the compact size of the optimization problem, a Levenberg-Marquardt algorithm is used.

2. In the next step new system parameters are estimated $(v_{x,j}, v_{y,j})$, using the adapted X_i, determined in the previous step. This step is also solved by a Levenberg-Marquardt algorithm and can be written as a vector equation (vectors are overlined):

$$\left(\overline{v}_r - v_{x,j}\cos(\overline{X}) - v_{y,j}\sin(\overline{X})\right)^2 + \eta\left(\overline{\theta} - \overline{X}\right)^2 \tag{5}$$

3. A convergence criterion, which considers the change in the orthogonal error, is checked and the optimization is stopped if it falls below a threshold.

Step 1. can be used as distance criterion in the velocity profile extraction (Section B) instead of the radial velocity.

An example is shown in Fig. 6. Ten targets and ten micro-Dopplers are received with the sensor specific accuracy ($\sigma_\theta = 1°$ and $\sigma_{vr} = 1$ m/s) from a crossing car ($\alpha = 0°$, $v = 5$ m/s). The RANSAC solution shows only minor differences ($\Delta v_x = 0.1$ m/s and $\Delta v_y = 0.3$ m/s) to the ground truth, whereas the results of the LSQ solution differ significantly ($\Delta v_x = 2.4$ m/s and $\Delta v_y = 4.9$ m/s).

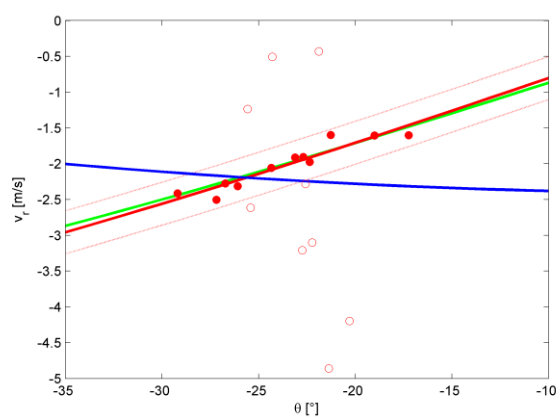

Fig. 6. Simulation of a car with v = 5 m/s and α = 0° with 10 targets and 10 random micro-Dopplers. Correct velocity profile (green), LSQ solution (blue) and RANSAC solution (red) with corridor (red-dotted) and inliers (filled).

D. *Parameter extraction*

The movement of the vehicle is determined using the estimated system parameters v_x and v_y. Its absolute velocity and orientation can be calculated out of these parameters. Furthermore, some quality values such as orthogonal distance error, number of inliers, ratio of inliers to outliers etc., can be calculated.

V. FUSION OF MULTIPLE RADAR SENSORS

It is obvious that a combination of multiple radars could significantly improve the accuracy of the proposed system. The requirements for the fusion of the targets received from different radar sensors are discussed in this section. In general, all available radar sensors with a known orientation to the ego-vehicle and the ability to determine the radial velocity and azimuth angle of a target can be fused.

In a pre-processing step, all targets of the same vehicle received by different sensors have to be fused. This can be done on a target or a cluster level. The clustering is not discussed further here and it is assumed, that all targets of a vehicle measured by all sensors are clustered.

In case of a different mounting orientation of the sensors, the azimuth position angles of the received targets have to be aligned on a common axis, e.g. the x-axis of the coordinate system of the ego-vehicle. Then the velocity profile can be determined independent of the target origin, as described in the previous section.

The advantage of using two or more sensors is shown in Fig. 7. Not only is the number of received targets increased, but the total azimuth area as well, resulting in a more reliable estimation of the velocity profile properties.

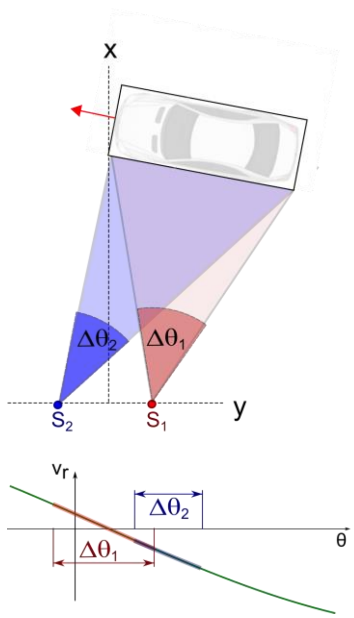

Fig. 7. Fusion of two radar sensors, resulting in an extended total azimuth area of the velocity profile (different aspect angles).

VI. SIMULATION RESULTS

A. *Simulation Overview*

It is difficult to evaluate the system using experimental results, since the quality of the results strongly depend on a number of parameters. They can be separated into three groups:

- Sensor parameters such as azimuth accuracy, Doppler velocity accuracy, field of view and sampling rate
- Velocity profile properties such as number of reflections and their distribution, covered azimuth area, micro-Dopplers and clutter
- Motion parameters of the vehicle such as velocity and orientation

Therefore, the performance of the algorithm is simulated under well-defined conditions. The measurement uncertainties of the radar sensors are modeled as an azimuth accuracy ($\sigma_\theta = 1°$) and a speed accuracy ($\sigma_{vr} = 1$ m/s). Both errors are zero-mean Gaussian distributed (Fig 4.).

A Monte-Carlo simulation is used to determine the accuracy of the proposed method compared to the LSQ solution for a car (length 5 m, width 2 m, v = 5 m/s) at a distance of 15m with 10 targets on the vehicle. It is assumed that the targets are equally spaced over the vehicle. The simulation was carried out 100000 times. The results are considered in terms of the estimation of the vehicle orientation (α_{est}) and absolute velocity (v_{est}). The mean error (μ) and standard deviation (σ) are investigated over the azimuth position angle of the vehicle. Except for the first simulation, only the accuracy of the velocity estimation is shown since the behavior of the orientation estimation is similar

B. *Influence of Motion Parameters*

The first simulation investigates the estimation accuracy depending on the vehicle orientation, e.g., a crossing car ($\alpha = 90°$) or a car heading towards the sensor ($\alpha = 0°$). The simulation is also performed for the fusion of two sensors (position: +/- 0.9 m). Both sensors receive 5 targets, so their sum is equal to the simulation with one sensor (10 targets).

Fig. 8 shows the accuracy of the velocity estimation and Fig. 9 the accuracy of the orientation estimation. The standard deviations of the ODR-solution are slightly higher than for the LSQ-solution. The orientation estimation is most precise for $\alpha = 0°$ and the velocity estimation for $\alpha = 90°$. The LSQ-solution has a significant bias error in the velocity estimation for $\alpha = 0°$ and for the orientation estimation for $\alpha \sim 40°$-$80°$. The results show no bias error for the ODR-solution. The use of two radar sensors increases the accuracy of the velocity estimation. The two sensor solution reduces the bias error of the LSQ-solution slightly.

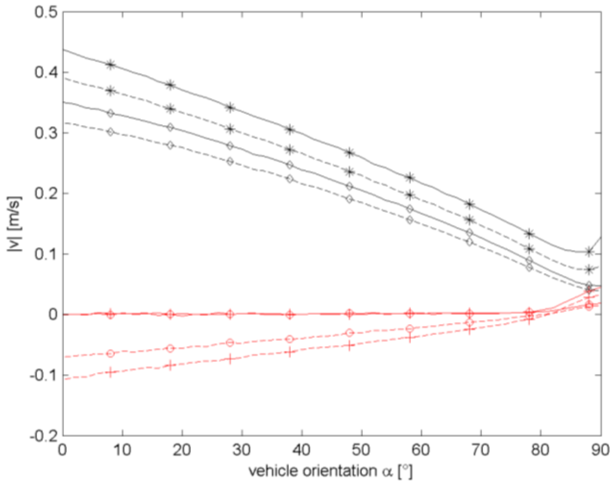

Fig. 8. Accuracy of the velocity estimation depending on the vehicle orientation for LSQ-solution (dotted) and ODR-solution (solid) in terms of mean error with 1 Sensor (red +) and 2 Sensors (red o) and standard deviation with 1 Sensor (black *) and 2 Sensors (black diamond).

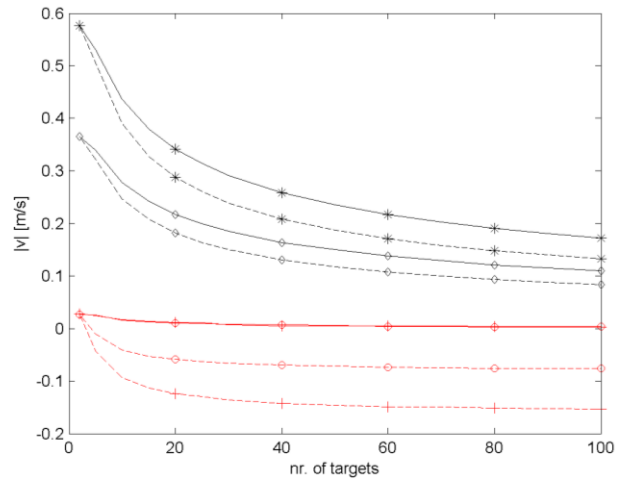

Fig. 10. Accuracy of the velocity estimation depending on the number of received targets for LSQ-solution (dotted) and ODR-solution (solid) in terms of mean error with α = 0° (red +) and α = 45° (red o) and standard deviation with α = 0° (black *) and α = 45° (black diamond).

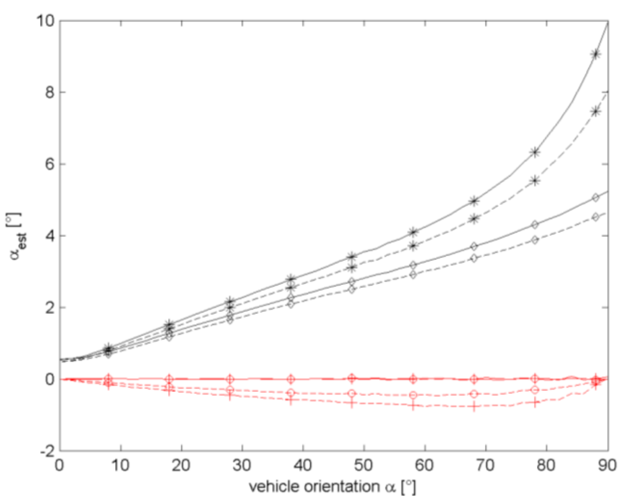

Fig. 9. Accuracy of the orientation estimation depending on the vehicle orientation for LSQ-solution (dotted) and ODR-solution (solid) in terms of mean error with 1 Sensor (red +) and 2 Sensors (red o) and standard deviation with 1 Sensor (black *) and 2 Sensors (black diamond).

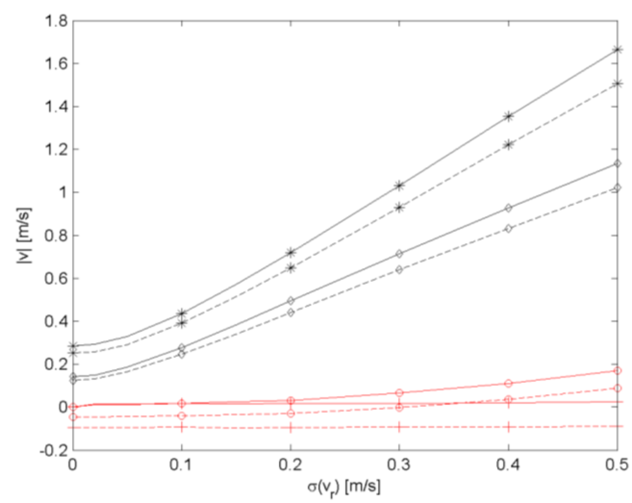

Fig. 11. Accuracy of the velocity estimation depending on the radial velocity accuracy of the sensor - LSQ-solution (dotted) and ODR-solution (solid) in terms of mean error with α = 0° (red +) and α = 45° (red o) and standard deviation with α = 0° (black *) and α = 45° (black diamond).

C. Influence of Velocity Profile Properties

The next simulation examines the influence of the number of received targets for α = 0° and α = 45° with one sensor. The result for the velocity estimation is shown in Fig. 10. The standard deviations decrease with the inverse square route of the number of targets. The ODR-solution shows only a small bias for less than 10 targets, whereas the LSQ-solution shows a significant bias.

D. Influence Sensor Parameters

The next simulation shows the dependencies of the sensor on the radial velocity accuracy for α = 0° and α = 45° with one sensor. Fig. 11 shows the accuracy of the velocity estimation. The standard deviations increase almost linearly. There is always a bias-error for the LSQ-solution. For the ODR-solution, a small bias error occurs for a radial velocity accuracy larger than 0.2 m/s. The last simulation shows the dependencies on the azimuth angle accuracy of the sensor for α = 0° and α = 45° with one sensor. Fig. 12 shows the accuracy of the velocity estimation. The increase of the standard deviations is smaller than for the radial velocity accuracy. The error caused by a radial velocity accuracy of 0.25 m/s is approximately as large as for an azimuth angle accuracy of 3°. The bias error increases over proportional for the LSQ-solution and gets even larger than the associated standard deviation. There is a small increase in the bias-error of the ODR solution for accuracies larger than 1.5°.

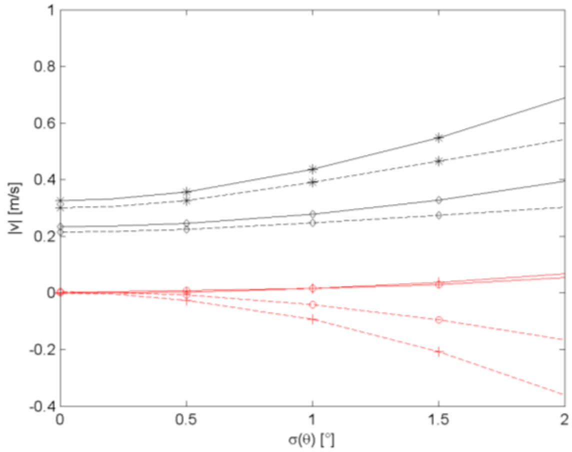

Fig. 12. Accuracy of the velocity estimation depending on the azimuth angle accuracy of the sensor - LSQ-solution (dotted) and ODR-solution (solid) in terms of mean error with $\alpha = 0°$ (red +) and $\alpha = 45°$ (red o) and standard deviation with $\alpha = 0°$ (black *) and $\alpha = 45°$ (black diamond).

E. *Simulation Interpretation*

Considering the velocity profile shown in Fig. 3, the results can be analyzed in detail. In general it could be stated that the closer the measured velocity profile part is to the amplitude and zero-crossing, the better is its respective estimation. The reason is that a larger extrapolation distances increases the corresponding standard deviations. This explains the good results of the velocity estimation for a car heading towards the sensor and of the orientation estimation for a crossing car. There is a large dependency on the radial velocity accuracy and the number of received targets.

The simulations show a large bias-error of the LSQ-solution, which mainly depends on the azimuth angle accuracy of the sensor. It also depends on the extrapolation distance, as well as on the properties of the velocity profile, like asymmetry or curvature. Fig. 13 shows the displacement of the mean value caused by an azimuth position error and the curvature of the velocity profile.

Another influence is the width of the velocity segment, which can be enlarged by the fusion of two sensors. Due to the different aspect angles, the azimuth segment is enlarged. Furthermore the number of targets is usually doubled, which results in a further improvement.

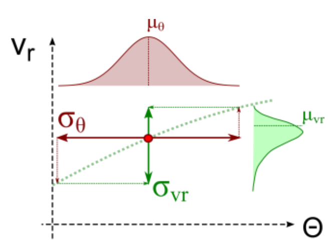

Fig. 13. Effect of the azimuth position error σ_{vr} on the LSQ-solution, which interprets this error as a radial velocity error σ_{θ}. This results in a deviation of the mean error.

VII. Experimental Results

The system is tested with commercial 76 GHz radar sensors for automotive safety applications. The sampling and computation time is approximately 50 ms. The parameters of the sensor are:

- speed accuracy: 0.1 m/s
- azimuth accuracy: 1°
- update rate: 20 Hz
- field of view: +/- 45°

A. *Fusion of two radar sensors*

In the first scene an observed vehicle is passing the ego-vehicle in a distance of approximate 15 m (linear motion, constant velocity). The ego-vehicle is equipped with two sensors both turned outwards to each side with 30° at both front corners, respectively. In Fig. 14 the orientation and absolute velocity estimation for the proposed algorithm (RANSAC + ODR) are shown. The standard deviations are examined for the complete scene and for a position of the observed vehicle inside both mounting angles ($-30° < \theta < 30°$) and outside:

TABLE I. Standard Deviations – Sensor Fusion

	v [m/s]	**α_{est} [°]**	**avg. nr. targets**
complete scene	0.25	1.05	10
($-30° < \theta < 30°$)	0.17	0.35	20
$\theta < -30°$ or $\theta > 30°$	0.32	1.27	4

The results confirm the simulation results in Fig. 8 ($\sigma_{vr} = 0.3$ m/s) and Fig. 9 ($\sigma_{\alpha,est} = 1.2°$) for the average orientation of $\alpha = 12°$.

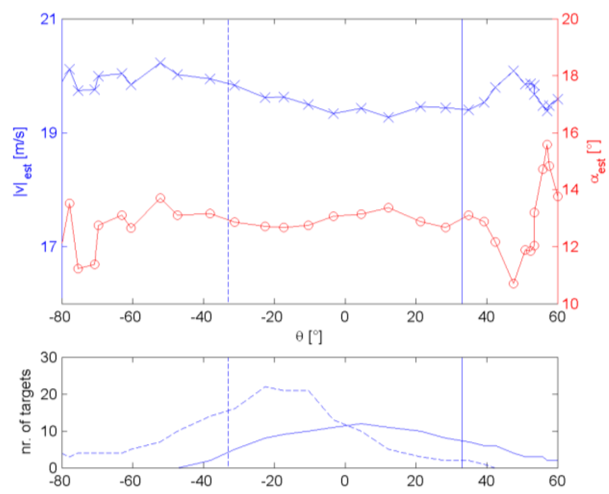

Fig. 14. Top: Orientation estimation (red o) and absolute velocity (blue o) estimation for a crossing car $\alpha \sim 13°$. Botton: Number of received targets of left sensor (dotted) and right sensor (solid). Sensor mouting angle are vertical lines.

The standard deviations get smaller the closer the observed vehicle gets to the ego-vehicle. The fusion of both sensors increases the accuracy significantly, if the observed vehicle is inside the common field of view, as discussed in Section V and seen in the simulation (Fig. 10). The results of the simulation are with $\sigma_{vr} = 0.23$ (nr. targets = 20) and $\sigma_{vr} = 0.31$ (nr. targets = 4) close to the experimental results.

B. Algorithm comparison

In the second scene, a vehicle is passing one sensor (mounting angle = 0°) at a distance of app. 15 m. In Fig. 15 the orientation and absolute velocity estimation for the LSQ-solution and the proposed algorithm (RANSAC + ODR) are shown. The vehicle was passing the sensor with a constant velocity of approximately 10 m/s and an orientation of $\alpha = 0°$.

It can be clearly seen that the RANSAC algorithm stabilizes the result considerably. The LSQ solution is unstable especially at both edges of the field of view. Outliers have a strong impact there due to the small number of received targets and the small azimuth area. Furthermore, the mentioned bias is visible, particularly for the orientation estimation of the LSQ-solution. For a negative azimuth angle, the orientation estimation error is almost continuously negative, whereas for a positive azimuth angle it is always positive. The estimated absolute velocity is lower than 10 m/s most of the time. Both findings are consistent with the simulation in the previous section. Table II shows the standard deviations.

TABLE II. STANDARD DEVIATIONS - ALGORITHM COMPARISON

	v [m/s]	α_{est} [°]
RANSAC + ODR	0.4	2.5
LSQ	2.7	6.4

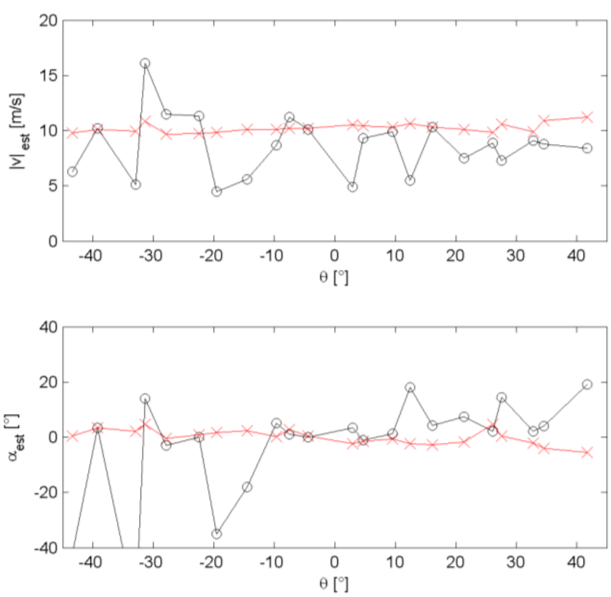

Fig. 15. Orientation estimation (bottom) and absolute velocity (top) estimation for a crossing car $\alpha = 0°$, with LSQ-estimator (black o) and RANSAC+ODR-estimator (red x)

VIII. CONCLUSION

This paper presented an algorithm to instantly determine the orientation and absolute velocity of a linear moving vehicle on a single frame. It uses the radial velocity information contained in multiple reflection of one vehicle to estimate the underlying velocity vector. It was shown that this is a nonlinear error-in-variables optimization problem and the LSQ-estimator contains a significant bias-error. Therefore, an optimized ODR-estimator was designed for fitting a cosine. A simulation proved that the results are bias-free. However, the real data can contain outlier targets, such as micro-Doppler from the wheels or clutter, and a stable result can only be obtained by using a robust algorithm. Therefore, a RANSAC was implemented and a significant improvement of the stability obtained in the results for a real scenario. The fusion of two or more radar sensors improves the results considerably.

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