Spatial RDD: Black (1999) and BFM (2007)

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The Regression Discontinuity Design

- Let us start by reviewing the traditional regression discontinuity design (RDD), as first suggested in Thistlethwaite and Campbell (1960).
- It is based on the assumption that at a particular threshold of an observed running variable, the treatment status either changes from zero to 1 for everyone (sharp design) or at least for a subpopulation (fuzzy design).
- As an example, let us assume that the treatment of interest is extended eligibility to unemployment benefits, to which only individuals aged 50 or older are entitled.
- The idea of the RDD is to compare the outcomes (like unemployment duration) of treated and non-treated subjects close to the age threshold, such as individuals aged 50 and 49.
- Such individuals slightly above and below the age threshold are arguably similar in characteristics

The Regression Discontinuity Design

- Another example is admission to a gifted program in which all students that have an IQ score over a threshold are admitted to the program.
- it might be argued that subjects just slightly below the score are very similar to subjects just slightly above in terms of background characteristics also affecting the outcome, such as ability, motivation, or other personality traits.
- The RDD, therefore, aims at imitating the experimental context at the threshold to evaluate the treatment effect locally for the subpopulation at the threshold.

The Sharp Regression Discontinuity Design

- Let R denote the running variable and r_0 the known threshold value.
- If the treatment is deterministic in R such that it is one whenever the threshold is reached or exceeded:

$$D = 1\{R \geq r_0\}$$

- The RDD is sharp in the sense that all individuals change their treatment status exactly at r_0.

A Key Smoothness Assumption

- The evaluation of causal effects in the sharp RDD relies on the assumption that the mean potential outcomes given the running variable, $E[Y_1|R]$ and $E[Y_0|R]$ are continuous and smooth around $R = r_0$ (Hahn, Todd, and van der Klaauw, 2001).
- This requires that any (and possibly unobserved) background characteristics other than D that affect the outcome are continuously distributed at the threshold.
- Such a continuity implies that if treated and non-treated populations with values of R exactly equal to r_0 existed, the treatment would be as good as randomly assigned with regard to mean potential outcomes.

Local Average Treatment Effects

- Let $e > 0$ denote a small positive number. Then we have in the constant treatment case:

$$
\begin{aligned}
& \lim_{e \to 0} E[Y|R \in [r_0, r_0 + e]) - E[Y|R \in [r_0 - e, r_0] \\
= & \lim_{e \to 0} E[Y_1|R \in [r_0, r_0 + e]) - E[Y_0|R \in [r_0 - e, r_0] \\
= & E[Y_1 - Y_0| \; R = r_0]
\end{aligned}
$$

- We can identify a conditional or local average treatment effect (LATE) at the threshold based on treated and non-treated outcomes in a neighborhood.
- Note that there is no common support in R across the treatment groups that would allow comparing treated and non-treated outcomes with exactly the same values in the running variable.

Estimation

The LATE can be estimate using a local linear regression

$$Y = \alpha + \beta_R R + \beta_D 1\{R \geq r_0\} + \beta_{RD} 1\{R \geq r_0\} R + \epsilon$$

where $\beta_D = E[Y|R = r_0, D = 1] - E[Y|R = r_0, D = 0]$.

Imperfect Compliance

- In analogy to the discussion of instrumental variables and experiments with imperfect compliance, treatment take- up as a function of being above or below the threshold of the running variable might not be perfect.
- This implies that in contrast to the sharp RDD, D is not deterministic in R such that noncompliance in the treatment participation occurs.
- In this case, a fuzzy RDD approach permits assessing the causal effect on compliers at $R = r_0$. who are induced to switch their treatment state at the threshold.

The Fuzzy Regression Discontinuity Design

- denote by $D(z)$ the potential treatment state as a function of the binary threshold indicator $Z = 1\{R \geq r_0\}$, which now serves as an instrument for actual treatment participation.
- Furthermore, let us assume that around the threshold, defiers do not exist and that the shares of compliers, always takers, and never takers, as well as their mean potential outcomes under treatment and non-treatment, are continuous,
- Under these conditions, we can assess the first-stage effect of instrument Z on treatment participation D at the threshold $R = r_0$.

Discontinuity in the Treatment Probability at r_0

Under the assumptions above we:

$$
\begin{aligned}
& \lim_{e\to 0} \; E[D|R \in [r_0, r_0+e]) \; - \; E[D|R \in [r_0 - e, r_0] \\
= \;& \lim_{e\to 0} \; E[D(1)|R \in [r_0, r_0+e]) \; - \; E[D(0)|R \in [r_0 - e, r_0] \\
= \;& E[D(1) - D(0)| \; R = r_0] \\
> \;& 0
\end{aligned}
$$

Hence we can identify the discontinuity in the treatment probability at r_0.

Local Average Treatment Effect in the Fuzzy Design

- As a consequence, the local average treatment for compliers at r_0 is identified by

$$
\begin{aligned}
& E[\Delta |\ D(1) = 1, D(0) = 0, R = r_0] \\
= \quad & \frac{\lim_{e \to 0}\ E[Y|R \in [r_0, r_0 + e]) \ - \ E[Y|R \in [r_0 - e, r_0]}{\lim_{e \to 0}\ E[D|R \in [r_0, r_0 + e]) \ - \ E[D|R \in [r_0 - e, r_0]}
\end{aligned}
$$

- We need to estimate the numerator and the denominator of the estimator separately.
- Again we can use local linear regressions or other non-parametric techniques.

Spatial RDD

- Black, S. (1999). Do Better Schools Matter? Parental Valuation of Elementary Education The Quarterly Journal of Economics, 1999, 114, 2, 577-599.
- She studies the impact of school quality on housing prices within a school district.
- She looks at houses located on the opposite side of attendance district boundaries.
- Houses then differ only by the elementary school the child attends.
- We can interpret this approach as using “distance to attendance district boundaries” as running variable in an RDD.

Attendance Zones

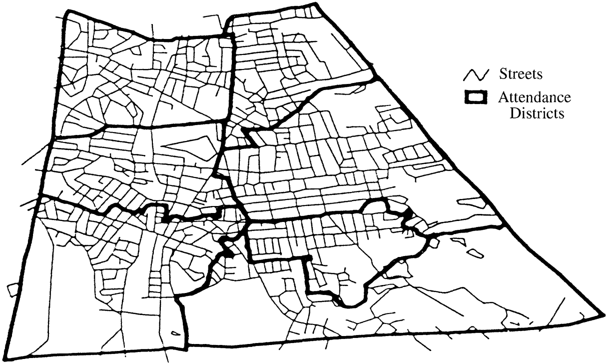

FIGURE I
Example of Data Collection for One City: Melrose Streets, and Attendance District Boundaries

Implementation

- Consider a house i in attendance zone a that is within close proximity to a boundary b

$$\ln(p_{iab}) = \alpha + \beta X_{iab} + \phi K_b + \gamma test_a + \epsilon_{iab}$$

where X_{iab} denotes structural characteristics, $test_a$ denotes test scores in elementary school in attendance district a, and K_b are boundary dummies.

Comments

- Conceptually, this methodology is equivalent to calculating differences in mean house prices on opposite sides of attendance district boundaries (controlling for house characteristics) and relating this to differences in test scores.3 Boundary dummies account for any unobserved characteristics shared by houses on either side of the boundary.
- If neighborhoods change continuously over space, by looking at houses very close to attendance district boundaries-where there is a discrete change in school quality, we can alleviate the pitfalls associated with omitted neighborhood characteristics.

Data

- The housing price data cover all purchases and sales from 1993 through 1995 for Middlesex, Essex, and Norfolk counties in Massachusetts, all suburbs of Boston.
- Massachusetts as sample because its school districts are small.
- She focuses on elementary schools because only these schools allow for enough within-district variation.
- The mean housing price in her sample is approximately $188,000.
- Mean test scores are 27.1 with a standard deviation of 1.4.

TABLE II

REGRESSION RESULTS[a]

(ADJUSTED STANDARD ERRORS ARE IN PARENTHESES[b])

DEPENDENT VARIABLE = ln (HOUSE PRICE)

Distance from boundary:	(1) All houses[d]	(2) 0.35 mile from boundary (616 yards)	(3) 0.20 mile from boundary (350 yards)	(4) 0.15 mile from boundary (260 yards)	(5) 0.15 mile from boundary (260 yards)
Elementary school test score[c]	.035	.016	.013	.015	.031
	(.004)	(.007)	(.0065)	(.007)	(.006)
Bedrooms	.033	.038	.037	.033	.035
	(.004)	(.005)	(.006)	(.007)	(.007)
Bathrooms	.147	.143	.135	.167	.193
	(.014)	(.018)	(.024)	(.027)	(.028)
Bathrooms squared	−.013	−.017	−.015	−.024	−.025
	(.003)	(.004)	(.005)	(.006)	(.007)
Lot size (1000s)	.003	.005	.005	.005	.004
	(.0003)	(.0005)	(.0005)	(.0007)	(.0006)
Internal square footage (1000s)	.207	.193	.191	.195	.191
	(.007)	(.01)	(.01)	(.02)	(.012)
Age of building	−.002	−.002	−.003	−.003	−.002
	(.0003)	(.0002)	(.0005)	(.0006)	(.0004)
Age squared	.000003	.000003	.00001	.000009	.000005
	(.000001)	(.0000006)	(.000002)	(.000003)	(.000002)
Boundary fixed effects	NO	YES	YES	YES	NO
Census variables	Yes	No	No	No	Yes
N	22,679	10,657	6,824	4,594	4,589
Number of boundaries	N/A	175	174	172	N/A
Adjusted R^2	0.6417	0.6745	0.6719	0.6784	.6564

a. Each regression includes quarter year dummies. Dummies are also included to indicate missing bedroom data, bathroom data, lot size data, and age of establishment data.

b. Standard errors are adjusted for clustering at the attendance district level.

c. Test scores are measured at the elementary school level and represent the sum of the reading and math scores from the fourth grade MEAP test averaged over three years (1988, 1990, and 1992). *Source:* Massachusetts Department of Education.

d. This regression also includes neighborhood characteristics such as the percentage of Hispanics, the percentage of non-Hispanic blacks, the age distribution of the neighborhood, the percentage of female-headed

Summary of Findings

- Parents do care about school peers and other unmeasured components of school quality.
- However, much the observed correlation between test scores and housing prices is driven by the correlation of school quality with other aspects of housing or neighborhood quality.
- Nevertheless, they are willing to pay about 2.1 percent-or $3948-more for houses associated with test scores that are 5 percent higher at the mean.
- The findings also suggest that a move from a school that scores in the twenty-fifth percentile of my sample to a school in the seventy-fifth percentile would result in a house price increase of $5452.

BFM Critique of Black (1999)

- Bayer, Ferreira, Mc Millan (2007). A Unified Framework for Measuring Preferences for Schools and Neighborhoods, Journal of Political Economy, 2007, vol. 115, no. 4, 588-638.
- BFM point out that sorting at school attendance zone boundaries is an important phenomenon as suggested by Black (1999).
- Even when boundary fixed effects are included in the analysis, failing to control for such sorting leads to a significant overstatement of the capitalization of average test scores into house prices.
- Observed neighborhood characteristics systematically differ on either side of the boundary.

TABLE 3

KEY COEFFICIENTS FROM BASELINE HEDONIC PRICE REGRESSIONS

	SAMPLE			
	Within 0.20 Mile of Boundary (N = 27,548)		Within 0.10 Mile of Boundary (N = 15,122)	
Boundary fixed effects included	No	Yes	No	Yes
	A. Excluding Neighborhood Sociodemographic Characteristics			
	(1)	(2)	(5)	(6)
Average test score (in standard deviations)	123.7	33.1	126.5	26.1
	(13.2)	(7.6)	(12.4)	(6.6)
R^2	.54	.62	.54	.62
	B. Including Neighborhood Sociodemographic Characteristics			
	(3)	(4)	(7)	(8)
Average test score (in standard deviations)	34.8	17.3	44.1	14.6
	(8.1)	(5.9)	(8.5)	(6.3)
% census block group black	−99.8	1.5	−123.1	4.3
	(33.4)	(38.9)	(32.5)	(39.1)
% block group with college degree or more	220.1	89.9	204.4	80.8
	(39.9)	(32.3)	(40.8)	(39.7)
Average block group income (/10,000)	60.0	45.0	55.6	42.9
	(4.0)	(4.6)	(4.3)	(6.1)
R^2	.59	.64	.59	.63

NOTE.—All regressions shown in the table also include controls for whether the house is owner-occupied, the number of rooms, year built (1980s, 1960–79, pre-1960), elevation, population density, crime, and land use (% industrial, % residential, % commercial, % open space, % other) in 1-, 2-, and 3-mile rings around each location. The dependent variable is the monthly user cost of housing, which equals monthly rent for renter-occupied units and a monthly user cost for owner-occupied housing, calculated as described in the text. Standard errors corrected for clustering at the school level are reported in parentheses.

The Sorting Model

- BMR apply the micro BLP model to estimate household preferences for school and neighborhood attributes in the presence of sorting.
- The conditional indirect utility function is given by

$$
\begin{aligned}
V_h^i &= \alpha_X^i X_h - \beta_p^i p_h + \alpha_d^i d_h^i + \theta_{bh} + \xi_h + \epsilon_{ih} \\
&= \delta_h + \lambda_h^i + \epsilon_{ih}
\end{aligned}
$$

  where

  - X_h are structural characteristics of the house and the neighborhood,
  - p_h is the price price (rent or monthly user cost),
  - d_h^i is distance to primary work,
  - θ_{bh} are boundary fixed effects,
  - ξ_h is an unobserved fixed effect,
  - $\delta_h = \alpha_{0X} X_h - \beta_{0p} p_h + \theta_{bh} + \xi_h$
  - $\alpha_j^i = \alpha_{0j} + \sum \alpha_{kj} z_k^i$ vary with observed households characteristics z_i such as income, race, ...

Structural "RDD"

Rearranging terms in the mean utility gives us:

$$p_h + \frac{\delta_h}{\alpha_{0p}} = \frac{\alpha_{0X}}{\alpha_{0p}} X_h \; + \; \frac{1}{\alpha_{0p}} \theta_{bh} \; + \; \frac{1}{\alpha_{0p}} \xi_h$$

Consequently, in the presence of heterogeneous preferences, the mean indirect utility estimated in the first stage of the estimation procedure provides an adjustment to the hedonic price equation so that the price regression accurately returns mean preferences.

Implementation

- The model is estimated using restricted access micro-level Census data from the SF metropolitan area using the subsample of houses that within 0.2 miles of a school district boundary.
- They develop an instrument for the price that is based on the exogenous attributes of houses and neighborhoods that are located more than 3 miles away from a given house, while allowing the attributes of houses and neighborhoods within 3 miles of the house to directly affect utility.
- The boundary fixed effects to account for potential correlation between school quality, neighborhood characteristics and unobserved characteristics In that sense they embed a boundary discontinuity design (Black, 1999) into the model.
- There are no separate instruments for peer effects.

Some Technical Issues

- Asymptotic theory requires that the number of products (houses) goes to infinity at a much slower pace than the number of individuals in the sample (Berry, Linton, and Pakes, 2004).
- If you use a logit model then you only need to observe a random subsample of the houses in the choice set because of the IIA property of the logit model (McFadden, 1978).
- See also the technical appendix of BFM (2007) for some additional discussion of these issues.

TABLE 7

DELTA REGRESSIONS: IMPLIED MEAN WILLINGNESS TO PAY

SAMPLE: WITHIN 0.20 MILE OF BOUNDARY (N = 27,458)

Boundary fixed effects included	No	Yes
	A. Excluding Neighborhood Sociodemographic Characteristics	
	(1)	(2)
Average test score (in standard deviations)	97.3	40.8
	(14.0)	(5.5)
	B. Including Neighborhood Sociodemographic Characteristics	
	(3)	(4)
Average test score (in standard deviations)	18.0	19.7
	(8.3)	(7.4)
% block group black	−404.8	−104.8
	(41.4)	(36.9)
% census block group Hispanic	−88.4	−3.5
% block group with college degree or more	183.5	104.6
	(26.4)	(31.8)
Average block group income (/10,000)	30.7	36.3
	(3.7)	(6.6)

NOTE.—All regressions shown in the table also include controls for whether the house is owner-occupied, the number of rooms, year built (1980s, 1960–79, pre-1960), elevation, population density, crime, and land use (% industrial, % residential, % commercial, % open space, % other) in 1-, 2-, and 3-mile rings around each location. The dependent variable is the monthly user cost of housing, which equals monthly rent for renter-occupied units and a monthly user cost for owner-occupied housing, calculated as described in the text. Standard errors corrected for clustering at the school level are reported in parentheses.

BFM Findings

- MWTP is in Dollars per month in Table 7.
- Households are willing to pay less than 1 percent more in house prices—substantially lower than previous estimates—when the average performance of the local school increases by 5 percent.
- Much of the apparent willingness to pay for more educated and wealthier neighbors is explained by the correlation of these sociodemographic measures with unobserved neighborhood quality.
- Neighborhood race is not capitalized directly into housing prices; instead, the negative correlation of neighborhood percent black and housing prices is due entirely to the fact that blacks live in neighborhoods with unobserved lower-quality.
- Finally, there is considerable heterogeneity in preferences for schools and neighbors, with households preferring to self-segregate on the basis of both race and education.