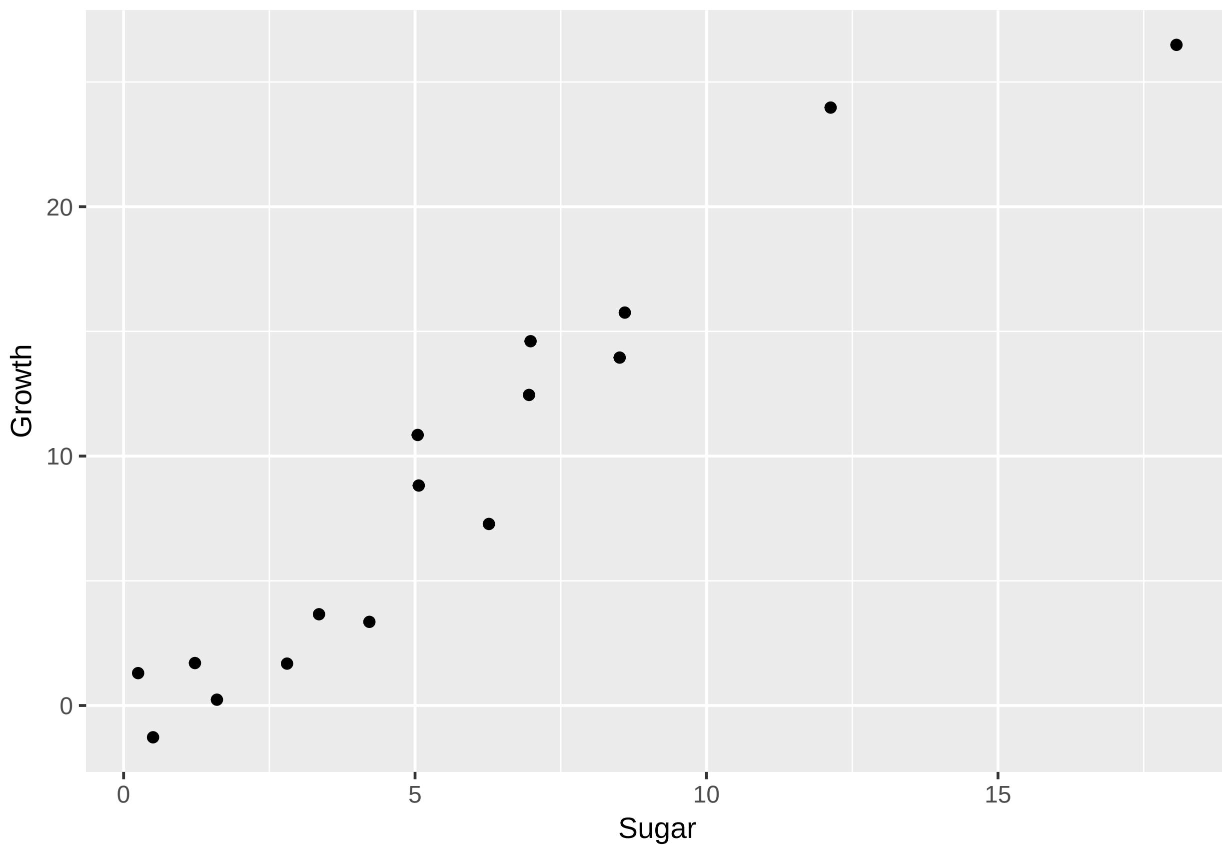
Sugar Regression

We have data from an experiment evaluating the effect of sugar concentration on growth of *E. Coli* in lab cultures. An overview of the data is presented below, both in tabular and graphical form:

	N	Mean	SD	Min	Q1	Median	Q3	Max
sugar	16	5.72	4.65	0.25	2.20	5.05	7.75	18.06
growth	16	9.05	8.40	-1.27	1.69	8.05	14.28	26.49



Superficially, at least, this appears to indicate some sort of linear relationship. We will test this by comparing the negative log likelihood functions of a constant model and a linear model using a likelihood ratio test. First, however, we need hypotheses:

- **Null Hypothesis:** The slope of a linear model fitted to the data is 0.
- **Alternate Hypothesis:** The slope of a linear model fitted to the data is not 0.

Let G be a variable denoting the values for growth, S be a variable denoting the values for sugar, and ϵ be an error term which is normally distributed with standard deviation σ. Then we fit models of the form $G = \beta_0 + \epsilon$ and $G = \beta_0 + \beta_1 S + \epsilon$ to the data. An optimization run on the first model returned values of $\beta_0 \approx 9.0559$ and $\sigma \approx 8.134$; an optimization run on thes second model returend values of $\beta_0 \approx -0.85101$, $\beta_1 \approx 1.7304$, and $\sigma \approx 2.3363$. The negative log-likelihood for the constat model was approximately 56.2412; for the linear model it was approximately 36.27864.

To implement the likelihood ratio test, we found the absolute values of the difference between the negative log-likelihood values, multiplied this by two. As the size of the sample grows, this value approaches a chi-square distribution with 1 degree of freedom, so we used a chi-square distribution with 1 degree of freedom to find the p-value for the experiment.

The p-value was $p \approx 2.6389 \times 10^{-10}$. This value is quite small, so we reject the null hypothesis, and conclude that a linear fit correctly explains the patterns we see in the data. We summarize this with another graph: