Discrete Choice Models and the Demand for Differentiated Products

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How Do We Estimate Demand Models?

- The study of demand is perhaps the most common example of structural modeling in empirical microeconomics.
- The basic goal: estimate the own- and cross-price elasticities for the set of goods in the market under study.
- One common problem in most settings is that there are too many goods.
- In many product markets, there are many varieties of products.
- This means there are many many own- and cross-price elasticities.
- In practice, there are often too many goods to practicably estimate price elasticities without imposing additional restrictions.

Aggregate Demand

- To see why this is a problem, consider estimating a simple linear demand system:

$$
\begin{aligned}
Q_1 &= \beta_{0,1} + \beta_{1,1} p_1 + \cdots + \beta_{J,1} p_J + \varepsilon_1 \\
&\vdots \\
Q_J &= \beta_{0,J} + \beta_{1,J} p_1 + \cdots + \beta_{J,J} p_J + \varepsilon_J
\end{aligned}
$$

- With J products, there are J^2 elasticities to estimate.
- Thus with J=140 types of butter and margarine, there are almost 20,000 elasticities to estimate.
- Even *without* endogeneity concerns, one would need *lots* of data to try to pin down so many parameters.
- We need to impose some structure on the problem.

The Problem of Differentiated Products

- Almost all consumer goods are not homogeneous goods.
- Instead they are differentiated products that differ along observed and potentially unobserved dimensions.
- Product differentiation provides limited market power that allows firms to set prices exceeding marginal costs. This is necessary to recover fixed costs.
- We use a characteristics approach that goes back to Gorman and Lancaster.
- We treat a product as a bundle of characteristics that can be described by a finite dimensional vector of attributes.
- We then aggregate the individual demand functions to generate the market level demand.

Discrete Choice Fundamentals

Consider a general specification of a discrete-continuous choice problem:

- A consumer:

  $$\max_{q_1,q_2,c} U(q_1, q_2, c)$$

  subject to

  $$\begin{aligned} p_1 q_1 + p_2 q_2 + c &= m \\ q_1\ q_2 &= 0 \end{aligned}$$

  where

  - q_1 and q_2 are the quantities of the two discrete alternatives the consumer is choosing,
  - c is the numeraire good (with its price normalized to 1),
  - m is income.
- The second constraint embodies the discreteness of the decision: either the consumer buys good 1 or she buys good 2, but not both.

Conditional Indirect Utility Functions

- Suppose that we *condition* on $q_1 = 0$.
- In this case, it is a standard utility maximization problem:

$$\max_{q_2, c} U(0, q_2, c)$$

subject to

$$p_2\ q_2 + c = m$$

- There is a standard solution for the demand functions $q_2(p_2, m)$ and $c(p_2, m)$. Plugging these demand functions back into the utility function yields the the conditional indirect utility function:

$$V_2(p_2, m) = U(0, q_2(p_2, m), c(p_2, m))$$

Optimal Choices

- Doing the same *conditioning* on $q_2 = 0$, we obtain an analogous expression:

$$V_1(p_1, m) = U(q_1(p_1, m), 0, c(p_1, m))$$

- The solution to the discrete choice part of the decision problem is then given by a choice between the two conditional indirect utility functions:

$$\max_{j=\{1,2\}} V_j(p_j, m)$$

An Example: Epple and Sieg (1999)

- Epple and Sieg (1999). "Estimating Equilibrium Models of Local Jurisdictions," Journal of Political Economy, 107(4), 645-681.
- This paper is the first to estimate am equilibrium model in which households chose among J communities.
- Each community differs by public good provision g_j and housing prices p_j.
- The conditional indirect utility function is given by:

$$V(g_j, p_j, m, \alpha) = \left\{ \alpha g_j^{\rho} + \left[e^{\frac{m^{1-\nu}-1}{1-\nu}} \; e^{-\frac{B p_j^{\eta+1}-1}{1+\eta}} \right]^{\rho} \right\}^{\frac{1}{\rho}}$$

- α is weight that a household assigns to the public goods and is treated as a random coefficient.

Epple and Sieg (cont)

- Roy's identify implies that the housing demand function is given by:

$$h = B \, p_j^{\eta} \, m^{\nu}$$

- η is the price elasticity, and ν is the income elasticity.
- How do we measure housing prices?
- Assuming heterogeneity in α this model gives rise to a sorting model that is similarto the quality latter model in IO.

Pure Discrete Choice Models

- The problem becomes significantly easier if we assume that q_1 and q_2 can only be zero or one.
- For example, you can only live in one city, buy one car, live in one house, or go on one vacation at a time.
- In that case, we have:

$$\begin{aligned} V_1(p_1, m) &= U(1, 0, m - p_1) \\ V_2(p_2, m) &= U(0, 1, m - p_2) \end{aligned}$$

- Consequently, the consumer will choose alternative 1 if and only if

$$V_1(p_1, m) \geq V_2(p_2, m)$$

Note that optimality conditions are given by inequalities.

Idiosyncratic Preferences

- In many settings, it is desirable to have a model in which consumers who look identical make different decisions.
- For example, we often observe two consumers with the same income and the same observe characteristics buying different cars.
- For example, one may buy a Honda Accord and the other one buys a Toyota Camry, these are very similar, but not identical choices.
- The differences in the choices must be due to differences in tastes or preferences which are purely idiosyncratic.

Random Utility Models

- We can capture these idiosyncratic taste differences by adding random shocks to the conditional indirect utility function (McFadden, 1974).
- Let ϵ_j thus denote the random shock associated with the conditional indirect utility of product j.
- If these shocks are additively separable to the conditional indirect utility function, then we obtain the following specification of the discrete choice problem:

$$\max_{j=\{1,2\}} \; [V_j(p_j, m) \; + \; \epsilon_j]$$

- Utility functions that arise when we add random preference shocks are called random utility functions.

Optimal Choices Revised

- Let's reconsider a simple example with two products.
- Here, a consumer will choose alternative 1 over alternative 2 if and only if:

$$V_1(p_1, m) + \epsilon_1 \geq V_2(p_2, m) + \epsilon_2$$

which we can rewrite as

$$\epsilon_1 - \epsilon_2 \geq V_2(p_2, m) - V_1(p_1, m)$$

- So the difference in the idiosyncratic random preference shocks $\epsilon_1 - \epsilon_2$ must larger than the difference in the common components $V_2(p_2, m) - V_1(p_1, m)$.

Market Shares

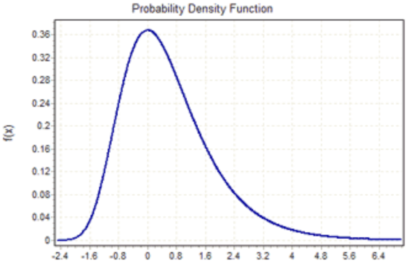
- Suppose we now want to compute the market share of product 1. Let's assume that there are a large number of consumers.
- The market share of product 1 is given by conditional choice probability:

$$s_1 = Pr\{\epsilon_1 - \epsilon_2 \geq V_2(p_2, m) - V_1(p_1, m)\}$$

- The conditional choice probability is the probability that an individual will choose product 1.
- It depends on the distribution of the idiosyncratic preference shocks ϵ_1 and ϵ_2. It also depends on the magnitude of the common components, $V_1(p_1, m)$ and $V_2(p_2, m)$.

Type I Extreme Value Distribution

- Following McFadden (1974) we assume that ϵ_j are Type I extreme value errors. The distribution function is illustrated in Figure 1.



- This distribution may look strange, but it is not that different than a normal distribution.

Market Shares with Type I Extreme Value Distribution

- With this functional form, the market share of product 1 has a closed form solution and is given by:

$$s_1 \quad = \quad \frac{\exp(V_1(p_1, m))}{\exp(V_1(p_1, m)) + \exp(V_2(p_2, m))}$$

- Hence the market share of product 1 increases in $V_1(\cdot)$ and decreases in $V_2(\cdot)$.
- However, even when the common utility of 1 is much larger than 2, some people will still choose 2 because of their idiosyncratic preferences.

Mixed Discrete Continuous Choice Models

- Dubin and McFadden (1984) discuss how to extend the pure discrete choice random utility models and estimate these mixed discrete-continuous choice models.
- You need two errors in the indirect utility function: one for the discrete part of the choice problem and one for the continuous choice.
- They consider an application in which individuals purchase refrigerators, and the choice of the refrigerator determines the demand for electricity.

Observed and Unobserved Product Characteristics

- We need to add observed and unobserved product characteristics to get a decent demand model for differentiated products (Berry,1994).
- Let x_j denote a vector of observed characteristics, and let ξ_j denote a scalar unobserved characteristic.
- A compelling linear approximation of a conditional indirect utility is

$$V_{ij} = x_j'\beta + \alpha(m - p_j) + \xi_j + \epsilon_{ij}$$

- An even more compelling model may treat the βs random coefficients which we will consider below.

A Linear Specification without Income Heterogeneity

- To illustrate the basic ideas, let us ignore differences in income.
- Let us assume that the indirect utility of individual i of product j is given by

$$\begin{aligned} V_{ij} &= x_j'\beta + \alpha p_j + \xi_j + \epsilon_{ij} \\ &= \delta_j + \epsilon_{ij} \end{aligned}$$

- δ_j is also called the "mean utility" of product j.
- Note that downward sloping demand requires that $\alpha < 0$.
- For goods in which prices are small relative to income (and income effects can be safely ignored), we often prefer this specification.

Logit Model Notes

- There is an *unobserved* (to the econometrician) product characteristic (or demand shock), ξ_j.
- This is very important for two reasons:
  1. It will be our econometric error in any regression/estimation.
  2. It is the source of our endogeneity problem.
- Why the endogeneity?
- Because it is observed by both consumers and firms.
- Products that have high ξ_j are more highly valued by consumers.
- Firms recognize this and price them higher.

A Normalization

We can set the mean utility of one of the goods to zero.

- Note all choices only depend on the *differences* in utility.
- Equivalently, if we added K "utils" to each product, no choices would change.
- Thus we set one utility to zero and measure all other utility *relative* to this baseline.
- Which j to choose? Usually the "outside option" which typically means buying nothing.
- If we denote the outside option is $j = 0$, we have $\delta_0 = 0$ $(= x_0 = p_0 = u_0)$

Conditional Choice Probabilities and Market Shares

- Let d_{ij} be an indicator variable that is equal to one if individual i purchases good j.
- Under the type one extreme value assumption, the conditional choice probability that individual i purchases good j is then given by:

$$
\begin{aligned}
Pr\{d_{ij} = 1 | x, p, \xi\} &= \frac{exp\{x_j'\beta + \alpha p_j + \xi_j\}}{1 + \sum_{i=1}^{J} exp\{x_i'\beta + \alpha p_i + \xi_i\}} \\
&= \frac{exp\{\delta_j\}}{1 + \sum_{i=1}^{J} exp\{\delta_i\}} \\
&= s_j(\delta)
\end{aligned}
$$

which is equal to the aggregate market share of product i.

- Note that $\delta_0 = 0$ and hence $exp(\delta_0) = 1$

Estimation

- Suppose we only have access to aggregate data, that is, we only observe $\{s_j, x_j, p_j\}_{j=0}^{J}$.
- Rewriting the equations characterizing the market shares yields the following regression model:

$$\delta_j = \ln(s_j) - \ln(s_0) = x_j'\beta + \alpha p_j + \xi_j$$

- Hence, we can recover the v_j's from the observed market shares.
- Note that the ξ_j is the error terms of the regression model above.

Identifying Assumptions

- To estimate this demand model we need one more assumption.
- $E[\xi_j | x_j, p_j] = 0$ is not a plausible restriction since p_j is likely to be correlated with ξ_j.
- Optimal pricing of firms will imply a correlation between prices and unobserved product characteristics.
- If there exists an instrument z_j such that $E[\xi_j | x_j, z_j] = 0$, we can then estimate the parameters of the model using a linear IV estimator.
- What are good instruments? Cost shifters, characteristics of products of close competitors, etc.

Some Comments on Aggregation

- To use aggregate data, we need to solve an aggregation problem.
- We start with a model of individual demand and derive the aggregate market shares by aggregating the individual conditional choice probabilities.
- In this simple version of the model, the market share is equal to the conditional choice probability since there is no individual heterogeneity besides the idiosyncratic errors.
- Let's look at some results from BLP (1995).

TABLE III

RESULTS WITH LOGIT DEMAND AND MARGINAL COST PRICING
(2217 OBSERVATIONS)

Variable	OLS Logit Demand	IV Logit Demand	OLS ln (*price*) on *w*
Constant	−10.068	−9.273	1.882
	(0.253)	(0.493)	(0.119)
*HP / Weight**	−0.121	1.965	0.520
	(0.277)	(0.909)	(0.035)
Air	−0.035	1.289	0.680
	(0.073)	(0.248)	(0.019)
MP$	0.263	0.052	—
	(0.043)	(0.086)	
*MPG**	—	—	−0.471
			(0.049)
*Size**	2.341	2.355	0.125
	(0.125)	(0.247)	(0.063)
Trend	—	—	0.013
			(0.002)
Price	−0.089	−0.216	—
	(0.004)	(0.123)	
No. Inelastic Demands	1494	22	*n.a.*
(+/−2 *s.e.'s*)	(1429–1617)	(7–101)	
R^2	0.387	*n.a.*	.656

Notes: The standard errors are reported in parentheses.

*The continuous product characteristics—hp/weight, size, and fuel efficiency (*MP$* or *MPG*)—enter the demand equations in levels, but enter the column 3 price regression in natural logs.

Logit Elasticities

- Own-price derivatives / elasticities:

$$\frac{\partial s_j(p)}{\partial p_j} = \frac{\partial s_j(p)}{\partial \delta_j}\frac{\partial \delta_j}{\partial p_j} = s_j(1 - s_j)\alpha$$

$$\boxed{\Rightarrow \eta_{jj} = \frac{\partial s_j(p)}{\partial p_j}\frac{p_j}{s_j} = \alpha p_j(1 - s_j)}$$

- Cross-price derivatives / elasticities:

$$\frac{\partial s_j(p)}{\partial p_k} = \frac{\partial s_j(p)}{\partial \delta_k}\frac{\partial \delta_k}{\partial p_k} = -s_j s_k \alpha$$

$$\boxed{\Rightarrow \eta_{jk} = \frac{\partial s_j(p)}{\partial p_k}\frac{p_k}{s_j} = -\alpha p_k s_k}$$

Implications of Logit Elasticities

- Note that the cross-price elasticity of good j with respect to good k *is independent of* j!
- This means that the proportionate change in demand for any two products with respect to good k is the same.
- For example, consider a Mercedes SL and a Honda Civic.
- Then the cross-price elasticity of each with respect to a change in the price of a Mini Cooper *is the same*, ...
- ... even though the Mini is "more like" the Civic.

A Final Thought about the Logit Model

- I have said we will consider alternative parameterizations of own- and cross-price elasticities that reduce the number of free parameters.
- Question: How many free parameters does a logit demand model have? Let's look again at the elasticity formulas:

$$\eta_{jj} = \alpha p_j(1 - s_j)$$
$$\eta_{jk} = -\alpha p_k s_k$$

- Answer: One free parameter. This is *not* very flexible!

Moving Forward: Generalized Logit Models

- Fortunately, we can generalize the logit model by allowing for observed heterogeneity among consumers, such as income, age, gender, etc.
- We can also allow for unobserved heterogeneity in the tastes for product characteristics, by treating the β's as random coefficients.
- These generalized logit models then generate more realistic demand patterns.
- We can also estimate marginal costs by specifying a supply model and using the first order conditions from the optimal pricing decision.
- Now we are in business. Let's look at some tables from BLP (1995).

TABLE IV

ESTIMATED PARAMETERS OF THE DEMAND AND PRICING EQUATIONS: BLP SPECIFICATION, 2217 OBSERVATIONS

Demand Side Parameters	Variable	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Means ($\bar{\beta}$'s)	*Constant*	−7.061	0.941	−7.304	0.746
	HP/Weight	2.883	2.019	2.185	0.896
	Air	1.521	0.891	0.579	0.632
	MP$	−0.122	0.320	−0.049	0.164
	Size	3.460	0.610	2.604	0.285
Std. Deviations (σ_β's)	*Constant*	3.612	1.485	2.009	1.017
	HP/Weight	4.628	1.885	1.586	1.186
	Air	1.818	1.695	1.215	1.149
	MP$	1.050	0.272	0.670	0.168
	Size	2.056	0.585	1.510	0.297
Term on Price (α)	$\ln(y-p)$	43.501	6.427	23.710	4.079
Cost Side Parameters					
	Constant	0.952	0.194	0.726	0.285
	ln(*HP/Weight*)	0.477	0.056	0.313	0.071
	Air	0.619	0.038	0.290	0.052
	ln(*MPG*)	−0.415	0.055	0.293	0.091
	ln(*Size*)	−0.046	0.081	1.499	0.139
	Trend	0.019	0.002	0.026	0.004
	$\ln(q)$			−0.387	0.029

TABLE VI

A SAMPLE FROM 1990 OF ESTIMATED OWN- AND CROSS-PRICE SEMI-ELASTICITIES: BASED ON TABLE IV (CRTS) ESTIMATES

	Mazda 323	Nissan Sentra	Ford Escort	Chevy Cavalier	Honda Accord	Ford Taurus	Buick Century	Nissan Maxima	Acura Legend	Lincoln Town Car	Cadillac Seville
323	−125.933	1.518	8.954	9.680	2.185	0.852	0.485	0.056	0.009	0.012	0.002
Sentra	0.705	−115.319	8.024	8.435	2.473	0.909	0.516	0.093	0.015	0.019	0.003
Escort	0.713	1.375	−106.497	7.570	2.298	0.708	0.445	0.082	0.015	0.015	0.003
Cavalier	0.754	1.414	7.406	−110.972	2.291	1.083	0.646	0.087	0.015	0.023	0.004
Accord	0.120	0.293	1.590	1.621	−51.637	1.532	0.463	0.310	0.095	0.169	0.034
Taurus	0.063	0.144	0.653	1.020	2.041	−43.634	0.335	0.245	0.091	0.291	0.045
Century	0.099	0.228	1.146	1.700	1.722	0.937	−66.635	0.773	0.152	0.278	0.039
Maxima	0.013	0.046	0.236	0.256	1.293	0.768	0.866	−35.378	0.271	0.579	0.116
Legend	0.004	0.014	0.083	0.084	0.736	0.532	0.318	0.506	−21.820	0.775	0.183
TownCar	0.002	0.006	0.029	0.046	0.475	0.614	0.210	0.389	0.280	−20.175	0.226
Seville	0.001	0.005	0.026	0.035	0.425	0.420	0.131	0.351	0.296	1.011	−16.313
LS400	0.001	0.003	0.018	0.019	0.302	0.185	0.079	0.280	0.274	0.606	0.212
735i	0.000	0.002	0.009	0.012	0.203	0.176	0.050	0.190	0.223	0.685	0.215

Note: Cell entries i, j, where i indexes row and j column, give the percentage change in market share of i with a \$1000 change in the price of j.

TABLE VIII

A SAMPLE FROM 1990 OF ESTIMATED PRICE-MARGINAL COST MARKUPS AND VARIABLE PROFITS: BASED ON TABLE 6 (CRTS) ESTIMATES

	Price	Markup Over MC $(p - MC)$	Variable Profits (in \$'000's) $q*(p - MC)$
Mazda 323	\$5,049	\$ 801	\$18,407
Nissan Sentra	\$5,661	\$ 880	\$43,554
Ford Escort	\$5,663	\$1,077	\$311,068
Chevy Cavalier	\$5,797	\$1,302	\$384,263
Honda Accord	\$9,292	\$1,992	\$830,842
Ford Taurus	\$9,671	\$2,577	\$807,212
Buick Century	\$10,138	\$2,420	\$271,446
Nissan Maxima	\$13,695	\$2,881	\$288,291
Acura Legend	\$18,944	\$4,671	\$250,695
Lincoln Town Car	\$21,412	\$5,596	\$832,082
Cadillac Seville	\$24,353	\$7,500	\$249,195
Lexus LS400	\$27,544	\$9,030	\$371,123
BMW 735*i*	\$37,490	\$10,975	\$114,802