ITU - ECN 407

Time Series Econometrics

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HOMEWORK ASSIGNMENT 1

1) Question 1

1.a) Replication of the Graph

The graph below presents the employment levels in agriculture and related industries in the USA over time.

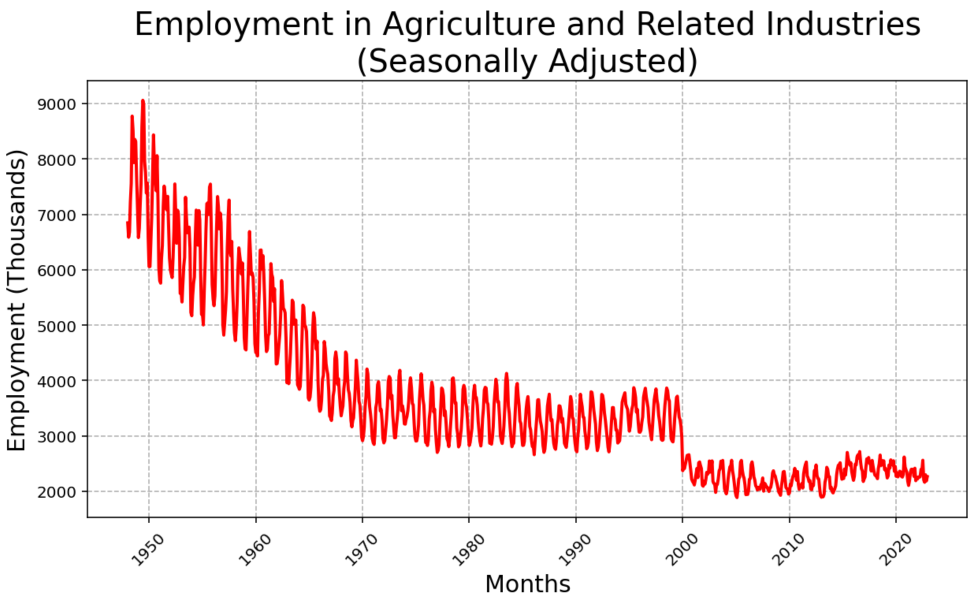

Figure 1.1

1.b) Trend Analysis

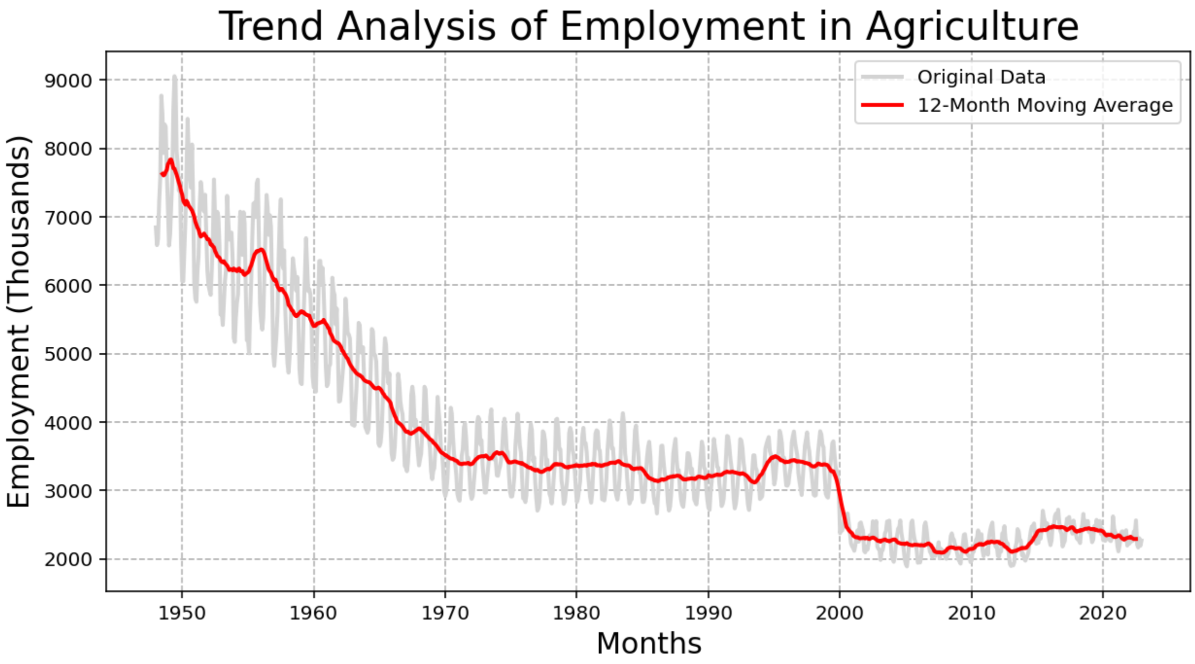

Figure 2.2

The employment level in the agricultural sector shows a declining trend over time.

• **1948-2000:** The data indicates a general decrease in employment levels, with periodic seasonal fluctuations.

• **2000-Present:** Employment levels appear to have stabilized at lower levels compared to earlier decades.

This can be explained by factors such as mechanization in agriculture, technological development, reduced need for sectoral labor, and concentration on other developing sectors. aAccording to the data, agriculture is not a growing sector in terms of employment.

1.c) Sample Restriction

For short-term analysis, the data was restricted to January 2018 - December 2018. The following graph displays the employment variations across months:

Figure 1.3

Analyzing 2018 data, seasonal fluctuations are clearly visible. While employment increases in spring and summer, it decreases in fall and winter. This shows that the agricultural sector depends on seasonal labor needs. Employment increases especially during the planting and harvesting periods, while it declines at other times of the year.

2) Question 2

2.a) Variables and Graphs Generation

GDP and tax are converted to real values by dividing them by the consumer price index**:**

$$r_{gdp} = \frac{gdp}{cpi}, \quad r_{tax} = \frac{tax}{cpi}$$

Then, the logarithm of these real values is taken to create new variables:

$$lrgdp = \log(rgdp), \quad lrtax = \log(rtax)$$

Table 2.1 (First 5 rows of data)

GDP	Tax	CPI	RGDP	RTAX	LRGDP	LRTAX
542.382	75.555	12.478	43.46703	6.055053	3.772003	1.800893
562.209	76.771	12.612	44.57868	6.087326	3.797256	1.806209
603.922	82.476	12.763	47.31894	6.46222	3.856911	1.865973
637.45	87.71	12.921	49.33436	6.788167	3.898621	1.915181
684.46	86.151	13.086	52.3037	6.583316	3.957067	1.884539

- A time series graph, where both **LRGDP** and **LRTAX** are plotted together:

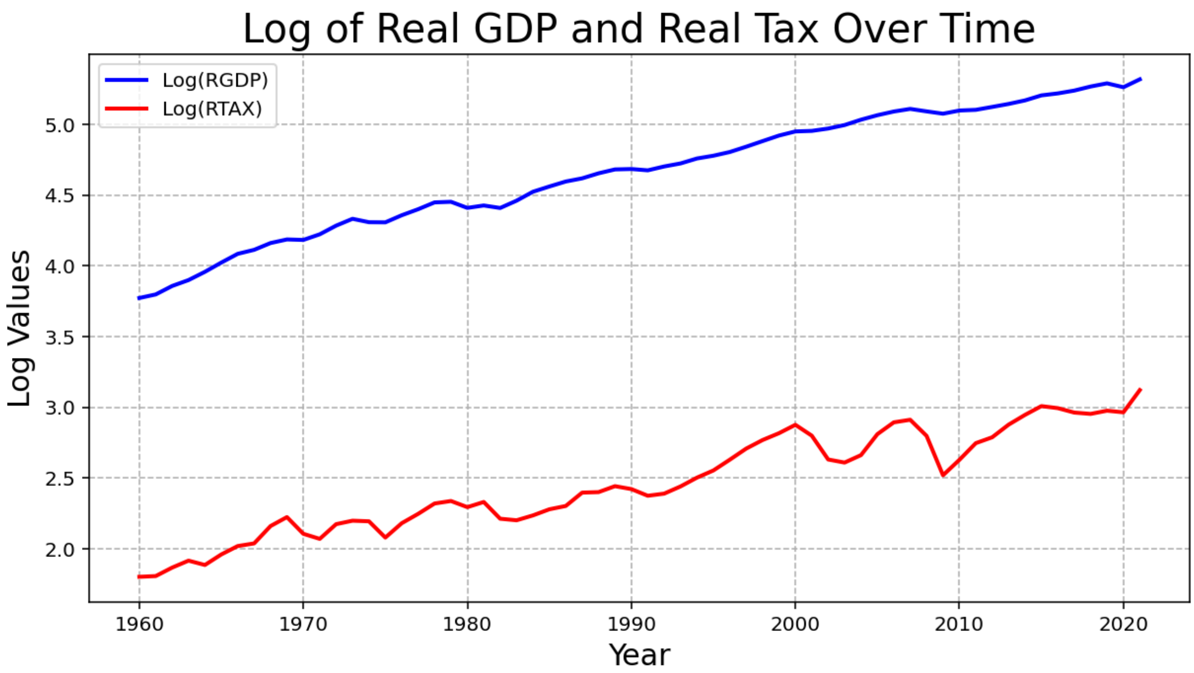

Figure 2.1

- A scatter plot**,** where **LRTAX** is on the vertical axis and **LRGDP** is on the horizontal axis:

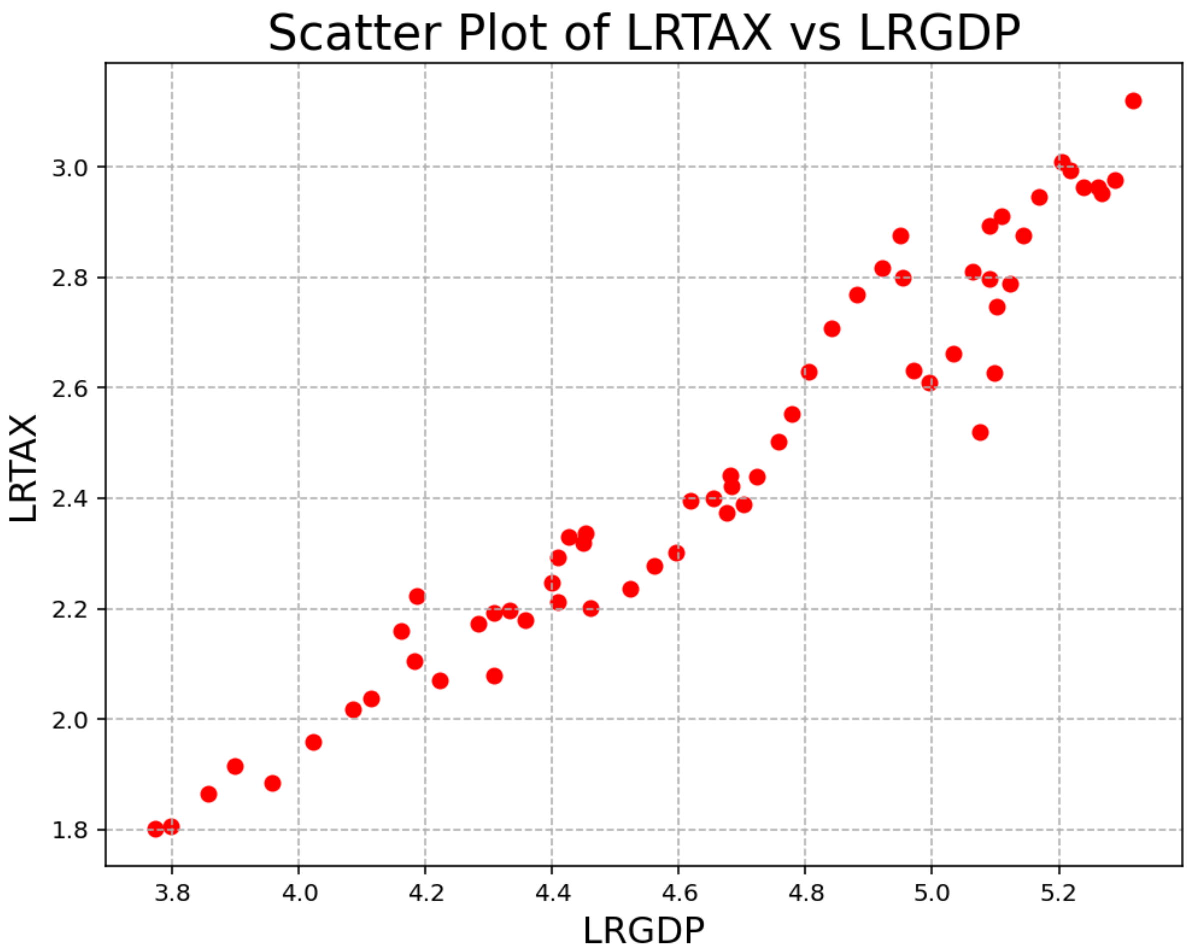

Figure 2.2

2.b) Relationship Between LRGDP and LRTAX

From the graphs, it can be seen that both variables move in the same direction over time. The time series graph shows a general increasing trend for both GDP and tax revenues. However, some fluctuations can be observed.

The scatter plot suggests a positive relationship between GDP and tax revenues. As GDP increases, tax revenues also tend to increase. This supports the expectation that higher economic activity leads to higher tax collection.

2.c) Regression Analysis

To examine this relationship further, a simple linear regression model is estimated:

$$\text{lrtax} = \beta_0 + \beta_1 \cdot \text{lrgdp} + \varepsilon$$

Python regression results

```
lrgdp    32
lrtax    32
dtype: int64
           lrgdp      lrtax
count  62.000000  62.000000
mean    4.661276   2.463201
std     0.432418   0.355663
min     3.772003   1.800893
25%     4.338194   2.198434
50%     4.682299   2.410331
75%     5.072198   2.794683
max     5.317810   3.121001
                            OLS Regression Results
==============================================================================
Dep. Variable:                  lrtax   R-squared:                       0.942
Model:                            OLS   Adj. R-squared:                  0.941
Method:                 Least Squares   F-statistic:                     968.4
Date:                Mon, 03 Mar 2025   Prob (F-statistic):           1.01e-38
Time:                        15:55:12   Log-Likelihood:                 64.709
No. Observations:                  62   AIC:                            -125.4
Df Residuals:                      60   BIC:                            -121.2
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.2572      0.120    -10.472      0.000      -1.497      -1.017
lrgdp          0.7981      0.026     31.120      0.000       0.747       0.849
==============================================================================
Omnibus:                        3.260   Durbin-Watson:                   0.623
Prob(Omnibus):                  0.196   Jarque-Bera (JB):                2.356
Skew:                          -0.411   Prob(JB):                        0.308
Kurtosis:                       3.487   Cond. No.                         53.4
==============================================================================
```

The results show:

- **β1 = 0.7981** → GDP and tax revenues have a strong positive relationship.
- **R2 = 0.942** → The model explains 94.2% of the variation in tax revenues.
- **p-value < 0.05** → The relationship is statistically significant.

A residual plot is created to analyze the errors in the regression model. The graph shows that some fluctuations exist, especially in certain periods. These deviations suggest that other factors, such as government policies or economic shocks, may also affect tax revenues. Adding additional variables may be the solution for a more stable result.

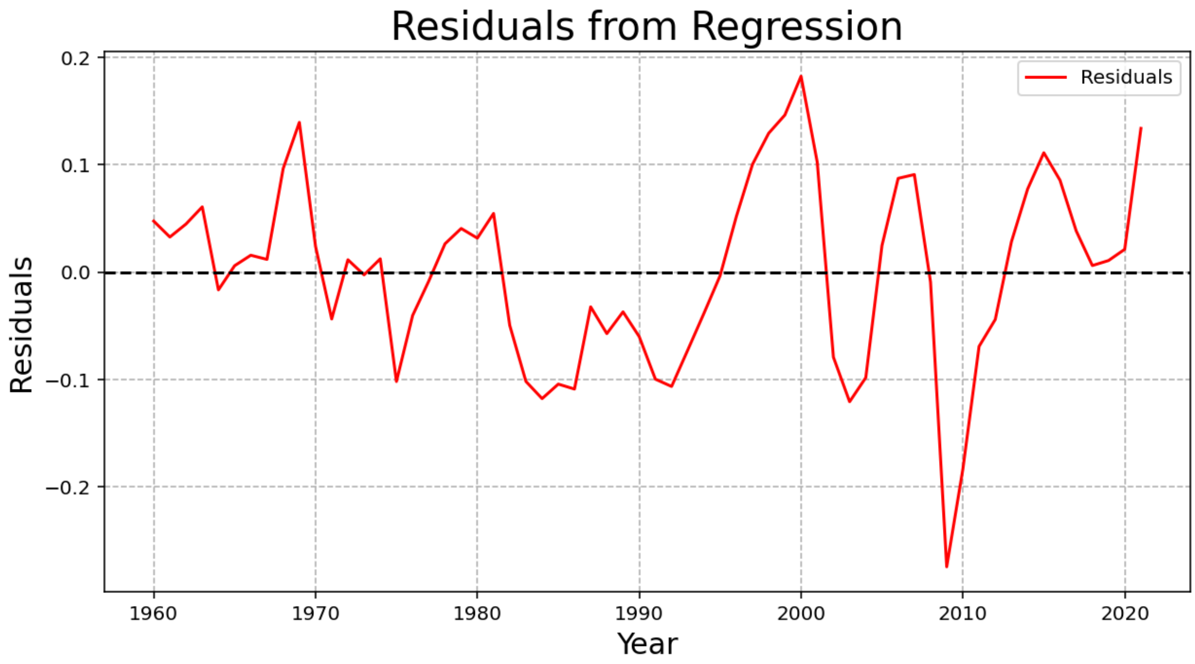

Figure 2.3