Sampling Distribution:

If we draw all possible samples of same size and for each sample we calculate a statistic then the value of statistic may or may not be varying sample to sample. If we arrange all possible values of that statistic with its corresponding probabilities then this arrangement is known as sampling distribution of that statistic.

Table 1.4: Sampling Distribution of Sample Means

S. No.	$\bar{X}$	Frequency(f)	Probability(p)
1	1.0	1	1/16 = 0.0625
2	1.5	2	2/16 = 0.1250
3	2.0	3	3/16 = 0.1875
4	2.5	4	4/16 = 0.2500
5	3.0	3	3/16 = 0.1875
6	3.5	2	2/16 = 0.1250
7	4.0	1	1/16 = 0.0625

So the arrangement of all possible values of sample mean with their corresponding probabilities is called the sampling distribution of mean.

The sampling distribution of sample means itself has mean, variance, etc.

Note that the mean of the sample means is equal to the population mean:

$$\bar{\bar{X}} = \mu$$

Standard Error:

The standard deviation of a sampling distribution of a statistic is known as standard error and it is denoted by SE.

If X_1 ,X_2 , ..., X_n is a random sample of size n taken from a population with mean μ and variance σ^2 then the standard errors of sample mean (X) is given by:

$$SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

Central Limit Theorem:

The central limit theorem is the most important theorem of Statistics. It was first introduced by De Movers in the early eighteenth century.

According to the central limit theorem, if 1 2 n X ,X , ..., X is a random sample of size n taken from a population with mean μ and variance σ^2 then the sampling distribution of the sample mean tends to normal distribution with mean μ and variance σ^2/n as sample size tends to large (n > 30) whatever the form of parent population. That is,

The mean

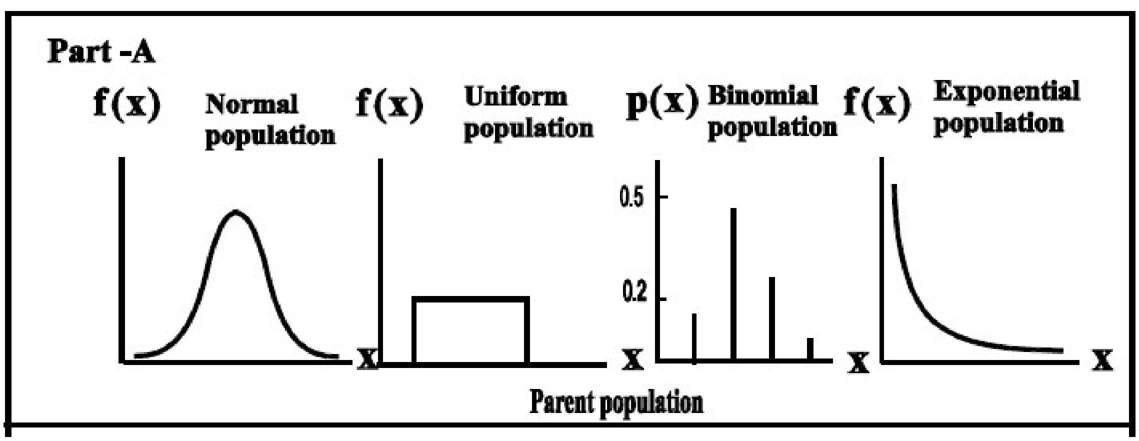
$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

and the variate

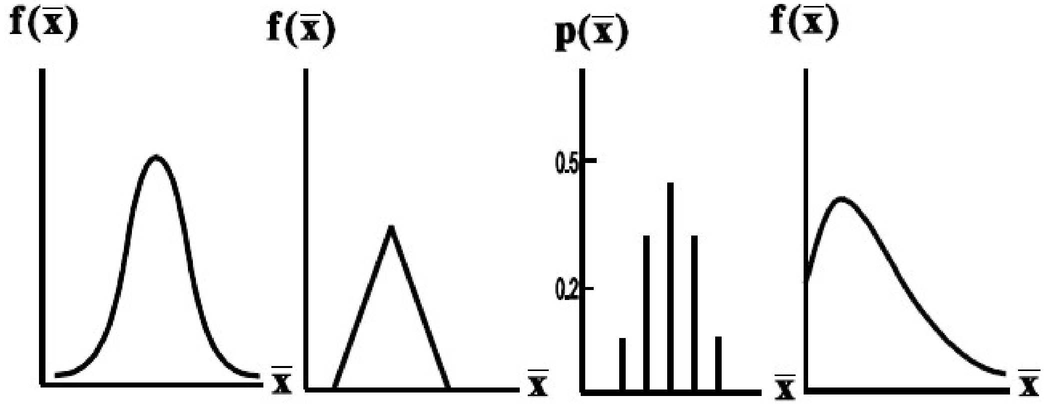
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

follow normal distribution with mean 0 and variance unity.

Here, we will also try to show that how large must the sample size be for which we can assume that the central limit theorem applies?



Part -B



Sampling distribution of sample mean for size n = 2

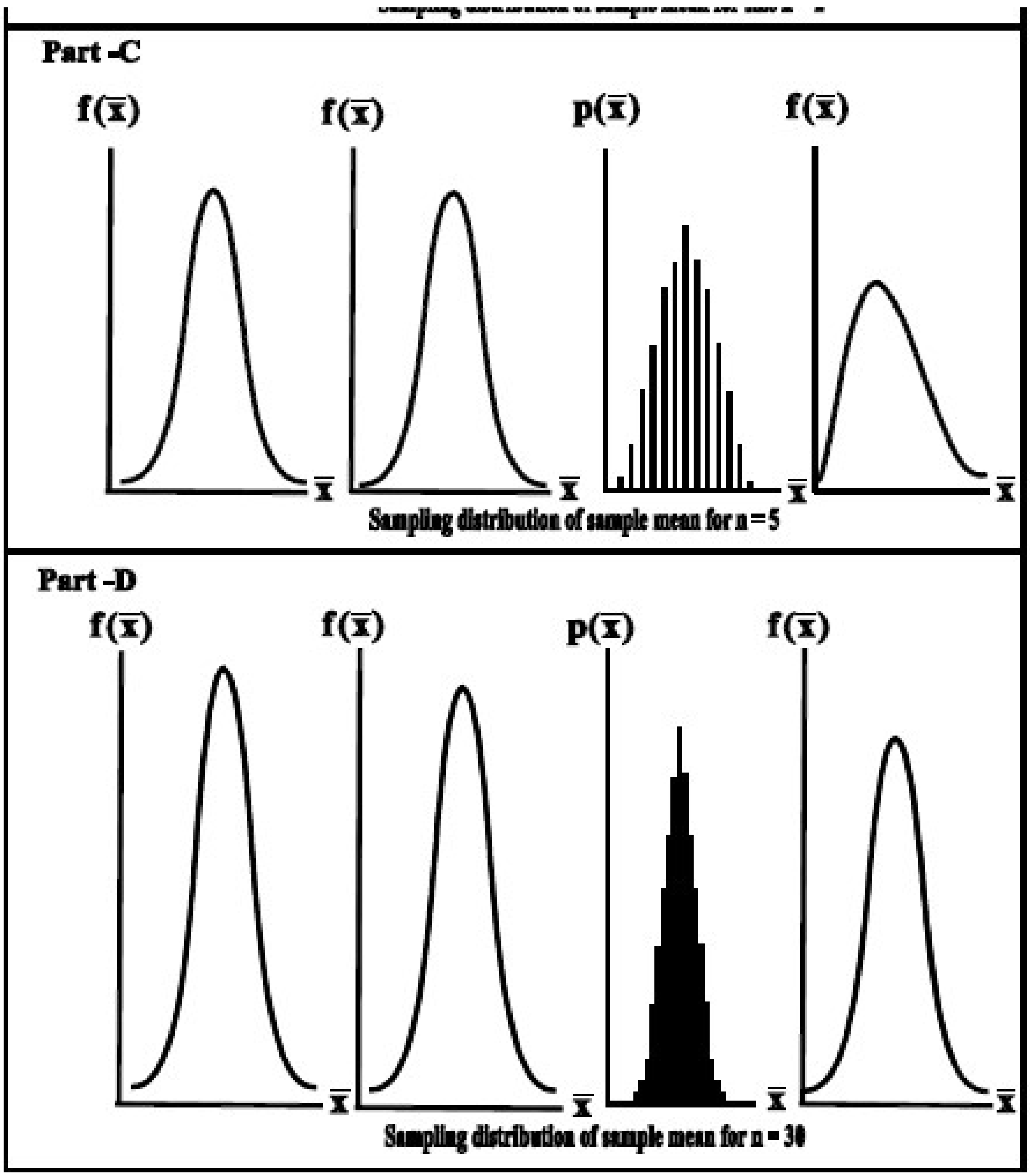


Fig. 1.1: Sampling distribution of sample means for various populations when n = 2, n = 5 and n = 30