New Track Fitting

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Abstract

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1. Equation of Motion

Notation.

$$
\begin{aligned}
step\ length &: S && [cm] \\
path\ length &: s && [cm] \\
position &: \vec{p} && [cm] \\
unit\ direction &: \hat{u} && [1] \\
orthogonal\ direction &: \hat{n}_\tau \ and\ \hat{n}_\rho && [1] \\
eta &: \tilde{\eta} = \tilde{q}/\left(\gamma\beta\right) && [1] \\
rigidity &: R = (m/q) \cdot \gamma\beta && [GV] \\
mass &: m && [GeV] \\
charge &: q && [e] \\
charge\ sign &: \tilde{q} = q/\left|q\right| && [1] \\
electron\ mass &: m_e = 0.510999 && [MeV] \\
electron\ charge &: q_e = 1 && [e] \\
velocity &: \beta && [1] \\
Lorentz\ factor &: \gamma = 1/\sqrt{1-\beta^2} && [1] \\
magnetic\ field &: \vec{B} && [kGauss] \\
lambda &: \lambda = 2.99792458 \cdot 10^{-4} && [GV \cdot kGauss^{-1} \cdot cm^{-1}] \\
radiation\ length &: X_0 && [g \cdot cm^{-2}] \\
mass\ density &: D && [g \cdot cm^{-3}] \\
cross\ length &: L = D/X_0 && [cm^{-1}] \\
passage\ length &: l && [cm] \\
Rydberg\ constant &: C_{\mathrm{Rydberg}} = 0.0136 && [GeV] \\
K &: K = 0.307075 && [MeV \cdot mol^{-1} \cdot cm^2] \\
atomic\ mass &: A && [g \cdot mol^{-1}] \\
atomic\ number &: Z && [1] \\
density\ effect &: \delta(\gamma\beta) && [MeV] \\
mean\ excitation\ energy &: I && [eV] \\
Landau\ factor &: j = 0.200 && [1]
\end{aligned}
\tag{1}
$$

Lorentz Force.

$$
\begin{aligned}
\frac{d\hat{u}}{ds} &= \Lambda \cdot (\hat{u} \times \vec{B}) \cdot \tilde{\eta} \\
\Lambda &= \lambda \cdot \left|q/m\right| \\
(\hat{u} \times \vec{B}) &= \begin{pmatrix} \hat{u}_y \vec{B}_z - \hat{u}_z \vec{B}_y \\ \hat{u}_z \vec{B}_x - \hat{u}_x \vec{B}_z \\ \hat{u}_x \vec{B}_y - \hat{u}_y \vec{B}_x \end{pmatrix}
\end{aligned} \tag{2}
$$

Multiple Scattering.

$$
\begin{aligned}
\frac{d\hat{u}}{ds} &= (\omega_\tau \cdot \hat{n}_\tau + \omega_\rho \cdot \hat{n}_\rho) \cdot \Omega_\theta \\
\hat{n}_\tau &= (\vec{B} - (\hat{u} \cdot \vec{B}) \cdot \hat{u}) / \left|\hat{u} \times \vec{B}\right| \\
\hat{n}_\rho &= (\hat{u} \times \vec{B}) / \left|\hat{u} \times \vec{B}\right| \\
\Omega_\theta &= C_{\text{Rydberg}} \cdot \left|q/m\right| \cdot (\left|\tilde{\eta}\right| \sqrt{\tilde{\eta}^2 + 1}) \cdot \sqrt{l \cdot L} \cdot (1 + 0.038 \cdot \ln(l \cdot L)) \cdot (1/S) \\
\omega_\tau, \omega_\rho &\sim Normal\,(0, 1) \\
f\,(\omega) &= \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{\omega^2}{2}\right)
\end{aligned} \tag{3}
$$

Ionisation Energy Loss.

$$
\begin{aligned}
\frac{d\tilde{\eta}}{ds} &= \tilde{\eta} \cdot (\omega_I \cdot \Omega_\delta + \Omega_\Delta) \\
\Omega_I &= (K/2) \cdot (D \cdot l \cdot Z/A) \cdot (q^2) \cdot (\tilde{\eta}^2 + 1) \\
\Omega_\delta &= \Omega_I \cdot (1/m) \cdot (\tilde{\eta}^2 \sqrt{\tilde{\eta}^{-2} + 1}) \cdot (1/S) \\
\Omega_\Delta &= \Omega_\delta \cdot \left[\ln\left(\frac{2m_e \cdot \tilde{\eta}^{-2}}{I}\right) + \ln\left(\frac{\Omega_I}{I}\right) + j - \beta^2 - \delta(\gamma\beta)\right] \\
\omega_I &\sim Landau\,(0, 1) \;\; require \;\; [\omega_I >= (-\Omega_\Delta/\Omega_\delta)] \\
f\,(\omega_I) &\simeq \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\,(\omega_I + \exp(-\omega_I))\right)
\end{aligned} \tag{4}
$$

Bremsstrahlung Energy Loss.

$$\begin{aligned}
\frac{d\tilde{\eta}}{ds} &= \tilde{\eta} \cdot (\omega_B \cdot \Omega_B) \\
\Omega_B &= \left(\frac{q}{q_e} \cdot \frac{m_e}{m}\right)^2 \cdot (\tilde{\eta}^2 + 1) \cdot \left(\frac{l \cdot L}{\ln 2}\right) \cdot (1/S) \\
\omega_B &\sim \Gamma(t, t) \quad require \;\; [0 \leq \omega_B < \inf] \\
f(\omega_B) &= \left(\frac{t^t}{\Gamma(t)}\right) \cdot \left(\omega_B^{t-1} \cdot \exp(-\omega_B \cdot t)\right) \;\; where \; t = \left(\frac{l \cdot L}{\ln 2}\right)
\end{aligned} \tag{5}$$

Summary.

$$\begin{aligned}
\frac{d\vec{p}}{ds} &= \hat{u} \\
\frac{d\hat{u}}{ds} &= \Lambda \cdot (\hat{u} \times \vec{B}) \cdot \tilde{\eta} + (\omega_\tau \cdot \hat{n}_\tau + \omega_\rho \cdot \hat{n}_\rho) \cdot \Omega_\theta \\
\frac{d\tilde{\eta}}{ds} &= \tilde{\eta} \cdot [(\omega_I \cdot \Omega_\delta + \Omega_\Delta) + (\omega_B \cdot \Omega_B)]
\end{aligned} \tag{6}$$

2. Propagation

Status.

$$\begin{aligned}
\xi_{\vec{p}}^T &= \begin{pmatrix} x & y & z \end{pmatrix} \\
\xi_{\hat{u}}^T &= \begin{pmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \end{pmatrix} \\
\xi_{\tilde{\eta}}^T &= \begin{pmatrix} \tilde{\eta} \end{pmatrix} \\
\xi_{\omega}^T &= \begin{pmatrix} \omega_\tau & \omega_\rho & \omega_I & \omega_B \end{pmatrix} \\
\xi^T &= \begin{pmatrix} \xi_{\vec{p}}^T & \xi_{\hat{u}}^T & \xi_{\tilde{\eta}}^T & \xi_{\omega}^T \end{pmatrix} \\
\xi_G^T &= \begin{pmatrix} \xi_{\vec{p}}^T & \xi_{\hat{u}}^T & \xi_{\tilde{\eta}}^T \end{pmatrix} \\
\xi_L^T &= \begin{pmatrix} \xi_{\omega}^T \end{pmatrix}
\end{aligned} \tag{7}$$

Differential Status.

$$\begin{aligned}
\zeta_{\vec{p}} = \frac{d\xi_{\vec{p}}}{ds} \quad &= \hat{u} \\
\zeta_{\vec{u}} = \frac{d\xi_{\vec{u}}}{ds} \quad &= \Lambda \cdot (\hat{u} \times \vec{B}) \cdot \tilde{\eta} + (\omega_\tau \cdot \hat{n}_\tau + \omega_\rho \cdot \hat{n}_\rho) \cdot \Omega_\theta \\
\zeta_{\tilde{\eta}} = \frac{d\xi_{\tilde{\eta}}}{ds} \quad &= \tilde{\eta} \cdot [(\omega_I \cdot \Omega_\delta + \Omega_\Delta) + (\omega_B \cdot \Omega_B)]
\end{aligned} \tag{8}$$

Transfer Status A.

$$
\begin{aligned}
\kappa_{\vec{p}} = \frac{d\zeta_{\vec{p}}}{d\xi} &= \begin{pmatrix} \frac{d\zeta_{\vec{p}}}{d\xi_{\vec{p}}} & \frac{d\zeta_{\vec{p}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\vec{p}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\vec{p}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\hat{u}} = \frac{d\zeta_{\hat{u}}}{d\xi} &= \begin{pmatrix} \frac{d\zeta_{\hat{u}}}{d\xi_{\vec{p}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\tilde{\eta}} = \frac{d\zeta_{\tilde{\eta}}}{d\xi} &= \begin{pmatrix} \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\vec{p}}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\vec{p}} = \frac{d\zeta_{\vec{p}}}{d\xi} &= \begin{pmatrix} 0 & \frac{d\zeta_{\vec{p}}}{d\xi_{\hat{u}}} & 0 & 0 \end{pmatrix} \\
\kappa_{\hat{u}} = \frac{d\zeta_{\hat{u}}}{d\xi} &= \begin{pmatrix} 0 & \frac{d\zeta_{\hat{u}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\tilde{\eta}} = \frac{d\zeta_{\tilde{\eta}}}{d\xi} &= \begin{pmatrix} 0 & 0 & \frac{d\zeta_{\tilde{\eta}}}{d\tilde{\eta}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega}} \end{pmatrix}
\end{aligned}
\tag{9}
$$

Transfer Status B.

$$
\begin{aligned}
\frac{d\zeta_{\vec{p}_i}}{d\xi_{\hat{u}_j}} &= \delta_{ij} \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\hat{u}_j}} &= \Lambda \cdot \frac{\partial(\hat{u} \times \vec{B})_i}{\partial \hat{u}_j} \cdot \tilde{\eta} \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\tilde{\eta}}} &= \Lambda \cdot (\hat{u} \times \vec{B})_i \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\omega_\tau}} &= (\Omega_\theta \cdot \hat{n}_\tau)_i \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\omega_\rho}} &= (\Omega_\theta \cdot \hat{n}_\rho)_i \\
\frac{d\zeta_{\tilde{\eta}}}{d\xi_{\tilde{\eta}}} &= (\omega_I \cdot \Omega_\delta + \Omega_\Delta) + (\omega_B \cdot \Omega_B) \\
\frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega_I}} &= \tilde{\eta} \cdot \Omega_\delta \\
\frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega_B}} &= \tilde{\eta} \cdot \Omega_B \\
\frac{\partial(\hat{u} \times \vec{B})}{\partial \hat{u}} &= \begin{pmatrix} 0 & \vec{B}_z & -\vec{B}_y \\ -\vec{B}_z & 0 & \vec{B}_x \\ \vec{B}_y & -\vec{B}_x & 0 \end{pmatrix}
\end{aligned}
\tag{10}
$$

Euler Method.

$$
\begin{aligned}
\xi^0 &= \xi\left(0\right) \\
\xi^f &= \xi\left(S\right) \\
\xi_G^f &= \xi_G^0 + \left(\begin{array}{l} S\cdot\zeta_{\vec{p}}^0 + \frac{S^2}{2}\cdot\zeta_{\hat{u}}^0 \\ S\cdot\zeta_{\hat{u}}^0 \\ S\cdot\zeta_{\tilde{\eta}}^0 \end{array}\right) \\
J^f &= \left(\begin{array}{cc} J_{GG}^f & J_{GL}^f \end{array}\right) \\
J_{GG}^f &= I + \left(\begin{array}{l} S\cdot\kappa_{\vec{p}G}^0 + \frac{S^2}{2}\cdot\kappa_{\hat{u}G}^0 \\ S\cdot\kappa_{\hat{u}G}^0 \\ S\cdot\kappa_{\tilde{\eta}G}^0 \end{array}\right) \\
J_{GL}^f &= \left(\begin{array}{l} \frac{S^2}{2}\cdot\kappa_{\hat{u}L}^0 \\ S\cdot\kappa_{\hat{u}L}^0 \\ S\cdot\kappa_{\tilde{\eta}L}^0 \end{array}\right)
\end{aligned}
\tag{11}
$$

Euler-Heun Method.

$$
\begin{aligned}
\xi^0 &= \xi\left(0\right)\\
\xi^1 &= \xi\left(S\right)\\
\xi^1_G &= \xi^0_G + \begin{pmatrix} S \cdot \zeta^0_{\vec{p}} + \frac{S^2}{2} \cdot \zeta^0_{\hat{u}} \\ S \cdot \zeta^0_{\hat{u}} \\ S \cdot \zeta^0_{\tilde{\eta}} \end{pmatrix}\\
J^1 &= \begin{pmatrix} J^1_{GG} & J^1_{GL} \end{pmatrix}\\
J^1_{GG} &= I + \begin{pmatrix} S \cdot \kappa^0_{\vec{p}G} + \frac{S^2}{2} \cdot \kappa^0_{\hat{u}G} \\ S \cdot \kappa^0_{\hat{u}G} \\ S \cdot \kappa^0_{\tilde{\eta}G} \end{pmatrix}\\
J^1_{GL} &= \begin{pmatrix} \frac{S^2}{2} \cdot \kappa^0_{\hat{u}L} \\ S \cdot \kappa^0_{\hat{u}L} \\ S \cdot \kappa^0_{\tilde{\eta}L} \end{pmatrix}\\
\xi^f &= \xi\left(S\right)\\
\xi^f_G &= \xi^0_G + \begin{pmatrix} S \cdot \zeta^0_{\vec{p}} + \frac{S^2}{6} \cdot \left(2\zeta^0_{\hat{u}} + \zeta^1_{\hat{u}}\right) \\ \frac{S}{2} \cdot \left(\zeta^0_{\hat{u}} + \zeta^1_{\hat{u}}\right) \\ \frac{S}{2} \cdot \left(\zeta^0_{\tilde{\eta}} + \zeta^1_{\tilde{\eta}}\right) \end{pmatrix}\\
J^f &= \begin{pmatrix} J^f_{GG} & J^f_{GL} \end{pmatrix}\\
J^f_{GG} &= I + \begin{pmatrix} S \cdot \kappa^0_{\vec{p}G} + \frac{S^2}{6} \cdot \left(2\kappa^0_{\hat{u}G} + \kappa^1_{\hat{u}G} J^1_{GG}\right) \\ \frac{S}{2} \cdot \left(\kappa^0_{\hat{u}G} + \kappa^1_{\hat{u}G} J^1_{GG}\right) \\ \frac{S}{2} \cdot \left(\kappa^0_{\tilde{\eta}G} + \kappa^1_{\tilde{\eta}G} J^1_{GG}\right) \end{pmatrix}\\
J^f_{GL} &= \begin{pmatrix} \frac{S^2}{6} \cdot \left(2\kappa^0_{\hat{u}L} + \left(\kappa^1_{\hat{u}G} J^1_{GL} + \kappa^1_{\hat{u}L}\right)\right) \\ \frac{S}{2} \cdot \left(\kappa^0_{\hat{u}L} + \left(\kappa^1_{\hat{u}G} J^1_{GL} + \kappa^1_{\hat{u}L}\right)\right) \\ \frac{S}{2} \cdot \left(\kappa^0_{\tilde{\eta}L} + \left(\kappa^1_{\tilde{\eta}G} J^1_{GL} + \kappa^1_{\tilde{\eta}L}\right)\right) \end{pmatrix}
\end{aligned}
\tag{12}
$$

Runge-Kutta-Nystrom Method A.

$$
\begin{aligned}
\xi^0 &= \xi\left(0\right) \\
\xi^1 &= \xi\left(\frac{S}{2}\right) \\
\xi^1_G &= \xi^0_G + \left(\begin{array}{l} \frac{S}{2}\cdot\zeta^0_{\vec{p}} + \frac{S^2}{8}\cdot\zeta^0_{\hat{u}} \\ \frac{S}{2}\cdot\zeta^0_{\hat{u}} \\ \frac{S}{2}\cdot\zeta^0_{\tilde{\eta}} \end{array}\right) \\
J^1 &= \left(\begin{array}{ll} J^1_{GG} & J^1_{GL} \end{array}\right) \\
J^1_{GG} &= I + \left(\begin{array}{l} \frac{S}{2}\cdot\kappa^0_{\vec{p}G} + \frac{S^2}{8}\cdot\kappa^0_{\hat{u}G} \\ \frac{S}{2}\cdot\kappa^0_{\hat{u}G} \\ \frac{S}{2}\cdot\kappa^0_{\tilde{\eta}G} \end{array}\right) \\
J^1_{GL} &= \left(\begin{array}{l} \frac{S^2}{8}\cdot\kappa^0_{\hat{u}L} \\ \frac{S}{2}\cdot\kappa^0_{\hat{u}L} \\ \frac{S}{2}\cdot\kappa^0_{\tilde{\eta}L} \end{array}\right) \\
\xi^2 &= \xi\left(\frac{S}{2}\right) \\
\xi^2_G &= \xi^i_G + \left(\begin{array}{l} \frac{S}{2}\cdot\zeta^0_{\vec{p}} + \frac{S^2}{8}\cdot\zeta^0_{\hat{u}} \\ \frac{S}{2}\cdot\zeta^1_{\hat{u}} \\ \frac{S}{2}\cdot\zeta^1_{\tilde{\eta}} \end{array}\right) \\
J^2 &= \left(\begin{array}{ll} J^2_{GG} & J^2_{GL} \end{array}\right) \\
J^2_{GG} &= I + \left(\begin{array}{l} \frac{S}{2}\cdot\kappa^0_{\vec{p}G} + \frac{S^2}{8}\cdot\kappa^0_{\hat{u}G} \\ \frac{S}{2}\cdot\kappa^1_{\hat{u}G}J^1_{GG} \\ \frac{S}{2}\cdot\kappa^1_{\tilde{\eta}G}J^1_{GG} \end{array}\right) \\
J^2_{GL} &= \left(\begin{array}{l} \frac{S^2}{8}\cdot\kappa^0_{\hat{u}L} \\ \frac{S}{2}\cdot\left(\kappa^1_{\hat{u}G}J^1_{GL} + \kappa^1_{\hat{u}L}\right) \\ \frac{S}{2}\cdot\left(\kappa^1_{\tilde{\eta}G}J^1_{GL} + \kappa^1_{\tilde{\eta}L}\right) \end{array}\right)
\end{aligned}
\tag{13}
$$

Runge-Kutta-Nystrom Method B.

$$
\begin{aligned}
\xi^3 &= \xi\left(S\right) \\
\xi_G^3 &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_{\vec{p}}^0 + \frac{S^2}{2} \cdot \zeta_{\hat{u}}^2 \\ S \cdot \zeta_{\hat{u}}^2 \\ S \cdot \zeta_{\vec{\eta}}^2 \end{pmatrix} \\
J^3 &= \begin{pmatrix} J_{GG}^3 & J_{GL}^3 \end{pmatrix} \\
J_{GG}^3 &= I + \begin{pmatrix} S \cdot \kappa_{\vec{p}G}^0 + \frac{S^2}{2} \cdot \kappa_{\hat{u}G}^2 J_{GG}^2 \\ S \cdot \kappa_{\hat{u}G}^2 J_{GG}^2 \\ S \cdot \kappa_{\vec{\eta}G}^2 J_{GG}^2 \end{pmatrix} \\
J_{GL}^3 &= \begin{pmatrix} \frac{S^2}{2} \cdot \left(\kappa_{\hat{u}G}^2 J_{GL}^2 + \kappa_{\hat{u}L}^2\right) \\ S \cdot \left(\kappa_{\hat{u}G}^2 J_{GL}^2 + \kappa_{\hat{u}L}^2\right) \\ S \cdot \left(\kappa_{\vec{\eta}G}^2 J_{GL}^2 + \kappa_{\vec{\eta}L}^2\right) \end{pmatrix} \\
\xi^f &= \xi\left(S\right) \\
\xi_G^f &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_{\vec{p}}^0 + \frac{S^2}{6} \cdot \left(\zeta_{\hat{u}}^0 + \zeta_{\hat{u}}^1 + \zeta_{\hat{u}}^2\right) \\ \frac{S}{6} \cdot \left(\zeta_{\hat{u}}^0 + 2\zeta_{\hat{u}}^1 + 2\zeta_{\hat{u}}^2 + \zeta_{\hat{u}}^3\right) \\ \frac{S}{6} \cdot \left(\zeta_{\vec{\eta}}^0 + 2\zeta_{\vec{\eta}}^1 + 2\zeta_{\vec{\eta}}^2 + \zeta_{\vec{\eta}}^3\right) \end{pmatrix} \\
J^f &= \begin{pmatrix} J_{GG}^f & J_{GL}^f \end{pmatrix} \\
J_{GG}^f &= I + \begin{pmatrix} S \cdot \kappa_{\vec{p}G}^0 + \frac{S^2}{6} \cdot \left(\kappa_{\hat{u}G}^0 + \kappa_{\hat{u}G}^1 J_{GG}^1 + \kappa_{\hat{u}G}^2 J_{GG}^2\right) \\ \frac{S}{6} \cdot \left(\kappa_{\hat{u}G}^0 + 2\kappa_{\hat{u}G}^1 J_{GG}^1 + 2\kappa_{\hat{u}G}^2 J_{GG}^2 + \kappa_{\hat{u}G}^3 J_{GG}^3\right) \\ \frac{S}{6} \cdot \left(\kappa_{\vec{\eta}G}^0 + 2\kappa_{\vec{\eta}G}^1 J_{GG}^1 + 2\kappa_{\vec{\eta}G}^2 J_{GG}^2 + \kappa_{\vec{\eta}G}^3 J_{GG}^3\right) \end{pmatrix} \\
J_{GL}^f &= \begin{pmatrix} \frac{S^2}{6} \cdot \left(\kappa_{\hat{u}L}^0 + \left(\kappa_{\hat{u}G}^1 J_{GL}^1 + \kappa_{\hat{u}L}^1\right) + \left(\kappa_{\hat{u}G}^2 J_{GL}^2 + \kappa_{\hat{u}L}^2\right)\right) \\ \frac{S}{6} \cdot \left(\kappa_{\hat{u}L}^0 + 2\left(\kappa_{\hat{u}G}^1 J_{GL}^1 + \kappa_{\hat{u}L}^1\right) + 2\left(\kappa_{\hat{u}G}^2 J_{GL}^2 + \kappa_{\hat{u}L}^2\right) + \left(\kappa_{\hat{u}G}^3 J_{GL}^3 + \kappa_{\hat{u}L}^3\right)\right) \\ \frac{S}{6} \cdot \left(\kappa_{\vec{\eta}L}^0 + 2\left(\kappa_{\vec{\eta}G}^1 J_{GL}^1 + \kappa_{\vec{\eta}L}^1\right) + 2\left(\kappa_{\vec{\eta}G}^2 J_{GL}^2 + \kappa_{\vec{\eta}L}^2\right) + \left(\kappa_{\vec{\eta}G}^3 J_{GL}^3 + \kappa_{\vec{\eta}L}^3\right)\right) \end{pmatrix}
\end{aligned}
\tag{14}
$$

3. Track Fitting

Negative-Log-Likelihood.

$$\mathcal{L} = -\ln\left(all\ probability\ density\ function\ multiply\right) + Constant \tag{15}$$

$$
\begin{aligned}
\mathcal{L} = &\left(\frac{1}{2}\right) \cdot \sum \left(\left(\xi_{\vec{p}} - m_{\vec{p}}\right)^T \sigma_{\vec{p}}^{-1} \left(\xi_{\vec{p}} - m_{\vec{p}}\right) \right) \\
&+ \left(\frac{1}{2}\right) \cdot \sum \omega_\tau^2 \\
&+ \left(\frac{1}{2}\right) \cdot \sum \omega_\rho^2 \\
&+ \left(\frac{1}{2}\right) \cdot \sum \left(\omega_I + \exp(-\omega_I)\right) \\
&+ \sum \left(\omega_B t + (1-t) \cdot \ln\left(\omega_B\right)\right)
\end{aligned} \tag{16}
$$

Negative-Log-Likelihood.

$$
\begin{aligned}
H_\xi = \frac{\partial \mathcal{L}}{\partial \xi^T} = &\sum \left(\left(\frac{d\vec{p}_\xi}{d\xi} \right)^T \sigma_m^{-1} \left(\vec{p}_\xi - \vec{p}_m\right) \right) + \\
&\sum \left(\left(\frac{\partial \omega_L}{\partial \xi} \right)^T \cdot \omega_L \right) + \\
&\sum \left(\left(\frac{\partial \omega_\theta}{\partial \xi} \right)^T \cdot \omega_\theta \right) + \\
&\sum \left(\left(\frac{\partial \omega_I}{\partial \xi} \right)^T \cdot \left(\frac{1}{2}\right) \left(1 - \exp(-\omega_I)\right) \right) + \\
&\sum \left(\left(\frac{\partial \omega_B}{\partial \xi} \right)^T \cdot \left(t + (1-t) \cdot \omega_B^{-1}\right) \right)
\end{aligned} \tag{17}
$$

Negative-Log-Likelihood.

$$
\begin{aligned}
C_\xi^{-1} = \frac{\partial \mathcal{L}}{\partial \xi^T \partial \xi} = &\sum \left(\left(\frac{d\vec{p}_\xi}{d\xi} \right)^T \sigma_m^{-1} \left(\frac{d\vec{p}_\xi}{d\xi} \right) \right) + \\
&\sum \left(\left(\frac{\partial \omega_L}{\partial \xi} \right)^T \cdot \left(\frac{\partial \omega_L}{\partial \xi} \right) \right) + \\
&\sum \left(\left(\frac{\partial \omega_\theta}{\partial \xi} \right)^T \cdot \left(\frac{\partial \omega_\theta}{\partial \xi} \right) \right) + \\
&\sum \left(\left(\frac{\partial \omega_I}{\partial \xi} \right)^T \cdot \left(\frac{1}{2}\right) \exp(-\omega_I) \cdot \left(\frac{\partial \omega_I}{\partial \xi} \right) \right) + \\
&\sum \left(\left(\frac{\partial \omega_B}{\partial \xi} \right)^T \cdot \left((t-1) \cdot \omega_B^{-2}\right) \cdot \left(\frac{\partial \omega_B}{\partial \xi} \right) \right)
\end{aligned} \tag{18}
$$

Minimization. The Gradient Descent Method

$$\Delta\xi = c \cdot H_\xi \text{ } where\ c\ is\ constant \tag{19}$$

The Gauss-Newton Method

$$\Delta\xi = C_\xi \cdot H_\xi \tag{20}$$

The Levenberg-Marquardt Method

$$\Delta\xi = (C_\xi + c \cdot diag\,(C_\xi)) \cdot H_\xi \text{ } where\ c\ is\ constant \tag{21}$$

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6. Bibliography styles

There are various bibliography styles available. You can select the style of your choice in the preamble of this document. These styles are Elsevier styles based on standard styles like Harvard and Vancouver. Please use BibTEX to generate your bibliography and include DOIs whenever available.

References