Multi-Hypothesis

A clear explanation

Multi-Hypothesis representation

- local hypotheses(hypotheses trees) representation
  - global weights
  - hypotheses look-up table
  - local hypotheses forest
- global hypotheses representation
  - global weights
  - global hypotheses

global weights   lookup table   hypotheses forest

#mixture components

w1	1 1 1 1	h1 h1 h1 h1
w2	1 0 0 1	h2 h2 h2 h2
w3	1 3 2 1	h3 h3 h3
w4	2 1 4 3	h4
w5	1 2 1 1	
w6	2 0 1 1	

for MBM filter

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \sim Bernoulli\ RFS \end{cases}$$

for TO-MHT

$$hn = Gauss\ state \begin{cases} mean \\ covar \end{cases} \sim Gauss\ dist.$$

#hypotheses trees   #hypotheses trees

for general case
    $hn = $ *some generic valid state*

h1 h1 h1 h1   *multi Bernoulli*

w1   h1 h1 h1 h1   *weighted multi Bernoulli*

global weights *lookup table* *hypotheses forest*

w1	1	1	1	1	h1	h1	h1	h1	
w2	1	0	0	1	h2	h2	h2	h2	
w3	1	3	2	1		h3	h3	h3	
w4	2	1	4	3			h4		
w5	1	2	1	1					
w6	2	0	1	1					

#mixture components

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \end{cases} \sim \; Bernoulli \; RFS$$

#hypotheses trees   *#hypotheses trees*

h1		h1

multi Bernoulli

w1	h1 h1 h1 h1
w2	h1 h1

weighted multi Bernoulli

global weights   *lookup table*   *hypotheses forest*

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \end{cases} \sim Bernoulli\ RFS$$

#mixture components

#hypotheses trees   #hypotheses trees

multi Bernoulli

weighted multi Bernoulli

global weights　　lookup table　　hypotheses forest

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \end{cases} \sim Bernoulli\ RFS$$

#mixture components

#hypotheses trees　　#hypotheses trees

multi Bernoulli

weighted multi Bernoulli

global weights   lookup table   hypotheses forest

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \end{cases} \sim Bernoulli\ RFS$$

#mixture components

#hypotheses trees   #hypotheses trees

multi Bernoulli

weighted multi Bernoulli

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \end{cases} \sim Bernoulli\ RFS$$

global weights *lookup table* *hypotheses forest*

#mixture components

#hypotheses trees *#hypotheses trees*

multi Bernoulli

multi Bernoulli mixture(MBM)

global weights   lookup table   hypotheses forest

w1	1	1	1	1	h1	h1	h1	h1	
w2	1	0	0	1	h2	h2	h2	h2	
w3	1	3	2	1		h3	h3	h3	
w4	2	1	4	3			h4		
w5	1	2	1	1					
w6	2	0	1	1					

#mixture components

delete unused leaves

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \end{cases} \sim Bernoulli\ RFS$$

#hypotheses trees

#hypotheses trees

multi Bernoulli

	h2	h2	h1	h1

w1	h1	h1	h1	h1
w2	h1			h1
w3	h1	h3	h2	h1
w4	h2	h1	h4	h3
w5	h1	h2	h1	h1
w6	h2		h1	h1

multi Bernoulli mixture(MBM)
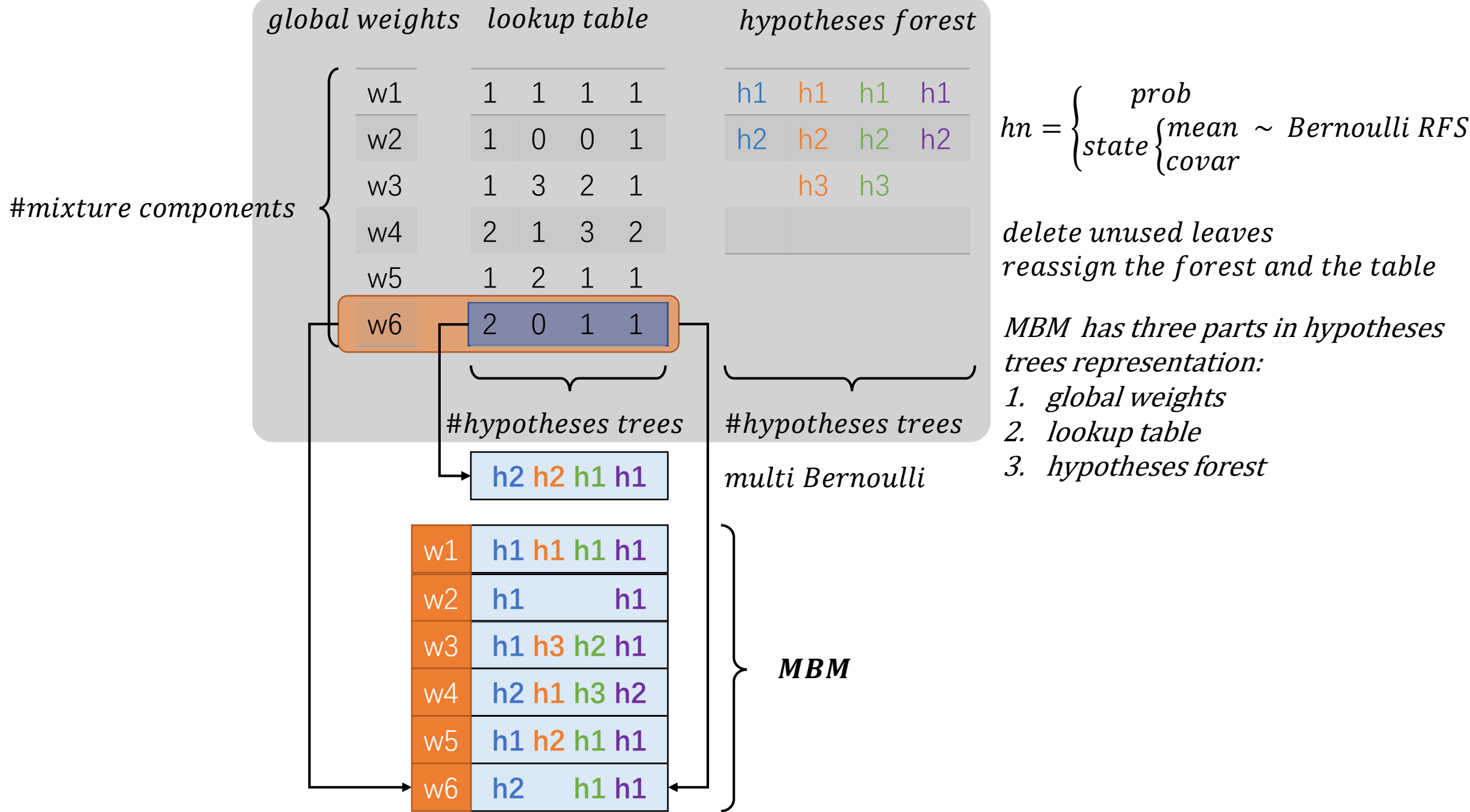
global weights   *lookup table*   *hypotheses forest*

w1	1	1	1	1		h1	h1	h1	h1
w2	1	0	0	1		h2	h2	h2	h2
w3	1	3	2	1			h3	h3	
w4	2	1	3	2					
w5	1	2	1	1					
w6	2	0	1	1					

#*mixture components*

$$hn = \begin{cases} prob \\ state \begin{cases} mean \\ covar \end{cases} \sim Bernoulli\ RFS \end{cases}$$

delete unused leaves
reassign the forest and the table

#*hypotheses trees*   #*hypotheses trees*

h2	h2	h1	h1

multi Bernoulli

MBM has three parts in hypotheses
trees representation:
1. global weights
2. lookup table
3. hypotheses forest

w1	h1	h1	h1	h1
w2	h1			h1
w3	h1	h3	h2	h1
w4	h2	h1	h3	h2
w5	h1	h2	h1	h1
w6	h2		h1	h1

MBM

MBM has two parts in global hypotheses
representation:
1. global weights
2. global hypotheses

Multi-Hypothesis predict(local hypo. repr.)

- local hypotheses(forest) predict
  - for each local hypo. in each hypo. tree
    - predict local hypo.
  - global weights / look-up table don't change.

timestamp = k

global weights

#mixture components

w1	1	1	1	1
w2	1	0	0	1
w3	1	3	2	1
w4	2	1	3	2
w5	1	2	1	1
w6	2	0	1	1

hypotheses forest

h1	h1	h1	h1
h2	h2	h2	h2
	h3	h3	

hn = generic state

multi hypotheses

timestamp = k + 1

global weights

#mixture components

w1	1	1	1	1
w2	1	0	0	1
w3	1	3	2	1
w4	2	1	3	2
w5	1	2	1	1
w6	2	0	1	1

hypotheses forest prediction

h1	h1	h1	h1
h2	h2	h2	h2
	h3	h3	

hn = generic state

multi hypotheses prediction

Multi-Hypothesis update(local hypo. repr.)

- local hypotheses(forest) update
  - for each local hypo. in each hypo. tree
    - gating(ellipsoidal gating in Gaussian state)
    - update hypo. and associated measurement log-likelihood with missed detection and associated measurements in gate
  - resulted in [#meas in gate + 1(missed hypo)] updated hypos and measurement log-likelihoods correspondingly
- global weights / look-up table / hypo. forest update
  - for each predicted global hypothesis
    - create cost matrix for data association(DA)(use the associated measurement log-likelihoods calculated before)
    - Murty DA
    - update global weights / look-up table / hypo. forest

timestamp = k

global weights	lookup table	hypotheses forest

hn = generic state

#mixture components

	global weights	lookup table				hypotheses forest			
	w1	1	1	1	1	h1	h1	h1	h1
	w2	1	0	0	1	h2	h2	h2	h2
	w3	1	3	2	1		h3	h3	
	w4	2	1	3	2				
	w5	1	2	1	1				
	w6	2	0	1	1				

multi hypotheses

timestamp = k + 1   new measurements: $z1, z2$

hypotheses forest

h1	h1	h1	h1
h2	h2	h2	h2
	h3	h3	

leaves update

h1,0	h1,0	h1,0	h1,0
h1+z1	h1+z1	h1+z1	h1+z1
h1+z2	h1+z2	h1+z2	h1+z2
h2,0	h2,0	h2,0	h2,0
h2+z1	h2+z1	h2+z1	h2+z1
h2+z2	h2+z2	h2+z2	h2+z2
	h3,0	h3,0	
	h3+z1	h3+z1	
	h3+z2	h3+z2	

association negative log likelihoods

L1,0	L1,1	L1,2
L2,0	L2,1	L2,2

L1,0	L1,1	L1,2
L2,0	L2,1	L2,2
L3,0	L3,1	L3,2

L1,0	L1,1	L1,2
L2,0	L2,1	L2,2
L3,0	L3,1	L3,2

L1,0	L1,1	L1,2
L2,0	L2,1	L2,2

*Lm, n means **negative** loglikelihood with the leaf m associated to the meas n at the blue tree, $n = 0$ means missed hypothesis*

timestamp = k + 1   *new measurements*: $z1, z2$

hypotheses forest

h1	h1	h1	h1
h2	h2	h2	h2
	h3	h3	

leaves update

h1	h1	h1	h1
h2	h2	h2	h2
h3	h3	h3	h3
h4	h4	h4	h4
h5	h5	h5	h5
h6	h6	h6	h6
	h7	h7	
	h8	h8	
	h9	h9	

association log likelihood

L1,0	L1,1	L1,2
L2,0	L2,1	L2.2

L1,0	L1,1	L1,2
L2,0	L2,1	L2,2
L3,0	L3,1	L3.2

L1,0	L1,1	L1,2
L2,0	L2,1	L2.2
L3,0	L3,1	L3.2

L1,0	L1,1	L1,2
L2,0	L2,1	L2.2

global weights *lookup table*   *cost matrix*: $size[\#trees, \#trees + \#meas] = [4, 4 + 2]$

global weights	lookup table			
w1	**1**	**1**	**1**	**1**
w2	1	0	0	1
w3	1	3	2	1
w4	2	1	3	2
w5	1	2	1	1
w6	2	0	1	1

L**1**,1	L1,2	L1,0	Inf	Inf	Inf
L**1**,1	L1.2	Inf	L1,0	Inf	Inf
L**1**,1	L1.2	Inf	Inf	L1,0	Inf
L**1**,1	L1.2	Inf	Inf	Inf	L1,0
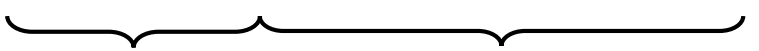
⎰_____⎱ ⎰_____⎱
obj. DA mat.   *missed DA mat.*

Murty DA →

DA matrix

1	2
4	1
2	5
6	6

NOTE: ele. in DA matrix > #trees
means missed DA

global weights   *lookup table*   *cost matrix*: $size[\#trees, \#trees + \#meas] = [4, 4+2]$

w1	1	1	1	1
w2	1	0	0	1
w3	1	3	2	1
w4	**2**	**1**	**3**	**2**
w5	1	2	1	1
w6	2	0	1	1

L**2**,1	L2,2	L2,0	Inf	Inf	Inf
L**1**,1	L1.2	Inf	L1,0	Inf	Inf
L**3**,1	L3.2	Inf	Inf	L3,0	Inf
L**2**,1	L2.2	Inf	Inf	Inf	L2,0

→ Murty DA →

1	2
4	1
2	5
6	6

$cost1 = L2,1 + L1,0 + L3,2 + L2,0$ NOTE: Lm, n *in negative logarithmic field*

$w4,1 = w4 - cost1$

w4,1	h(2-1)*3+2	h(1-1)*3+1	h(3-1)*3+3	h(2-1)*3+1

leaves update

h1	h1	h1	h1
h2	h2	h2	h2
h3	h3	h3	h3
h4	h4	h4	h4
h5	h5	h5	h5
h6	h6	h6	h6
	h7	h7	
	h8	h8	
	h9	h9	

NOTE:
wm, n *in negative logarithmic field*

$w4,1$ *means the* $4th$ *tree and the* $1st$ *meas DA,*
$h(2-1)*3 + 2$ *means the* $2nd$ *leaf*
(idx is $(2-1)*3, 3 = \#meas + 1)$ *in the* $4th$ *tree and the* **1st** *meas DA,*
+1 *means the missed DA.*
+2 *means the* 1st *meas DA.*
+3 *means the* 2nd *meas DA.*
and so on...

global weights *lookup table*   *cost matrix*: $size[\#trees, \#trees + \#meas] = [4, 4+2]$

w1		1	1	1	1
w2		1	0	0	1
w3		1	3	2	1
w4		2	1	3	2
w5		1	2	1	1
w6		2	0	1	1

L2,1	L2,2	L2,0	Inf	Inf	Inf
L1,1	L1.2	Inf	L1,0	Inf	Inf
L3,1	L3.2	Inf	Inf	L3,0	Inf
L2,1	L2.2	Inf	Inf	Inf	L2,0

$\rightarrow$ Murty DA $\rightarrow$

1	2
4	1
2	5
6	6

$cost2 = L2,2 + L1,1 + L3,5 + L2,6$ NOTE: Lm,n in negative logarithmic field

$w4,1 = w4 - cost1$
$w4,2 = w4 - cost2$

w4,1	h(2-1)*3+2	h(1-1)*3+1	h(3-1)*3+3	h(2-1)*3+1
w4,2	h(2-1)*3+3	h(1-1)*3+3	h(3-1)*3+1	h(2-1)*3+1

$\downarrow$

w4,1	h5	h1	h9	h4
w4,2	h6	h3	h7	h4

NOTE:
wm,n in negative logarithmic field

leaves update

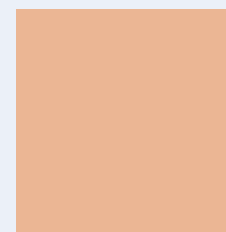| h1	h1	h1	h1
h2	h2	h2	h2
h3	h3	h3	h3
h4	h4	h4	h4
h5	h5	h5	h5
h6	h6	h6	h6
	h7	h7	
	h8	h8	
	h9	h9	

and so on … $\Rightarrow$

timestamp = k + 1
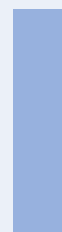
updated global weights   *updated lookup table*   *updated hypotheses forest*