

# New Track Fitting

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## Abstract

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## 1. Equation of Motion

*Notation.*

<i>step length</i> : $S$	[cm]
<i>path length</i> : $s$	[cm]
<i>position</i> : $\vec{p}$	[cm]
<i>unit direction</i> : $\hat{u}$	[1]
<i>orthogonal direction</i> : $\hat{n}_\tau$ and $\hat{n}_\rho$	[1]
<i>eta</i> : $\tilde{\eta} = \tilde{q}/(\gamma\beta)$	[1]
<i>rigidity</i> : $R = (m/q) \cdot \gamma\beta$	[GV]
<i>mass</i> : $m$	[GeV]
<i>charge</i> : $q$	[e]
<i>charge sign</i> : $\tilde{q} = q/ q $	[1]
<i>electron mass</i> : $m_e = 0.510999$	[MeV]
<i>electron charge</i> : $q_e = 1$	[e]
<i>velocity</i> : $\beta$	[1]
<i>Lorentz factor</i> : $\gamma = 1/\sqrt{1 - \beta^2}$	[1]
<i>magnetic field</i> : $\vec{B}$	[kGauss]
<i>lambda</i> : $\lambda = 2.99792458 \cdot 10^{-4}$	[GV · kGauss <sup>-1</sup> · cm <sup>-1</sup> ]
<i>radiation length</i> : $X_0$	[g · cm <sup>-2</sup> ]
<i>mass density</i> : $D$	[g · cm <sup>-3</sup> ]
<i>cross length</i> : $L = D/X_0$	[cm <sup>-1</sup> ]
<i>passage length</i> : $l$	[cm]
<i>Rydberg constant</i> : $C_{\text{Rydberg}} = 0.0136$	[GeV]
<i>K</i> : $K = 0.307075$	[MeV · mol <sup>-1</sup> · cm <sup>2</sup> ]
<i>atomic mass</i> : $A$	[g · mol <sup>-1</sup> ]
<i>atomic number</i> : $Z$	[1]
<i>density effect</i> : $\delta(\gamma\beta)$	[MeV]
<i>mean excitation energy</i> : $I$	[eV]
<i>Landau factor</i> : $j = 0.200$	[1]

*Lorentz Force.*

$$\begin{aligned}
\frac{d\hat{u}}{ds} &= \Lambda \cdot (\hat{u} \times \vec{B}) \cdot \tilde{\eta} \\
\Lambda &= \lambda \cdot |q/m| \\
(\hat{u} \times \vec{B}) &= \begin{pmatrix} \hat{u}_y \vec{B}_z - \hat{u}_z \vec{B}_y \\ \hat{u}_z \vec{B}_x - \hat{u}_x \vec{B}_z \\ \hat{u}_x \vec{B}_y - \hat{u}_y \vec{B}_x \end{pmatrix}
\end{aligned} \tag{2}$$

*Multiple Scattering.*

$$\begin{aligned}
\frac{d\hat{u}}{ds} &= (\omega_\tau \cdot \hat{n}_\tau + \omega_\rho \cdot \hat{n}_\rho) \cdot \Omega_\theta \\
\hat{n}_\tau &= (\vec{B} - (\hat{u} \cdot \vec{B}) \cdot \hat{u}) / |\hat{u} \times \vec{B}| \\
\hat{n}_\rho &= (\hat{u} \times \vec{B}) / |\hat{u} \times \vec{B}| \\
\Omega_\theta &= C_{\text{Rydberg}} \cdot |q/m| \cdot (|\tilde{\eta}| \sqrt{\tilde{\eta}^2 + 1}) \cdot \sqrt{l \cdot L} \cdot (1 + 0.038 \cdot \ln(l \cdot L)) \cdot (1/S) \\
\omega_\tau, \omega_\rho &\sim \text{Normal}(0, 1) \\
f(\omega) &= \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{\omega^2}{2}\right)
\end{aligned} \tag{3}$$

*Ionisation Energy Loss.*

$$\begin{aligned}
\frac{d\tilde{\eta}}{ds} &= \tilde{q} \cdot (\omega_I \cdot \Omega_\delta + \Omega_\Delta) \\
\Omega_I &= (K/2) \cdot (D \cdot l \cdot Z/A) \cdot (q^2) \cdot (\tilde{\eta}^2 + 1) \\
\Omega_\delta &= \Omega_I \cdot (1/m) \cdot (\tilde{\eta}^2 \sqrt{\tilde{\eta}^{-2} + 1}) \cdot (|\tilde{\eta}|/S) \\
\Omega_\Delta &= \Omega_\delta \cdot \left[ \ln\left(\frac{2m_e \cdot \tilde{\eta}^{-2}}{I}\right) + \ln\left(\frac{\Omega_I}{I}\right) + j - \beta^2 - \delta(\gamma\beta) \right] \\
\omega_I &\sim \text{Landau}(0, 1) \text{ require } [\omega_I \geq (-\Omega_\Delta/\Omega_\delta)] \\
f(\omega_I) &\simeq \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}(\omega_I + \exp(-\omega_I))\right)
\end{aligned} \tag{4}$$

*Bremsstrahlung Energy Loss.*

$$\begin{aligned}
\frac{d\tilde{\eta}}{ds} &= \tilde{q} \cdot (\omega_B \cdot \Omega_B) \\
\Omega_B &= \left( \frac{q}{q_e} \cdot \frac{m_e}{m} \right)^2 \cdot (\tilde{\eta}^2 + 1) \cdot \left( \frac{l \cdot L}{\ln 2} \right) \cdot (|\tilde{\eta}|/S) \\
\omega_B &\sim \Gamma(t, t) \text{ require } [0 \leq \omega_B < \inf] \\
f(\omega_B) &= \left( \frac{t^t}{\Gamma(t)} \right) \cdot (\omega_B^{t-1} \cdot \exp(-\omega_B \cdot t)) \text{ where } t = \left( \frac{l \cdot L}{\ln 2} \right)
\end{aligned} \tag{5}$$

*Summary.*

$$\begin{aligned}
\frac{d\vec{p}}{ds} &= \hat{u} \\
\frac{d\hat{u}}{ds} &= \Lambda \cdot (\hat{u} \times \vec{B}) \cdot \tilde{\eta} + (\omega_\tau \cdot \hat{n}_\tau + \omega_\rho \cdot \hat{n}_\rho) \cdot \Omega_\theta \\
\frac{d\tilde{\eta}}{ds} &= \tilde{q} \cdot [(\omega_I \cdot \Omega_\delta + \Omega_\Delta) + (\omega_B \cdot \Omega_B)]
\end{aligned} \tag{6}$$

## 2. Propagation

*Status.*

$$\begin{aligned}
\xi_{\vec{p}}^T &= \begin{pmatrix} x & y & z \end{pmatrix} \\
\xi_{\hat{u}}^T &= \begin{pmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \end{pmatrix} \\
\xi_{\tilde{\eta}}^T &= \begin{pmatrix} \tilde{\eta} \end{pmatrix} \\
\xi_\omega^T &= \begin{pmatrix} \omega_\tau & \omega_\rho & \omega_I & \omega_B \end{pmatrix} \\
\xi^T &= \begin{pmatrix} \xi_{\vec{p}}^T & \xi_{\hat{u}}^T & \xi_{\tilde{\eta}}^T & \xi_\omega^T \end{pmatrix} \\
\xi_G^T &= \begin{pmatrix} \xi_{\vec{p}}^T & \xi_{\hat{u}}^T & \xi_{\tilde{\eta}}^T \end{pmatrix} \\
\xi_L^T &= \begin{pmatrix} \xi_\omega^T \end{pmatrix}
\end{aligned} \tag{7}$$

*Differential Status.*

$$\begin{aligned}
\zeta_{\vec{p}} = \frac{d\xi_{\vec{p}}}{ds} &= \hat{u} \\
\zeta_{\hat{u}} = \frac{d\xi_{\hat{u}}}{ds} &= \Lambda \cdot (\hat{u} \times \vec{B}) \cdot \tilde{\eta} + (\omega_\tau \cdot \hat{n}_\tau + \omega_\rho \cdot \hat{n}_\rho) \cdot \Omega_\theta \\
\zeta_{\tilde{\eta}} = \frac{d\xi_{\tilde{\eta}}}{ds} &= \tilde{q} \cdot [(\omega_I \cdot \Omega_\delta + \Omega_\Delta) + (\omega_B \cdot \Omega_B)]
\end{aligned} \tag{8}$$

*Transfer Status A.*

$$\begin{aligned}
\kappa_{\vec{p}} = \frac{d\zeta_{\vec{p}}}{d\xi} &= \begin{pmatrix} \frac{d\zeta_{\vec{p}}}{d\xi_{\vec{p}}} & \frac{d\zeta_{\vec{p}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\vec{p}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\vec{p}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\hat{u}} = \frac{d\zeta_{\hat{u}}}{d\xi} &= \begin{pmatrix} \frac{d\zeta_{\hat{u}}}{d\xi_{\vec{p}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\tilde{\eta}} = \frac{d\zeta_{\tilde{\eta}}}{d\xi} &= \begin{pmatrix} \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\vec{p}}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\vec{p}} = \frac{d\zeta_{\vec{p}}}{d\xi} &= \begin{pmatrix} 0 & \frac{d\zeta_{\vec{p}}}{d\xi_{\hat{u}}} & 0 & 0 \end{pmatrix} \\
\kappa_{\hat{u}} = \frac{d\zeta_{\hat{u}}}{d\xi} &= \begin{pmatrix} 0 & \frac{d\zeta_{\hat{u}}}{d\xi_{\hat{u}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\tilde{\eta}}} & \frac{d\zeta_{\hat{u}}}{d\xi_{\omega}} \end{pmatrix} \\
\kappa_{\tilde{\eta}} = \frac{d\zeta_{\tilde{\eta}}}{d\xi} &= \begin{pmatrix} 0 & 0 & 0 & \frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega}} \end{pmatrix}
\end{aligned} \tag{9}$$

*Transfer Status B.*

$$\begin{aligned}
\frac{d\zeta_{\vec{p}_i}}{d\xi_{\hat{u}_j}} &= \delta_{ij} \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\hat{u}_j}} &= \Lambda \cdot \frac{\partial(\hat{u} \times \vec{B})_i}{\partial \hat{u}_j} \cdot \tilde{\eta} \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\tilde{\eta}}} &= \Lambda \cdot (\hat{u} \times \vec{B})_i \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\omega_\tau}} &= (\Omega_\theta \cdot \hat{n}_\tau)_i \\
\frac{d\zeta_{\hat{u}_i}}{d\xi_{\omega_\rho}} &= (\Omega_\theta \cdot \hat{n}_\rho)_i \\
\frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega_I}} &= \tilde{q} \cdot \Omega_\delta \\
\frac{d\zeta_{\tilde{\eta}}}{d\xi_{\omega_B}} &= \tilde{q} \cdot \Omega_B \\
\frac{\partial(\hat{u} \times \vec{B})}{\partial \hat{u}} &= \begin{pmatrix} 0 & \vec{B}_z & -\vec{B}_y \\ -\vec{B}_z & 0 & \vec{B}_x \\ \vec{B}_y & -\vec{B}_x & 0 \end{pmatrix}
\end{aligned} \tag{10}$$

*Euler Method.*

$$\begin{aligned}
\xi^0 &= \xi(0) \\
\xi^f &= \xi(S) \\
\xi_G^f &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_p^0 + \frac{S^2}{2} \cdot \zeta_u^0 \\ S \cdot \zeta_u^0 \\ S \cdot \zeta_{\tilde{\eta}}^0 \end{pmatrix} \\
J^f &= \begin{pmatrix} J_{GG}^f & J_{GL}^f \end{pmatrix} \\
J_{GG}^f &= I + \begin{pmatrix} S \cdot \kappa_{\tilde{p}G}^0 + \frac{S^2}{2} \cdot \kappa_{uG}^0 \\ S \cdot \kappa_{uG}^0 \\ S \cdot \kappa_{\tilde{\eta}G}^0 \end{pmatrix} \\
J_{GL}^f &= \begin{pmatrix} \frac{S^2}{2} \cdot \kappa_{uL}^0 \\ S \cdot \kappa_{uL}^0 \\ S \cdot \kappa_{\tilde{\eta}L}^0 \end{pmatrix}
\end{aligned} \tag{11}$$

*Euler-Heun Method.*

$$\begin{aligned}
\xi^0 &= \xi(0) \\
\xi^1 &= \xi(S) \\
\xi_G^1 &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_p^0 + \frac{S^2}{2} \cdot \zeta_u^0 \\ S \cdot \zeta_u^0 \\ S \cdot \zeta_{\eta}^0 \end{pmatrix} \\
J^1 &= \begin{pmatrix} J_{GG}^1 & J_{GL}^1 \end{pmatrix} \\
J_{GG}^1 &= I + \begin{pmatrix} S \cdot \kappa_{pG}^0 + \frac{S^2}{2} \cdot \kappa_{uG}^0 \\ S \cdot \kappa_{uG}^0 \\ S \cdot \kappa_{\eta G}^0 \end{pmatrix} \\
J_{GL}^1 &= \begin{pmatrix} \frac{S^2}{2} \cdot \kappa_{uL}^0 \\ S \cdot \kappa_{uL}^0 \\ S \cdot \kappa_{\eta L}^0 \end{pmatrix} \\
\xi^f &= \xi(S) \\
\xi_G^f &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_p^0 + \frac{S^2}{6} \cdot (2\zeta_u^0 + \zeta_u^1) \\ \frac{S}{2} \cdot (\zeta_u^0 + \zeta_u^1) \\ \frac{S}{2} \cdot (\zeta_{\eta}^0 + \zeta_{\eta}^1) \end{pmatrix} \\
J^f &= \begin{pmatrix} J_{GG}^f & J_{GL}^f \end{pmatrix} \\
J_{GG}^f &= I + \begin{pmatrix} S \cdot \kappa_{pG}^0 + \frac{S^2}{6} \cdot (2\kappa_{uG}^0 + \kappa_{uG}^1 J_{GG}^1) \\ \frac{S}{2} \cdot (\kappa_{uG}^0 + \kappa_{uG}^1 J_{GG}^1) \\ \frac{S}{2} \cdot (\kappa_{\eta G}^0 + \kappa_{\eta G}^1 J_{GG}^1) \end{pmatrix} \\
J_{GL}^f &= \begin{pmatrix} \frac{S^2}{6} \cdot (2\kappa_{uL}^0 + (\kappa_{uG}^1 J_{GL}^1 + \kappa_{uL}^1)) \\ \frac{S}{2} \cdot (\kappa_{uL}^0 + (\kappa_{uG}^1 J_{GL}^1 + \kappa_{uL}^1)) \\ \frac{S}{2} \cdot (\kappa_{\eta L}^0 + (\kappa_{\eta G}^1 J_{GL}^1 + \kappa_{\eta L}^1)) \end{pmatrix}
\end{aligned} \tag{12}$$



*Runge-Kutta-Nystrom Method A.*

$$\begin{aligned}
\xi^0 &= \xi(0) \\
\xi^1 &= \xi\left(\frac{S}{2}\right) \\
\xi_G^1 &= \xi_G^0 + \begin{pmatrix} \frac{S}{2} \cdot \zeta_p^0 + \frac{S^2}{8} \cdot \zeta_u^0 \\ \frac{S}{2} \cdot \zeta_u^0 \\ \frac{S}{2} \cdot \zeta_{\tilde{\eta}}^0 \end{pmatrix} \\
J^1 &= \begin{pmatrix} J_{GG}^1 & J_{GL}^1 \end{pmatrix} \\
J_{GG}^1 &= I + \begin{pmatrix} \frac{S}{2} \cdot \kappa_{pG}^0 + \frac{S^2}{8} \cdot \kappa_{uG}^0 \\ \frac{S}{2} \cdot \kappa_{uG}^0 \\ \frac{S}{2} \cdot \kappa_{\tilde{\eta}G}^0 \end{pmatrix} \\
J_{GL}^1 &= \begin{pmatrix} \frac{S^2}{8} \cdot \kappa_{uL}^0 \\ \frac{S}{2} \cdot \kappa_{uL}^0 \\ \frac{S}{2} \cdot \kappa_{\tilde{\eta}L}^0 \end{pmatrix} \\
\xi^2 &= \xi\left(\frac{S}{2}\right) \\
\xi_G^2 &= \xi_G^1 + \begin{pmatrix} \frac{S}{2} \cdot \zeta_p^0 + \frac{S^2}{8} \cdot \zeta_u^0 \\ \frac{S}{2} \cdot \zeta_u^1 \\ \frac{S}{2} \cdot \zeta_{\tilde{\eta}}^1 \end{pmatrix} \\
J^2 &= \begin{pmatrix} J_{GG}^2 & J_{GL}^2 \end{pmatrix} \\
J_{GG}^2 &= I + \begin{pmatrix} \frac{S}{2} \cdot \kappa_{pG}^0 + \frac{S^2}{8} \cdot \kappa_{uG}^0 \\ \frac{S}{2} \cdot \kappa_{uG}^1 J_{GG}^1 \\ \frac{S}{2} \cdot \kappa_{\tilde{\eta}G}^1 J_{GG}^1 \end{pmatrix} \\
J_{GL}^2 &= \begin{pmatrix} \frac{S^2}{8} \cdot \kappa_{uL}^0 \\ \frac{S}{2} \cdot (\kappa_{uG}^1 J_{GL}^1 + \kappa_{uL}^1) \\ \frac{S}{2} \cdot (\kappa_{\tilde{\eta}G}^1 J_{GL}^1 + \kappa_{\tilde{\eta}L}^1) \end{pmatrix}
\end{aligned} \tag{13}$$

*Runge-Kutta-Nystrom Method B.*

$$\begin{aligned}
\xi^3 &= \xi(S) \\
\xi_G^3 &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_p^0 + \frac{S^2}{2} \cdot \zeta_u^2 \\ S \cdot \zeta_u^2 \\ S \cdot \zeta_{\tilde{\eta}}^2 \end{pmatrix} \\
J^3 &= \begin{pmatrix} J_{GG}^3 & J_{GL}^3 \end{pmatrix} \\
J_{GG}^3 &= I + \begin{pmatrix} S \cdot \kappa_{pG}^0 + \frac{S^2}{2} \cdot \kappa_{uG}^2 J_{GG}^2 \\ S \cdot \kappa_{uG}^2 J_{GG}^2 \\ S \cdot \kappa_{\tilde{\eta}G}^2 J_{GG}^2 \end{pmatrix} \\
J_{GL}^3 &= \begin{pmatrix} \frac{S^2}{2} \cdot (\kappa_{uG}^2 J_{GL}^2 + \kappa_{uL}^2) \\ S \cdot (\kappa_{uG}^2 J_{GL}^2 + \kappa_{uL}^2) \\ S \cdot (\kappa_{\tilde{\eta}G}^2 J_{GL}^2 + \kappa_{\tilde{\eta}L}^2) \end{pmatrix} \\
\xi^f &= \xi(S) \\
\xi_G^f &= \xi_G^0 + \begin{pmatrix} S \cdot \zeta_p^0 + \frac{S^2}{6} \cdot (\zeta_u^0 + \zeta_u^1 + \zeta_u^2) \\ \frac{S}{6} \cdot (\zeta_u^0 + 2\zeta_u^1 + 2\zeta_u^2 + \zeta_u^3) \\ \frac{S}{6} \cdot (\zeta_{\tilde{\eta}}^0 + 2\zeta_{\tilde{\eta}}^1 + 2\zeta_{\tilde{\eta}}^2 + \zeta_{\tilde{\eta}}^3) \end{pmatrix} \\
J^f &= \begin{pmatrix} J_{GG}^f & J_{GL}^f \end{pmatrix} \\
J_{GG}^f &= I + \begin{pmatrix} S \cdot \kappa_{pG}^0 + \frac{S^2}{6} \cdot (\kappa_{uG}^0 + \kappa_{uG}^1 J_{GG}^1 + \kappa_{uG}^2 J_{GG}^2) \\ \frac{S}{6} \cdot (\kappa_{uG}^0 + 2\kappa_{uG}^1 J_{GG}^1 + 2\kappa_{uG}^2 J_{GG}^2 + \kappa_{uG}^3 J_{GG}^3) \\ \frac{S}{6} \cdot (\kappa_{\tilde{\eta}G}^0 + 2\kappa_{\tilde{\eta}G}^1 J_{GG}^1 + 2\kappa_{\tilde{\eta}G}^2 J_{GG}^2 + \kappa_{\tilde{\eta}G}^3 J_{GG}^3) \end{pmatrix} \\
J_{GL}^f &= \begin{pmatrix} \frac{S^2}{6} \cdot (\kappa_{uL}^0 + (\kappa_{uG}^1 J_{GL}^1 + \kappa_{uL}^1) + (\kappa_{uG}^2 J_{GL}^2 + \kappa_{uL}^2)) \\ \frac{S}{6} \cdot (\kappa_{uL}^0 + 2(\kappa_{uG}^1 J_{GL}^1 + \kappa_{uL}^1) + 2(\kappa_{uG}^2 J_{GL}^2 + \kappa_{uL}^2) + (\kappa_{uG}^3 J_{GL}^3 + \kappa_{uL}^3)) \\ \frac{S}{6} \cdot (\kappa_{\tilde{\eta}L}^0 + 2(\kappa_{\tilde{\eta}G}^1 J_{GL}^1 + \kappa_{\tilde{\eta}L}^1) + 2(\kappa_{\tilde{\eta}G}^2 J_{GL}^2 + \kappa_{\tilde{\eta}L}^2) + (\kappa_{\tilde{\eta}G}^3 J_{GL}^3 + \kappa_{\tilde{\eta}L}^3)) \end{pmatrix} \\
\end{aligned} \tag{14}$$

### 3. Track Fitting

*Negative-Log-Likelihood.*

$$\mathcal{L} = -\ln(\text{all probability density function multiply}) + \text{Constant} \tag{15}$$

$$\begin{aligned}
\mathcal{L} = & \left(\frac{1}{2}\right) \cdot \sum \left( (\xi_{\vec{p}} - m_{\vec{p}})^T \sigma_{\vec{p}}^{-1} (\xi_{\vec{p}} - m_{\vec{p}}) \right) \\
& + \left(\frac{1}{2}\right) \cdot \sum \omega_{\tau}^2 \\
& + \left(\frac{1}{2}\right) \cdot \sum \omega_{\rho}^2 \\
& + \left(\frac{1}{2}\right) \cdot \sum (\omega_I + \exp(-\omega_I)) \\
& + \sum (\omega_B t + (1-t) \cdot \ln(\omega_B))
\end{aligned} \tag{16}$$

*Negative-Log-Likelihood.*

$$\begin{aligned}
H_{\xi} = \frac{\partial \mathcal{L}}{\partial \xi^T} = & \sum \left( \left( \frac{d\vec{p}_{\xi}}{d\xi} \right)^T \sigma_m^{-1} (\vec{p}_{\xi} - \vec{p}_m) \right) + \\
& \sum \left( \left( \frac{\partial \omega_L}{\partial \xi} \right)^T \cdot \omega_L \right) + \\
& \sum \left( \left( \frac{\partial \omega_{\theta}}{\partial \xi} \right)^T \cdot \omega_{\theta} \right) + \\
& \sum \left( \left( \frac{\partial \omega_I}{\partial \xi} \right)^T \cdot \left( \frac{1}{2} \right) (1 - \exp(-\omega_I)) \right) + \\
& \sum \left( \left( \frac{\partial \omega_B}{\partial \xi} \right)^T \cdot (t + (1-t) \cdot \omega_B^{-1}) \right)
\end{aligned} \tag{17}$$

*Negative-Log-Likelihood.*

$$\begin{aligned}
C_{\xi}^{-1} = \frac{\partial \mathcal{L}}{\partial \xi^T \partial \xi} = & \sum \left( \left( \frac{d\vec{p}_{\xi}}{d\xi} \right)^T \sigma_m^{-1} \left( \frac{d\vec{p}_{\xi}}{d\xi} \right) \right) + \\
& \sum \left( \left( \frac{\partial \omega_L}{\partial \xi} \right)^T \cdot \left( \frac{\partial \omega_L}{\partial \xi} \right) \right) + \\
& \sum \left( \left( \frac{\partial \omega_{\theta}}{\partial \xi} \right)^T \cdot \left( \frac{\partial \omega_{\theta}}{\partial \xi} \right) \right) + \\
& \sum \left( \left( \frac{\partial \omega_I}{\partial \xi} \right)^T \cdot \left( \frac{1}{2} \right) \exp(-\omega_I) \cdot \left( \frac{\partial \omega_I}{\partial \xi} \right) \right) + \\
& \sum \left( \left( \frac{\partial \omega_B}{\partial \xi} \right)^T \cdot ((t-1) \cdot \omega_B^{-2}) \cdot \left( \frac{\partial \omega_B}{\partial \xi} \right) \right)
\end{aligned} \tag{18}$$

*Minimization.* The Gradient Descent Method

$$\Delta\xi = c \cdot H_\xi \text{ where } c \text{ is constant} \quad (19)$$

The Gauss-Newton Method

$$\Delta\xi = C_\xi \cdot H_\xi \quad (20)$$

The Levenberg-Marquardt Method

$$\Delta\xi = (C_\xi + c \cdot \text{diag}(C_\xi)) \cdot H_\xi \text{ where } c \text{ is constant} \quad (21)$$

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## 35 References