Balancing EduMiP

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EduMiP Modeling

Referring to Example 17.10 in Numerical Renaissance, the final equations of motion for the MIP are as follows.

$$(I_w + (m_w + m_b)r^2)\ddot{\phi} + (m_b r l \cos(\theta))\ddot{\theta} - (m_b r l \sin\theta)\dot{\theta}^2 = \tau \tag{1}$$

$$(m_b r l \cos\theta)\ddot{\phi} + (I_b + m_b l^2)\ddot{\theta} - m_b g l \sin\theta = -\tau \tag{2}$$

You will notice these are functions of torque, whereas we wish to design a controller that outputs a PWM duty cycle to our motors. Therefore we must also include the dynamics of the motors themselves. We can reasonably model a geared DC motor's output torque as a function of it's speed. Since there are two motors we include a coefficient of two in front of the equation. The motor specs listed below are for the simple DC motor without a gearbox, so we must include the gearbox ratio G in the motor model too. You can solve for the torque constant k with the below provided stall torque $\bar{s}$ and free run speed ω_f.

$$\tau = 2G(\bar{s} * u - k * \omega_m) \tag{3}$$

$$\omega_w = \dot{\phi} - \dot{\theta} = \omega_m / G \tag{4}$$

Where:
$\bar{s}$ = Motor Stall Torque
k = Motor Constant
ω_w = wheel speed
ω_m = motor armature speed
G = gearbox ratio
u = normalized motor duty cycle (value between -1 and 1)

Your assembled EduMiP has roughly the following physical properties:

1. Encoder disks have 15 slots and therefore provide 60 counts per motor armature revolution
2. Nominal battery voltage $V_b = 7.4V$
3. Motors (without gearbox) have free run speed of $\omega_f = 1760 rad/s$ at V_b
4. Motors (without gearbox) have stall torque $\bar{s} = 0.003Nm$ at V_b
5. Motors are connected to the wheel with a gearbox ratio $G = 35.\overline{5}$
6. The motor armature has inertia $I_m = 3.6 * 10^{-8} Kg * m^2$
7. Wheels have a radius $r = 34mm$
8. Wheels have a mass $m_w = 27g$ each
9. Total assembled MiP body has a mass $m_b = 180g$
10. MiP center of mass to wheel axis $l = 47.7mm$
11. MiP body inertia $I_b = 0.000263 Kg * m^2$

Just as we had to compensate for the torque of both motors accounting for the gearbox, we must also take the gearbox into account when modeling the inertia of the wheels. Attaching a rotating inertia through a gear ratio instead of directly attaching it to a shaft increases the effective inertia by the square of the gearbox. By modeling the wheels as disks and adding their inertia to the effective inertia of the motor armatures, we arrive at the following estimate of the combined wheel inertia.

$$I_w = 2 * ((m_w r^2)/2 + G^2 * I_m) \tag{5}$$