Wigner Function for Two Mode Squeezed Vacuum(TMSV)

Given in the *Squeezed Light*,

$$\psi_R(q_1, q_2) = \frac{1}{\sqrt{\pi}} e^{\frac{-(q_1+q_2)^2}{4R^2}} e^{\frac{-R^2(q_1-q_2)^2}{4}}$$

Where

$$R = e^r = e^{\zeta e^{-i\phi}}$$

Wigner Representation (assume $\phi = 0$)

$$W(p_1, p_2, q_1, q_2) = \left\langle q_2 - \frac{x_2}{2} \right| \left\langle q_1 - \frac{x_1}{2} \right| \rho \left| q_1 + \frac{x_1}{2} \right\rangle \left| q_2 + \frac{x_2}{2} \right\rangle$$

$$= \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi^*\left(q_1 - \frac{x_1}{2}, q_2 - \frac{x_2}{2}\right) \times \psi\left(q_1 + \frac{x_1}{2}, q_2 + \frac{x_2}{2}\right) e^{\frac{ip_1x_1}{\hbar}} e^{\frac{ip_2x_2}{\hbar}} dx_1 dx_2$$

where

$$\psi^*\left(q_1 - \frac{x_1}{2}, q_2 - \frac{x_2}{2}\right) = \frac{1}{\sqrt{\pi}} e^{\frac{-\left(q_1+q_2-\frac{x_1}{2}-\frac{x_2}{2}\right)^2}{4}} e^{\frac{-\left(q_1-q_2-\frac{x_1}{2}+\frac{x_2}{2}\right)^2}{4}}$$

$$\psi\left(q_1 + \frac{x_1}{2}, q_2 + \frac{x_2}{2}\right) = \frac{1}{\sqrt{\pi}} e^{\frac{-\left(q_1+q_2+\frac{x_1}{2}+\frac{x_2}{2}\right)^2}{4}} e^{\frac{-\left(q_1-q_2+\frac{x_1}{2}-\frac{x_2}{2}\right)^2}{4}}$$

$$W(p_1, p_2, q_1, q_2) = \frac{4}{\pi} e^{-{q_1}^2} e^{-{q_2}^2} e^{-\frac{{p_1}^2}{\hbar^2}} e^{-\frac{{p_2}^2}{\hbar^2}}$$