# PS 2

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2025-04-15

## Part 1

First, read in the data

```
library(tidyverse)
library(data.table)

data <- read_csv("data/middle_kink.csv")

# view the data
data %>% head()
```

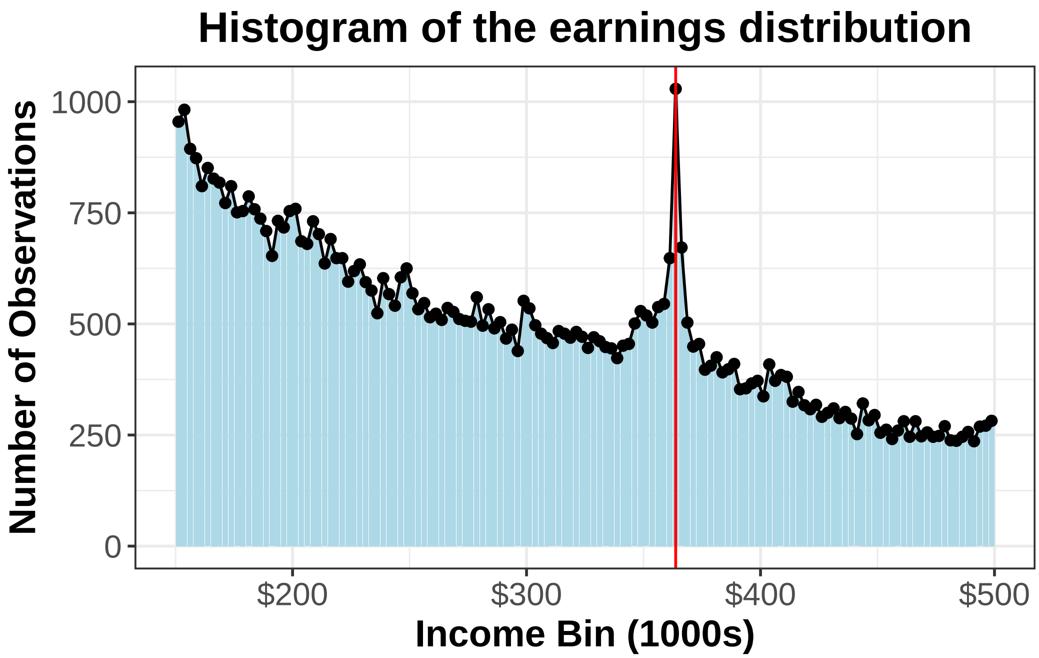
```
# A tibble: 6 x 2
  income_bin     n
       <dbl> <dbl>
1     151250   955
2     153750   982
3     156250   894
4     158750   873
5     161250   810
6     163750   851
```

### a.

Now we will plot the publication-quality histogram of the earnings distribution:

```r
library(ggtext)

earning_dist <- data %>%
  ggplot(aes(x=income_bin, y = n)) +
  geom_col(fill = "lightblue") +
  geom_point() +
  # geom_smooth(method = "lm", se = FALSE, color = "black") +
  geom_line() +
  scale_x_continuous(labels = scales::label_number(scale = 0.001, prefix =
  ↪  "$")) +
  geom_vline(xintercept = 363750, color="red", shape="solid") +
  # annotate("text", label = "kink", x = 380000, y = 800, size = 5, colour =
  ↪  "black") +
  labs(title = "**Histogram of the earnings distribution**",
       x = "**Income Bin (1000s)**",
       y = "**Number of Observations**") +
  theme_bw() +
  theme(
    plot.title = element_markdown(size = 16, hjust = 0.5),
    axis.title.x = element_markdown(size = 14),
    axis.title.y = element_markdown(size = 14),
    axis.text.x = element_text(size = 12),
    axis.text.y = element_text(size = 12)
  )
earning_dist
```




**b.**

We will be following Saez (2010) to construct the equation to retrieve the elasticity $e$. Note for our case, the kink happens at $z^* = 365000$ and marginal tax rate changes from 0.07 to 0.21. We need to use equation (5) in the paper to get the elasticity. The equation is as follows:

$$B = z^* \left[ \left( \frac{1 - t_0}{1 - t_1} \right)^e - 1 \right] \frac{h(z^*)_- + h(z^*)_+ \Big/ \left( \frac{1-t_0}{1-t_1} \right)^e}{2}.$$

In order to compute $B$, we need to decide $\delta$ to calculate the width we will use to calculate excess bunching. We will use the "simplest method" mentioned in the paper which is to select $\delta$ graphically such that the full excess bunching is included in the band $(z^* - \delta + z^* + \delta)$. In our case, it seems to be about $\delta = 4$ (Note that since our data is in income bin of width 2,500, this is equivalent to 10,000 difference). Numerically, it will be calculated as follows (this is just following the equation (6) in the paper):

```
A <- data %>% count(wt=n) %>% pull()

b <- function(delta) {

B1  <- data %>%
  filter(income_bin >= 365000 - 2500*delta & income_bin <= 365000 +
  ↪  2500*delta) %>%  # 25,000 differences
```

```
  count(wt=n) %>%
  pull() #8,574

B2 <- data %>%
  filter(income_bin >= 365000 - 2500*2*delta & income_bin <= 365000 -
  ↪  2500*delta) %>%
  count(wt=n) %>%
  pull() # 3615

B3 <- data %>%
  filter(income_bin >= 365000 + 2500*delta & income_bin <= 365000 +
  ↪  2500*2*delta) %>%
  count(wt=n) %>%
  pull() # 2982

B = (B1 - B2 - B3) / A

}

b1 <- function(delta) {

B1  <- data %>%
  filter(income_bin >= 365000 - 2500*delta & income_bin <= 365000 +
  ↪  2500*delta) %>%  # 25,000 differences
  count(wt=n) %>%
  pull() #8,574

}

b2 <- function(delta) {

B2 <- data %>%
  filter(income_bin >= 365000 - 2500*2*delta & income_bin <= 365000 -
  ↪  2500*delta) %>%
  count(wt=n) %>%
  pull() # 3615

}

b3 <- function(delta) {

B3 <- data %>%
```

```r
  filter(income_bin >= 365000 + 2500*delta & income_bin <= 365000 +
  ↪  2500*2*delta) %>%
  count(wt=n) %>%
  pull() # 2982
}

B <- b(4)
B1 <- b1(4)
B2 <- b2(4)
B3 <- b3(4)
```

Note that we computed $B$ in the equation by getting the total fraction of people that are in the excess bunching.

Now we also need to compute two $h$ in the main equation. Empirically we can calculate this by dividing B2, B3 by $\delta$ respectively. They are the left and right density point around the bunching.

```r
h_min = (B2 / 5000) / A
h_plus = (B3 / 5000) / A

h_min
```

```
[1] 5.891811e-06
```

```r
h_plus
```

```
[1] 4.648559e-06
```

Finally, we can plug in the values we got from the data and get the elasticity $e$. Here, we are just basically getting the solution by plugging in the empirical numbers we computed from the data into the main equation:

```r
# Define the function whose root we want to find
f <- function(e) {

  z  <- 365000
  ratio <- (1 - 0.07) / (1 - 0.21)

  z * (ratio^e - 1) * ( (h_min + (h_plus / ratio^e)) / 2) - B
```

```
}

# Use uniroot to solve f(e) = 0 in a reasonable range for e
result <- uniroot(f, lower = -1, upper = 1)

# Extract the solution
e_solution <- result$root
print(e_solution)
```

```
[1] 0.05340052
```

### c.

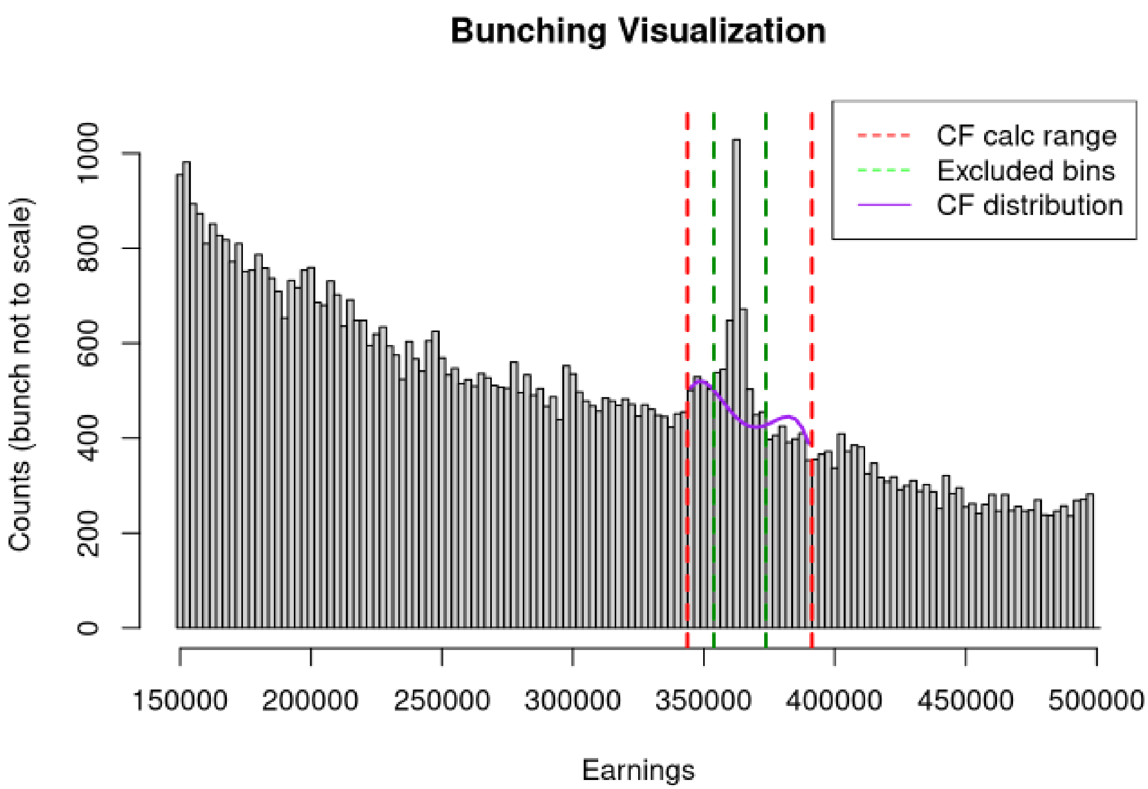
Using `bunchr` package in R, I plot the counterfactual density and get the estimate of the elasticity as follows:

```
library(bunchr)
# # analyzing a kink
# ability_vec <- 4000 * rbeta(100000, 2, 5)
# earning_vec <- sapply(ability_vec, earning_fun, 0.2, 0, 0.2, 0, 1000)
# earning_vec
# # bunch_viewer(earning_vec, 1000, 20, 20, 1, 1, binw = 20)
# estim <- bunch(earning_vec, 1000, 0, 0.2, Tax = 0, 20, 20, 1, 1,
# binw = 20, draw=TRUE, nboots = 0, seed = 16)
# estim$e

# Step 1: Expand binned data into a raw vector
z_vector <- data %>%
  rowwise() %>%
  summarise(vec = list(rep(income_bin, n))) %>%
  pull(vec) %>%
  unlist()

# Step 2: Estimate bunching
estim <- bunch(
  earnings = z_vector,
  zstar = 365000,
  binw = 2500,
  t1 = 0.07,
  t2 = 0.21,
  cf_start = 8,
```

```
  cf_end = 10,
  exclude_before = 4,
  exclude_after = 3,
  # poly_size = 2,
  draw = FALSE,
  nboots = 100
)
```




The elasticity estimate is as follows:

```
# Estimate of the elasticity from the package
estim$e
```

```
[1] 0.1189242
```

### d.

The elasticity in (b) does not match the one in (c). It may be due to several reasons. First, in (b) I set $\delta = 4$ which is different from (c) and I also set asymmetric bandwidth in `bunchr` package. Due to these different settings, the results would be different from each other. While both results are bit different from the working paper by Anagol et al. (2024), the result I got from (c) seems to be more close to the working paper. This is likely because, like Chetty et al. (2011), Anagol et al. (2024) proposes a method for estimating $B$ using the observed density, without requiring full knowledge of the counterfactual densities. However, Anagol et al. (2024)'s approach differs somewhat from that of Chetty et al. (2011) and other related work. While Chetty et al. (2011) estimates $B$ by integrating over the observed density relative to a counterfactual density estimated via polynomial fitting, Anagol et al. (2024) takes a more explicit approach to modeling the diffusion process—separately identifying the diffuse bunching expected near a kink from the underlying counterfactual distribution.

## Part 2

There is no straightforward answer to this question. Before we started to consider the dynamic nature of the taxation policy, the consensus was that we should not tax the capital. The reason was similar to our consensus on not wanting to tax the commodity. This is due to two main classical answers to this question. First, Chamley-Judd results told us that we should not tax capital in the long run.[1] Secondly, Atkinson-Stiglitz results told us that we should only be taxing through income as taxing through commodity or capital will further distort the behavior of people.

But our consensus could change if we move onto a dynamic setting. In fact, some papers have shown that the capital taxation should be positive in the dynamic setting. For example, savings (capital) wedge generally becomes positive in dynamic mechanism design solution for mirlees problem. The intuition for this is because in the dynamic setting people try to save more for the future to account for any possible risk in the future such as negative shock on their ability.Hence it leads to a negative fiscal externality for the government revenue. In order to alleviate this, the government should try to tax the capital. In this case, we could say it should tax the capital to offset the externality. However, setting positive capital taxation might not be that important in terms of policy perspective. This is because some papers (e.g. Farhi and Werning 2013) have shown that while it is true that capital tax is positive in dynamic setting, its actual magnitude seems to be small (close to zero). Thus in real life policy, it might not make much of a difference whether you tax capital (very slightly) or not.

Furthermore, there are some other methodological approaches to think about optimal capital taxation. For instance, there are some papers such as dynamic Ramsey setup where they

[1] But note that Chamley-Judd results were recently overturned in Straub and Werning (AER, 2020) so we should be cautious about their results.

impose certain parametric form for the tax function. In this case, optimal tax can be positive depending on the tax policy instrument restrictions. Also, capital taxation can be positive in sufficient statistics approach. Still, whether capital taxation has to be indeed positive will again depend on certain conditions that have no definitive answer.[2]

## References


Anagol, Santosh, Allan Davids, Benjamin B Lockwood, and Tarun Ramadorai. 2024. "Diffuse Bunching with Frictions: Theory and Estimation." Working Paper 32597. Working Paper Series. National Bureau of Economic Research. https://doi.org/10.3386/w32597.

Chetty, Raj, John N. Friedman, Tore Olsen, and Luigi Pistaferri. 2011. "Adjustment Costs, Firm Responses, and Micro Vs. Macro Labor Supply Elasticities: Evidence from Danish Tax Records *." *The Quarterly Journal of Economics* 126 (2): 749–804. https://doi.org/10.1093/qje/qjr013.

Saez, Emmanuel. 2010. "Do Taxpayers Bunch at Kink Points?" *American Economic Journal: Economic Policy* 2 (3): 180–212. https://doi.org/10.1257/pol.2.3.180.


[2] For example, we would need to consider whether capital taxation is just a workaround for some other features of dynamic constrained optimum for dynamic Ramsey. For sufficient statistics case, we would need to have a structural model for full calibration.