Question 1 :

Sunco processes;

1.*OIL* into

1.a *AVIATION FUEL*

1.b *HEATING OIL*

Output from distillation may be sold

A. *DIRECTLY* or
B. processed in the *CATALYTIC CRACKER*

Answer for A

Decision Variables

1. ***OIL***
2. ***AVIATION_DIRECTLY***
3. ***AVIATION_CCRACKER***
4. ***HEATING_OIL_DIRECTLY***
5. ***HEATING_OIL_CCRACKER***

Objective Function

Max Z : 40 HEATING _OIL_ DIRECTLY + 90 HEATING_OIL_CCRACKER + 60 AVIATION_DIRECTLY + 130 AVIATION_CCRACKER - 40 OIL

Constraints

OIL <= 20 ***MAX SUPPLY***

0.5 OIL = AVIATION_DIRECTLY+ AVIATION_CCRACKER ***MAX_AVIATION_FUEL***

0.5 OIL = HEATING _OIL_ DIRECTLY + HEATING_OIL_CCRACKER ***MAX_HEATING_OIL***

1*AVIATION_CCRACKER + 0.75*HEATING_OIL_CCRACKER <= 8 ***MAX_CRACKER_HOUR***

OIL>=0, AVIATION_DIRECTLY >=0, AVIATION_CCRACKER >=0, HEATING _OIL_ DIRECTLY >=0, HEATING_OIL_CCRACKER >=0 ***MINIMUM VALUES OF DECISION VARIABLES***

Answer for B

#Google Colab is used for solution

!pip install pulp

```python
from pulp import LpVariable, LpMaximize, LpStatus, LpProblem, LpInteger

prob= LpProblem("asdas",LpMaximize)

OIL = LpVariable("Purchased Barrels of Oil", lowBound=0, cat='Continuous')

AVIATION_DIRECTLY = LpVariable("Aviation Fuel Barrels Sold Directly", lowBound=0, cat='Continuous')
AVIATION_CCRACKER = LpVariable("Aviation Fuel Barrels Sold Processed In the Catalytic Cracker", 
lowBound=0, cat='Continuous')

HEATING_OIL_DIRECTLY = LpVariable("Heating Oil Barrels Sold Directly", lowBound=0, cat='Continuous')
HEATING_OIL_CCRACKER = LpVariable("Heating Oil Barrels Sold Processed In the Catalytic Cracker", 
lowBound=0, cat='Continuous')

#LP Model to maximize profit
prob += 60*AVIATION_DIRECTLY + 130*AVIATION_CCRACKER + 40*HEATING_OIL_DIRECTLY + 
90*HEATING_OIL_CCRACKER - 40*OIL
#available barrel supply
prob += OIL <= 20, "MAX SUPPLY"
#Total aviation fuel must be half of distillation
prob += 0.5*OIL - AVIATION_DIRECTLY - AVIATION_CCRACKER == 0, "MAX_AVIATION_FUEL"
#Total heating oil must be half of distillation
prob += 0.5*OIL - HEATING_OIL_DIRECTLY - HEATING_OIL_CCRACKER == 0, "MAX_HEATING_OIL"
#total hours is 8, we need to divide our factor to max barrel can produced per hour
#for AVIATION 1000 barrels per hour, HEATING 1000 barrels per 45 minutes which is 0.75
prob += AVIATION_CCRACKER + 0.75*HEATING_OIL_CCRACKER <= 8, "MAX_CRACKER_HOUR"

# The prob is solved using PuLP's choice of Solver
```

```python
prob.solve()



# The status of the solution is printed to the screen
print("Status:", LpStatus[prob.status])



# Each of the variables is printed with it's resolved optimum value
for v in prob.variables():
   print(v.name, "=", v.varValue)
# The optimised objective function value is printed to the screen
print("Total Cost of the Process = ", prob.objective.value(), '$')



import pandas as pd
#Report sensivity Analysis
print("\nSensivity Analysis")



Cons_Sensivity_Report = [{'Constraint_Name':name,'Slack':c.slack,"Shadow_Price":c.pi}
                    for name, c in prob.constraints.items()]
print(pd.DataFrame(Cons_Sensivity_Report))
print("\n")
Variable_Sensitivity_Report = [{'Variable_Name': v.name, 'Value':v.varValue,'Reduced_Cost': v.dj}
                         for v in prob.variables()]



print(pd.DataFrame(Variable_Sensitivity_Report)[['Variable_Name','Value','Reduced_Cost']])
```

OUTPUT:

```
Status: Optimal
Aviation_Fuel_Barrels_Sold_Directly = 2.0
Aviation_Fuel_Barrels_Sold_Processed_In_the_Catalytic_Cracker = 8.0
Heating_Oil_Barrels_Sold_Directly = 10.0
Heating_Oil_Barrels_Sold_Processed_In_the_Catalytic_Cracker = 0.0
Purchased_Barrels_of_Oil = 20.0
Total Cost of the Process =  760.0 $

Sensivity Analysis
     Constraint_Name  Slack  Shadow_Price
0         MAX_SUPPLY   -0.0          10.0
1  MAX_AVIATION_FUEL   -0.0         -60.0
2    MAX_HEATING_OIL   -0.0         -40.0
3   MAX_CRACKER_HOUR   -0.0          70.0


                                   Variable_Name  Value  Reduced_Cost
0                Aviation_Fuel_Barrels_Sold_Directly    2.0           0.0
1  Aviation_Fuel_Barrels_Sold_Processed_In_the_Ca...    8.0           0.0
2                  Heating_Oil_Barrels_Sold_Directly   10.0           0.0
3  Heating_Oil_Barrels_Sold_Processed_In_the_Cata...    0.0          -2.5
4                           Purchased_Barrels_of_Oil   20.0           0.0
```

Answer for C

Yes, it is profitable. If we look at Shadow Price of MAX_SUPPLY, we can see how it will affect how much more profit you would get by increasing the amount of that resource by one unit, which can be seen, its positive value.

Answer for D

Shadow Price of per oil purchase is 10$, so every x unit we purchase, our profit will increase 10*x unit.

Answer for E

Shadow Price of per hour for Catalytic Cracker is 70$, but our supply is not limitless, so it will increase for certain time, but when we used all of our barrels, it will became useless.

Question 2 :

Answer for A

#Decision Variables

1. regular1 => Week 1 Regular Production
2. regular2 => Week 2 Regular Production
3. regular3 => Week 3 Regular Production
4. regular4 => Week 4 Regular Production
5. regular5 => Week 5 Regular Production
6. regular6 => Week 6 Regular Production
7. overtime1 => Week 1 Overtime Production
8. overtime2 => Week 2 Overtime Production
9. overtime3 => Week 3 Overtime Production
10. overtime4 => Week 4 Overtime Production
11. overtime5 => Week 5 Overtime Production
12. overtime6 => Week 6 Overtime Production
13. extra1 => Week 1 Extra Production
14. extra2 => Week 2 Extra Production
15. extra3 => Week 3 Extra Production
16. extra4 => Week 4 Extra Production
17. extra5 => Week 5 Extra Production

#Objective Function for Minimize Z

Minimize Z = $190(regular1+regular2+regular3+regular4+regular5+regular6)+$260*(overtime1+overtime2+overtime3+overtime4+overtime5+overtime6)+$10*(extra1+extra2+extra3+extra4+extra5)*

#Constraints

regular1+overtime1-extra1 = 105	***Week 1 Orders***
regular2+overtime2+extra1-extra2 = 170	***Week 2 Orders***
regular3+overtime3+extra2-extra3 = 230	***Week 3 Orders***
regular4+overtime4+extra3-extra4 = 180	***Week 4 Orders***
regular5+overtime5+extra4-extra5 = 150	***Week 5 Orders***
regular6+overtime6+extra5 = 250	***Week 6 Orders***

regular1, regular2, regular3, regular4, regular5, regular6 <= 160 ***Upbound for Regular Production***

overtime1, overtime2, overtime3, overtime4, overtime5, overtime6 <= 50 ***Upbound for Overtime Production***

regular1, regular2, regular3, regular4, regular5, regular6, overtime1, overtime2, overtime3, overtime4, overtime5, overtime6, extra1+extra2+extra3+extra4+extra5 >= 0 ***Lowbound for All Variables***

Answer for B

```
#Google Colab is used for solution

#Adding install pulp in case of code run seperated from first question solution
!pip install pulp
from pulp import LpVariable, LpMinimize, LpStatus, LpProblem

prob = LpProblem("Minimize_Cost_Problem",LpMinimize)

#Decision Variables of LP
regular1 = LpVariable("Week 1 Regular Production", lowBound=0, upBound=160)
regular2 = LpVariable("Week 2 Regular Production", lowBound=0, upBound=160)
regular3 = LpVariable("Week 3 Regular Production", lowBound=0, upBound=160)
regular4 = LpVariable("Week 4 Regular Production", lowBound=0, upBound=160)
regular5 = LpVariable("Week 5 Regular Production", lowBound=0, upBound=160)
regular6 = LpVariable("Week 6 Regular Production", lowBound=0, upBound=160)

overtime1 = LpVariable("Week 1 Overtime Production", lowBound=0, upBound=50)
overtime2 = LpVariable("Week 2 Overtime Production", lowBound=0, upBound=50)
overtime3 = LpVariable("Week 3 Overtime Production", lowBound=0, upBound=50)
```

```
overtime4 = LpVariable("Week 4 Overtime Production", lowBound=0, upBound=50)

overtime5 = LpVariable("Week 5 Overtime Production", lowBound=0, upBound=50)

overtime6 = LpVariable("Week 6 Overtime Production", lowBound=0, upBound=50)



extra1 = LpVariable("Week 1 Extra Production", lowBound=0)

extra2 = LpVariable("Week 2 Extra Production", lowBound=0)

extra3 = LpVariable("Week 3 Extra Production", lowBound=0)

extra4 = LpVariable("Week 4 Extra Production", lowBound=0)

extra5 = LpVariable("Week 5 Extra Production", lowBound=0)



#Objective Function for Minimize Z

prob += 190*(regular1+regular2+regular3+regular4+regular5+regular6)+260*(overtime1+overtime2+overtime3+overtime4+overtime5+overtime6)+10*(extra1+extra2+extra3+extra4+extra5)



#Constraints of Problem

prob += regular1+overtime1-extra1 == 105, "Week 1 Orders"

prob += regular2+overtime2+extra1-extra2 == 170, "Week 2 Orders"

prob += regular3+overtime3+extra2-extra3 == 230, "Week 3 Orders"

prob += regular4+overtime4+extra3-extra4 == 180, "Week 4 Orders"

prob += regular5+overtime5+extra4-extra5 == 150, "Week 5 Orders"

prob += regular6+overtime6+extra5 == 250, "Week 6 Orders"



prob.solve()



print("Status:", LpStatus[prob.status])
```

```python
for v in prob.variables():
   print(v.name, "=", v.varValue)
print("Total Cost of the Process = ", prob.objective.value())


import pandas as pd
#Report sensivity Analysis
print("\nSensivity Analysis")



Cons_Sensivity_Report = [{'Constraint_Name':name,'Slack':c.slack,"Shadow_Price":c.pi}
                for name, c in prob.constraints.items()]
print(pd.DataFrame(Cons_Sensivity_Report))
print("\n")
Variable_Sensitivity_Report = [{'Variable_Name': v.name, 'Value':v.varValue,'Reduced_Cost': v.dj}
                   for v in prob.variables()]


print(pd.DataFrame(Variable_Sensitivity_Report)[['Variable_Name','Value','Reduced_Cost']])
```

OUTPUT :

```
Status: Optimal
Week_1_Extra_Production = 55.0
Week_1_Overtime_Production = 0.0
Week_1_Regular_Production = 160.0
Week_2_Extra_Production = 45.0
Week_2_Overtime_Production = 0.0
Week_2_Regular_Production = 160.0
Week_3_Extra_Production = 0.0
Week_3_Overtime_Production = 25.0
Week_3_Regular_Production = 160.0
Week_4_Extra_Production = 0.0
Week_4_Overtime_Production = 20.0
Week_4_Regular_Production = 160.0
Week_5_Extra_Production = 40.0
Week_5_Overtime_Production = 30.0
Week_5_Regular_Production = 160.0
Week_6_Overtime_Production = 50.0
Week_6_Regular_Production = 160.0
Total Cost of the Process =  216300.0

Sensivity Analysis
  Constraint_Name  Slack  Shadow_Price
0   Week_1_Orders   -0.0         240.0
1   Week_2_Orders   -0.0         250.0
2   Week_3_Orders   -0.0         260.0
3   Week_4_Orders   -0.0         260.0
4   Week_5_Orders   -0.0         260.0
5   Week_6_Orders   -0.0         270.0


                 Variable_Name  Value  Reduced_Cost
0      Week_1_Extra_Production   55.0           0.0
1   Week_1_Overtime_Production    0.0          20.0
2    Week_1_Regular_Production  160.0         -50.0
3      Week_2_Extra_Production   45.0           0.0
4   Week_2_Overtime_Production    0.0          10.0
5    Week_2_Regular_Production  160.0         -60.0
6      Week_3_Extra_Production    0.0          10.0
7   Week_3_Overtime_Production   25.0           0.0
8    Week_3_Regular_Production  160.0         -70.0
9      Week_4_Extra_Production    0.0          10.0
10  Week_4_Overtime_Production   20.0           0.0
11   Week_4_Regular_Production  160.0         -70.0
12     Week_5_Extra_Production   40.0           0.0
13  Week_5_Overtime_Production   30.0           0.0
14   Week_5_Regular_Production  160.0         -70.0
15  Week_6_Overtime_Production   50.0         -10.0
16   Week_6_Regular_Production  160.0         -80.0
```

Answer for C

Highest Shadow Price belongs to Week 6, so I would choose Week 6.

Question 3 :

Answer for A

DecisionVariables;

N => Necklace

B => Bracelet

Maximize Profit;

Max Z = N*400$ + B*300$

Constraints;

N*3 + B*2 <= 18

N*2 + B*4 <= 20

N <= 4

B >= 0, N >= 0

Answer for B

For 1st Constraint =>

Max N => N =6, B = 0

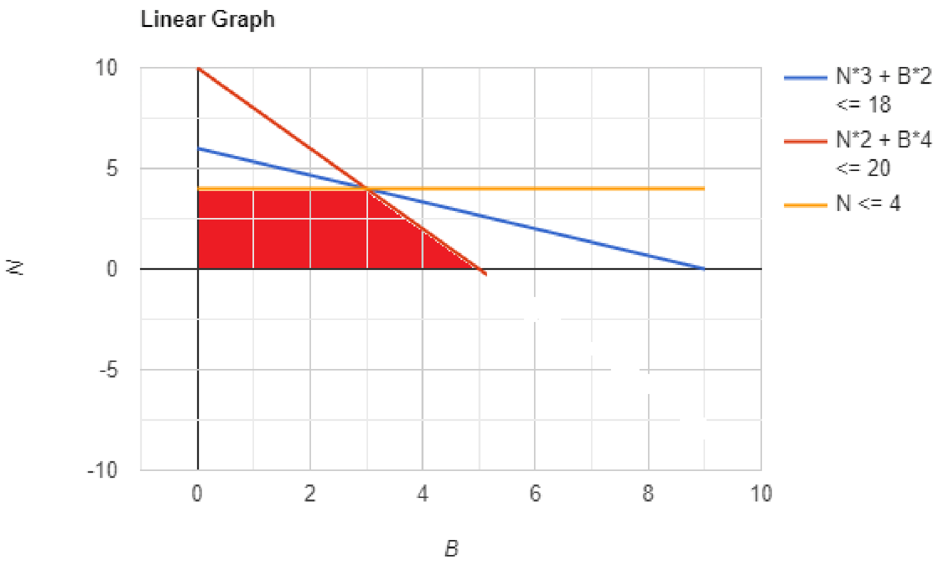
Max B => N = 0, B = 9

For 2nd Constraint =>

Max N => N = 10, B = 0

Max B => N = 0, B = 5

Graph Is


Linear Graph
10
5
0
-5
-10
N
0
2
4
6
8
10
B
N*3 + B*2 <= 18
N*2 + B*4 <= 20
N <= 4